Further Applications of the Atmospheric Fading Model to Sequential Decoding Performance

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The Deep Space Network is interested in predicting whether it will be able to achieve a satisfactory performance level on Pioneer Venus 1978 (PV'78) telemetry over a fading channel. In an earlier article, a theoretical model for the combined effects of log-normal fading and a noisy carrier reference on sequentially decoded phase-shift keyed (PSK) telemetry was used to analyze the PV'78 large probe link at 256 bps. This model is now applied to the 16-bps small probe telemetry mode.

I. Introduction

An analytical model was recently developed for predicting the combined effects of log-normal fading and a noisy carrier reference on convolutionally encoded/sequentially decoded phase-shift-keyed (PSK) telemetry (Ref. 1). In a previous article (Ref. 2), this model was used to analyze the Pioneer Venus 1978 (PV'78) large probe telemetry link at 256 bps. As part of a continuing DSN support study, a similar examination of the 16-bps small probe telemetry mode is reported here.

II. Assumptions

As in the 256-bps case, the desired frame deletion rate was set at 10^{-2}. The received signal-to-noise ratio P_T/N_0 required to achieve this level of performance was computed as a function of the severity of the channel fading, characterized by the variance σ_χ^2 of the log-amplitude fluctuations. Using experimental data from the Russian Venera space probes, Woo (Ref. 3, Eqs. 12 and 13) has shown that

$$\sigma_\chi^2 = 0.0142 \left(\frac{L}{55}\right)^{11/6} \tag{1}$$

where L is the atmospheric path length of the probe-to-Earth telemetry link, in km.

The fading/noisy reference model incorporates processing losses in the data and carrier tracking channels (Ref. 2, Eqs. 2 and 3); at 16 bps, these anticipated losses are

$$L_D = 2.32 \text{ dB} \tag{2}$$

$$L_C = 1.00 \text{ dB} \tag{3}$$

It was again assumed that a standard DSN sequential decoder could perform about 25,000 computations/second in real-time operations: this translates into a computational capacity $N = 1562.5$ computations/bit at 16 bps. A value of N an order of magnitude larger than this was also used to determine whether a significant improvement in performance could be achieved by resorting to non-real-time decoding.

As in the 256-bps analysis, we constrained the tracking loop to have a carrier reference phase jitter σ_ϕ^2 compatible with very long baseline interferometry (VLBI) requirements. This required that the received carrier signal-to-noise ratio in the (two-sided) threshold phase-locked loop bandwidth $2B_{L0}$ satisfy the constraint (Ref. 2, Eq. 4)

$$\eta \equiv \frac{P_T \cos^2 \theta}{N_0 2B_{L0} L_C} \gtrsim (10 + 14.1\,\sigma_\chi^2)\ \mathrm{dB} \tag{4}$$

where θ is the modulation angle, and $2B_{L0} = 12$ Hz.

Finally, despite previously expressed reservations (Ref. 1, "Commentary," p. 60; Ref. 2, "Results"), the analysis of the 16-bps link was performed using Layland's formula for the characteristic decoder memory time T_m (Ref. 4, Eq. 6) in the fading/noisy reference model. There are plans to determine this parameter empirically so as to maximize the agreement between our analytical model and experimental evidence.

III. Results

As a reference point for design control table purposes, in order to measure losses due to fading, noisy carrier reference, and receiver processing, an idealized 16-bps link with $\sigma_\chi^2 = 0$, $L_D = L_C = 0$, and a perfect carrier reference was analyzed. The results are contrasted in Table 1 with the 256-bps case (Ref. 2, Fig. 1) as a function of the decoding rate C. We find that the slower telemetry link requires a 0.50-dB smaller received bit energy-to-noise ratio ρ for real-time decoding, with a lesser savings in the non-real-time case.

Figure 1 still neglects the fading, but incorporates the noisy reference and processing losses into the analysis of the 16-bps link with real-time decoding. Strictly from a telemetry viewpoint, we see that our desired frame deletion rate of 10^{-2} could be achieved in this case with P_T/N_0 as low as 22.03 dB at a corresponding modulation angle $\theta = 46.8$ deg; however, the tracking loop parameter η would then be 6.96 dB, which would not satisfy the VLBI requirement, $\eta = 10$ dB in the absence of fading. The minimum received signal-to-noise ratio which simultaneously satisfies the data and carrier channel requirements is $P_T/N_0 = 23.19$ dB at $\theta = 31.7$ deg, an increase of 1.16 dB over the data requirement alone.

