A New Approach to the Evaluation and Prediction of Wet Tropospheric Zenith Range Refraction

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The best current model for predicting wet tropospheric zenith range refraction exhibits an uncertainty of approximately 3.0 cm (1σ). This report presents an approach that has the potentiality of reducing the uncertainty to the range of approximately 1.5 to 2.0 cm (1σ). Furthermore, the current model requires both surface parameters as well as tropospheric parameters, while the approach dealt with in this report requires only surface parameters.

I. Introduction

The last five years have seen a substantial effort at the Jet Propulsion Laboratory to evaluate and, hence, be able to predict zenith range refraction. Determination of the dry component of range refraction is quite straightforward; therefore, most activity has been expended in attempting to develop accurate methods to evaluate the wet component of range refraction. In 1970 this author proposed a model based on the assumption of constant relative humidity as follows (Ref. 3):

$$\Delta R_W = 10^{-6} \int_0^\infty NW(h)\, dh$$

$$= \left[\frac{C_1 C_2 (RH)_s}{\gamma}\right] \left\{\frac{(1 - \frac{C}{T_0})^2}{B - AC}\right\} \exp\left(\frac{AT_0 - B}{T_0 - C}\right)$$

where

ΔR_W	=	wet zenith range refraction (cm)
$NW(h)$	=	wet component of refractivity
h	=	height above observer (km)
C_1	=	77.6
C_2	=	29341.0
RH	=	relative humidity (100% = 1.0)
γ	=	temperature lapse rate (K/km)
C	=	38.45
T_0	=	extrapolated surface temperature, K
A	=	7.4475 (ln 10)
B	=	2034.28 (ln 10)
s	=	surface parameter value

This model exhibited a standard deviation of approximately:

$$1\sigma \approx 4.0 \text{ cm}$$

In 1973, Chao (Ref. 6), assuming an adiabatic atmosphere model, improved upon the evaluation of wet zenith range refraction with the following expression:

$$10^{-6} \int_0^\infty NW(h)\, dh = 1.63 \times 10^2 \left\{ \frac{e_0^{1.23}}{T_0^2} \right\} + 2.05 \times 10^2 \alpha \left\{ \frac{e_0^{1.46}}{T_0^3} \right\}$$

where

e_0 = surface vapor pressure, N/m²

T_0 = surface temperature, K

α = temperature lapse rate, K/km

This model produced a standard deviation for combined day and night cases of

$$1\sigma \approx 3.0 \text{ cm}$$

The uncertainty in even the Chao model is large, however, when compared to an average wet zenith range refraction of approximately 10 cm; thus efforts to improve upon this model would not be entirely academic.

II. New Approach to Evaluation of Wet Zenith Range Refraction

In the course of developing a new radio frequency angular tropospheric refraction model (Ref. 1), it was found necessary to evaluate the ratio of wet zenith range refraction to dry zenith range refraction. Letting

$N(h)$ = $ND(h) + NW(h)$, the total refractivity at radio frequencies

$ND(h)$ = dry or optical component of refractivity

$NW(h)$ = wet component of refractivity

h = height

h_0 = observer height

s = parameter surface value

$ND(h_0)$ = ND_s

$NW(h_0)$ = NW_s

$N(h_0)$ = N_s

and defining

$$\Delta R_W = 10^{-6} \int_0^\infty NW(h)\, dh$$

$$\Delta R_D = 10^{-6} \int_0^\infty ND(h)\, dh$$

It was necessary to determine some f such that:

$$f = \frac{\Delta R_W}{\Delta R_D} = \frac{\int_0^\infty NW(h)\, dh}{\int_0^\infty ND(h)\, dh}$$

It was empirically found that there existed strong correlation between the ratios of wet and dry zenith range refraction and of wet and dry surface refractivity, i.e. that

$$f = \frac{\Delta R_W}{\Delta R_D} \approx K \left[\frac{NW_s}{ND_s} \right]$$

with $K \approx 0.32$.

There is an immediate implication that, if indeed there existed strong correlation, then this (assumed) relationship might be useful in predicting wet zenith range refraction.