Using Fig. 1 and a similar set of curves for the non-real-time decoding case, the variation of P_T/N_0 with θ is shown in Fig. 2 for a deletion rate of 10^{-2} in the absence of fading. The solid curves indicate the regions where $\eta \geq 10$ dB.

The remaining graphs deal with the effects of atmospheric fading, which, we have already noted, is characterized by σ_χ^2. During its approximately 2-hour descent to the surface of Venus, σ_χ^2 will vary from an initial value of zero to a maximum value on the surface, when the atmospheric path length L in Eq. (1) is greatest. Referring to Fig. 3, suppose the probe-to-Earth transmission angle is ϕ relative to the radial direction; assuming Venus has a radius of 6050 km, and the atmosphere is homogeneous and ends abruptly 55 km above the surface of Venus (Ref. 5; Ref. 3, p. 8), a transmission from the surface has an atmospheric path length

$$L = \sqrt{(6105)^2 - (6050 \sin \phi)^2} - 6050 \cos \phi,\ \mathrm{km} \tag{5}$$

Equations (1) and (5) have been combined in Graph 3 to provide a curve of σ_χ^2 at the surface of Venus vs the transmission angle ϕ. Two values of ϕ have commonly been used as benchmarks for the PV'78 mission; the corresponding values of σ_χ^2 from Fig. 4 are

$$\sigma_\chi^2 = \begin{cases} 0.014; & \phi = 0^\circ \\ 0.049; & \phi = 60^\circ \end{cases} \tag{6}$$

Figure 5 represents the degradation of the link in Fig. 1 due to log-normal fading, when $\sigma_\chi^2 = 0.049$. As before, the optimum operating point with regard to our telemetry performance requirement only ($P_T/N_0 = 24.48$ dB; $\theta = 50.9$ deg) does not satisfy the VLBI requirement ($\eta \geq 10.69$ dB). To simultaneously meet both needs, the minimum received $P_T/N_0 = 24.94$ dB (an increase of 0.46 dB, which is less than in the nonfading case of Fig. 1) at $\theta = 41.1°$. In analyzing the 16-bps link over the probe lifetime, this represents the worst case requirement for $\phi = 60°$.

Figure 6 is an extension of Fig. 5 and demonstrates how our link requirements vary with σ_χ^2 for real-time decoding; Fig. 7 does the same for the non-real-time case. For the $\sigma_\chi^2 = 0.049$ case, Fig. 7 shows that increasing the computational capacity an order of magnitude lowers the required P_T/N_0 to 24.79 dB, which is a trivial savings of 0.15 dB over the real-time decoding requirement.

As an example, Table 1 and Figs. 2 and 6 were used to generate the design control table shown in Table 2 for the 16-bps link with $\sigma_\chi^2 = 0.049$ and real-time decoding. The entries for this table are derived in the manner described for the 256-bps link (Ref. 2). Unlike the 256-bps case, the carrier and data channel margins are equal at 16 bps. In the example of Table 2, the overall link margin (minimum of the carrier and data channel margins) is 5.78 dB; this can be compared with an overall margin of 2.45 dB for the equivalent 256-bps link. Note that the noisy reference and fading losses for these two links are virtually identical:

$$\text{noisy reference loss} = \begin{cases} 2.12\text{ dB; 16 bps} \\ 2.10\text{ dB; 256 bps} \end{cases} \tag{7}$$

$$\text{fading loss} = \begin{cases} 2.74\text{ dB; 16 bps} \\ 2.75\text{ dB; 256 bps} \end{cases} \tag{8}$$

References

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2. Levitt, B. K., "Pioneer Venus 1978: Telemetry Performance Predicts," in *The Deep Space Network Progress Report 42-24*, Jet Propulsion Laboratory, Pasadena, Calif., Dec. 15, 1974.
3. Woo, R., et al., *Effects of Turbulence in the Atmosphere of Venus on Pioneer Venus Radio – Phase 1*, Technical Memorandum 33-644, Jet Propulsion Laboratory, Pasadena, Calif., June 30, 1973.
4. Layland, J. W., "A Model for Sequential Decoding Overflow Due to a Noisy Carrier Reference," in the Proceedings of the International Telemetering Conference (ITC'74), held Oct. 15–17, 1974, Los Angeles, Calif.
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Table 1. Required signal-to-noise ratios for ideal lossless telemetry links at 16 and 256 bps (reference case)

No fading ($\sigma_x^2 = 0$); perfect carrier reference; no system losses ($L_D = L_C = 0$); frame deletion rate = 10^{-2}

Parameter	Real-time processing		Non-real-time processing	
C, comp/s	2.5×10^4		2.5×10^5	
R_B, bps	16	256	16	256
$N = C/R$, comp/bit	1562.5	97.7	15,625	977
$\rho = E_B/N_0$, dB	2.08	2.58	1.77	2.14

Table 2. Telecommunications design control table

Telemetry mode: PV'78 small probe-to-earth telemetry link; Convolutional coding ($K = 32$, $\nu = 1/2$) with sequential decoding; Real-time processing, $N = 1562.5$ comp/bit

Performance	No.	Parameter	Nominal value, dB	Comments
	1	Available P_T/N_0	30.72	Transmitter: 10 W Receiver: 26.1 °K, 30° elev
Carrier	2	Threshold snr, η	10.00	VLBI requirement
	3	Fading loss	0.69	$\sigma_x^2 = 0.049$
	4	System loss, L_C	1.00	
	5	$2B_{L0}$	10.79	12 Hz
	6	$\mathrm{Cos}^2\theta$	−2.46	$\theta = 41.1°$
	7	Required P_T/N_0	24.94	Nos. (2+3+4+5−6)
	8	Margin	5.78	Nos. (1−7)
Telemetry	9	Ideal ρ (lossless)	2.08	Deletion rate = 10^{-2}
	10	Noisy reference loss	2.12	
	11	Fading loss	2.74	$\sigma_x^2 = 0.049$
	12	System loss, L_D	2.32	
	13	Rate, R_B	12.04	16 bps
	14	$\mathrm{Sin}^2\theta$	−3.64	$\theta = 41.1°$
	15	Required P_T/N_0	24.94	Nos. (9+10+11+12+13−14)
	16	Margin	5.78	Nos. (1−15)

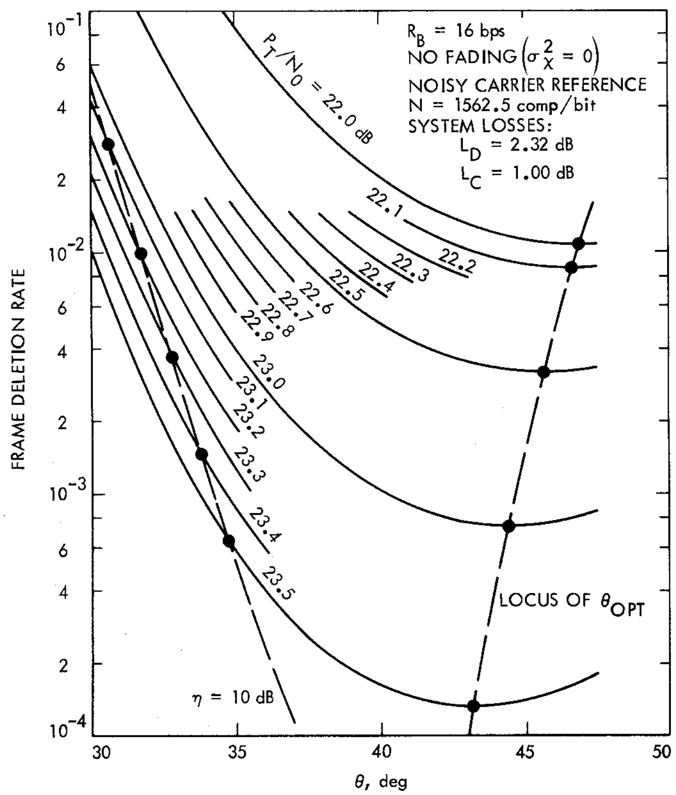

Fig. 1. Signal-to-noise requirements for 16-bps telemetry link in the absence of fading, with real-time decoding

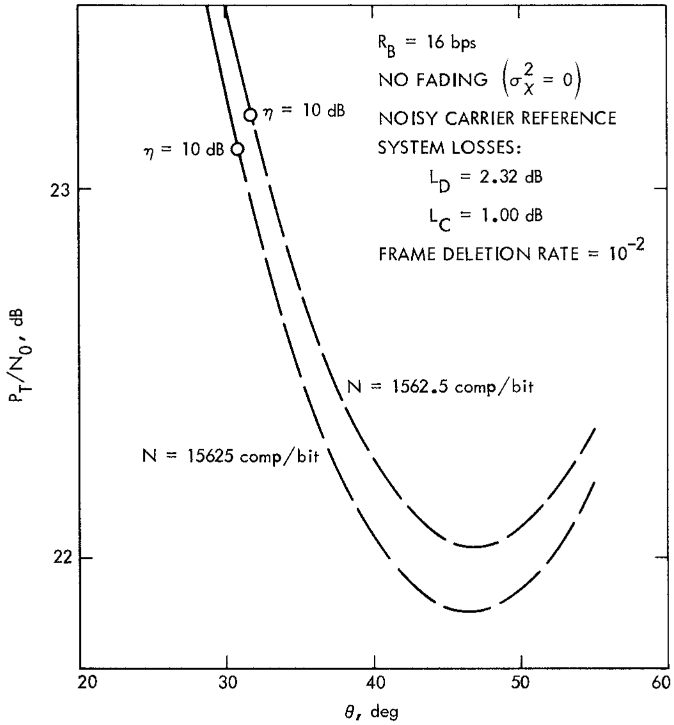

Fig. 2. Signal-to-noise ratios required to achieve a frame deletion rate of 10^{-2} in the absence of fading

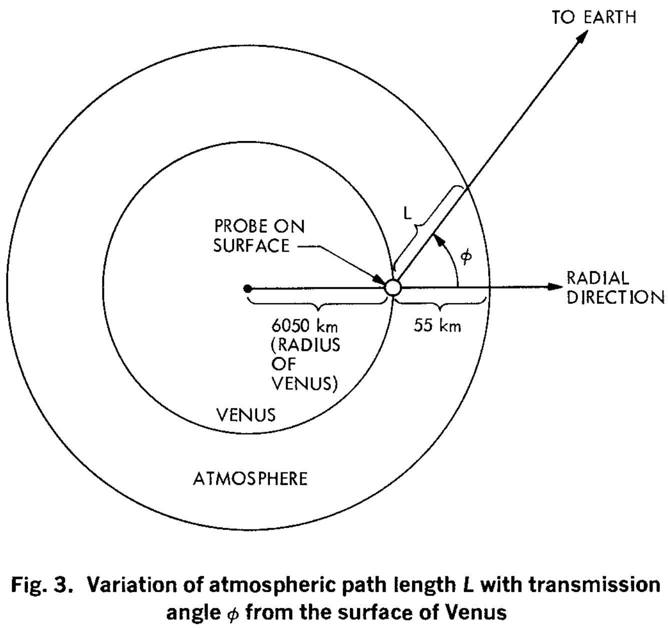

Fig. 3. Variation of atmospheric path length *L* with transmission angle ϕ from the surface of Venus

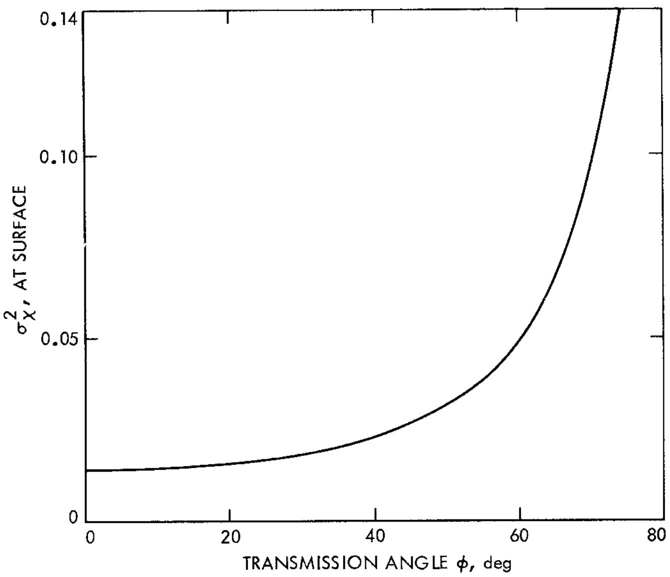

Fig. 4. Variation of log-normal fading parameter σ_χ^2 at the surface of Venus with probe transmission angle

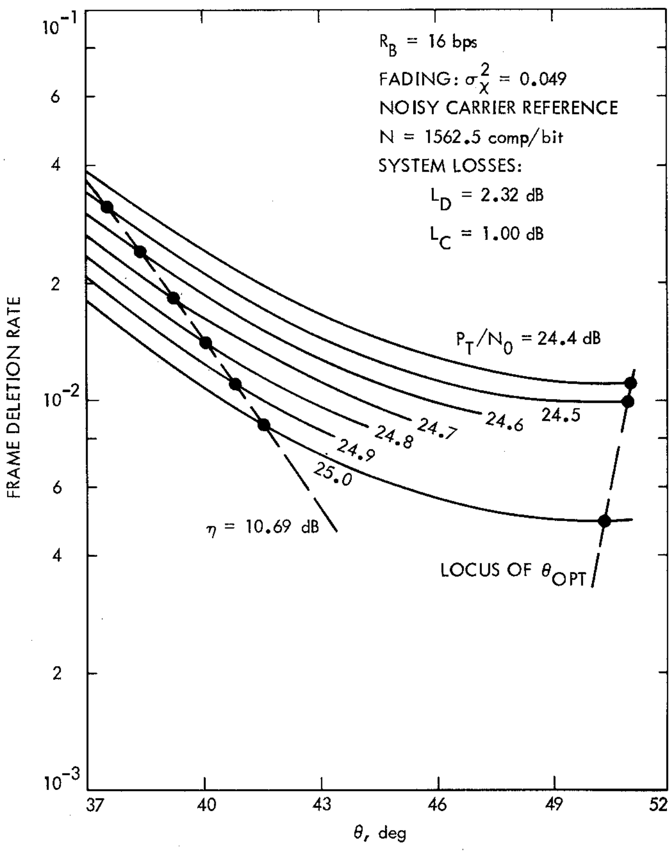

Fig. 5. Signal-to-noise requirements in the presence of fading (σ_χ^2 = 0.049) with real-time decoding

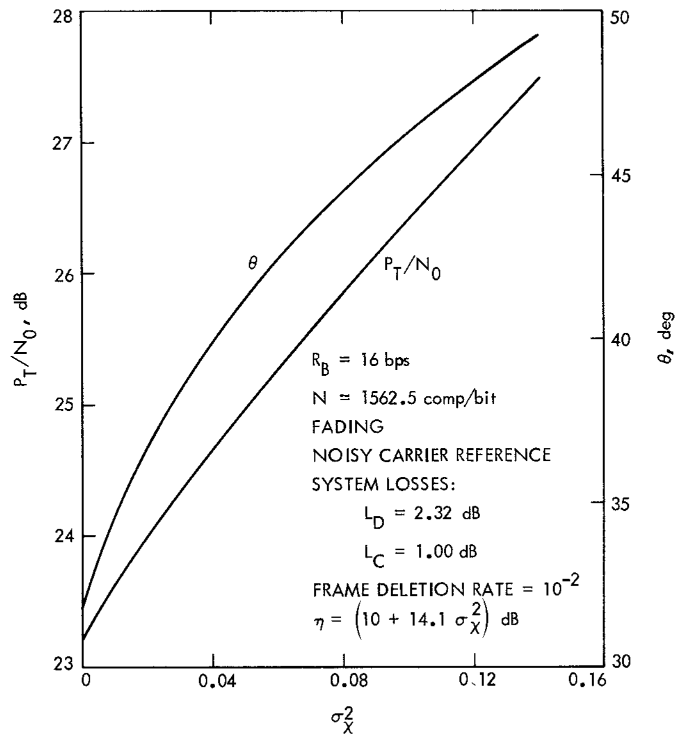

Fig. 6. Degradation of 16-bps telemetry link with real-time decoding as a function of atmospheric fading parameter σ_χ^2

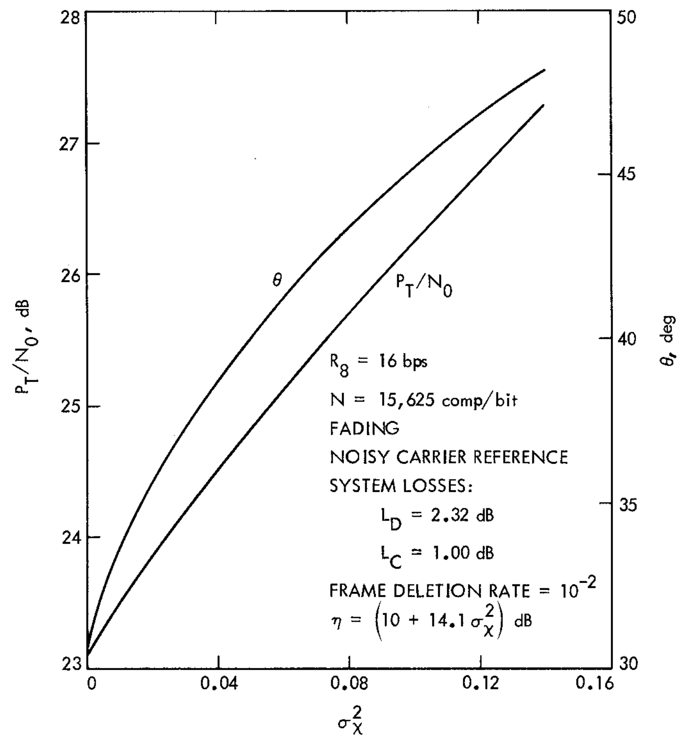

Fig. 7. Non-real-time decoding version of Fig. 5