If

$$\frac{\Delta R_W}{\Delta R_D} \approx K \left[\frac{NW_s}{ND_s} \right]$$

then

$$\Delta R_W \approx K \left[\frac{\Delta R_D}{ND_s} \right] (NW_s)$$

Defining

C_1 = 77.6

C_2 = 29341.0

RH = relative humidity

R = perfect gas constant

g = gravitational acceleration

$g/R = 34.1\ ^\circ\mathrm{C/km}$

P = pressure

T = temperature

s = surface parameter value

$A = 7.4475 \ln(10)$

$B = 2034.28 \ln(10)$

$C = 38.45$

one would have from Ref. 3:

$$\Delta R_D(\mathrm{cm}) = (10^{-1})\, C_1 P_s \left[\frac{R}{g}\right]$$

$$ND_s = C_1 \left[\frac{P_s}{T_s}\right]$$

$$NW_s = \frac{C_1 C_2 (RH)_s}{T_s^2} \exp\left(\frac{AT_s - B}{T_s - C}\right)$$

so that

$$\Delta R_W \approx K \left\{\frac{(10^{-1})\, C_1 P_s \left[\frac{R}{g}\right]}{C_1 \frac{P_s}{T_s}}\right\} \times \frac{C_1 C_2 (RH)_s}{T_s^2} \exp\left(\frac{AT_s - B}{T_s - C}\right)$$

$$\approx K \left\{\frac{R\, T_s}{g(10)}\right\} \frac{C_1 C_2 (RH)_s}{T_s^2} \exp\left(\frac{AT_s - B}{T_s - C}\right)$$

$$\approx K \left\{\frac{R}{g}\right\} \frac{C_1 C_2 (RH)_s}{(10)\, T_s} \exp\left(\frac{AT_s - B}{T_s - C}\right)$$

now

$$\left\{\frac{R}{g}\right\} \frac{C_1 C_2}{(10)} = \frac{(77.6)(29341.)}{34.1} = 6677.0$$

so that finally

$$\Delta R_W \approx K (RH)_s \left[\frac{6677.0}{T_s}\right] \exp\left(\frac{AT_s - B}{T_s - C}\right)$$

The above expression is simpler than those referred to previously, and more significantly, it is only dependent upon surface parameters, unlike previous expressions that depended upon one or more tropospheric parameters not directly measurable from the ground. In 1970, J. V. Ondrasik considered a very similar approach, that is, correlating zenith wet range refraction with surface wet refractivity (Ref. 4, P. 34), i.e.,

$$\Delta R_W \approx K_0 \{NW_s\} + K_1$$

but (apparently) dropped the idea as unpromising.

Table 1 presents a detailed listing of 10 selected atmospheric cases previously described in Refs. 1 and 3. The standard deviations described in the table are as follows:

$$\sigma(\mathrm{cm}) = \sigma\left(\Delta R_W - K\left[\frac{\Delta R_D}{ND_s}\right] NW_s\right), \mathrm{cm}$$

$$\sigma(\%) = 100 \times \sigma\left(\frac{\Delta R_W}{\Delta R_D} - K\left[\frac{NW_s}{ND_s}\right]\right), \%$$

The results of a least-squares curve fit of these 10 cases to:

$$\Delta R_W;\ K\left[\frac{\Delta R_D}{ND_s}\right] NW_s$$

yielded

$$K = 0.3224$$

$$\sigma(\mathrm{cm}) \sim 2.0\ \mathrm{cm}$$

This data can also be seen in Fig. 1.

As a check to the basic procedure, use was made of data presented by Ondrasik in 1970 (Ref. 4, p. 34, Fig. 11). Although this data consists of

$$\Delta R_W \text{ vs } NW_s$$

the process of predicting

$$\Delta R_W \approx K\, NW_s$$

should yield a substantially similar uncertainty. Using the average integrated wet refractivity relationship derived in Ref. 1 (and based on work done by Chao, Ref. 5), i.e.,

$$10^{-6} \int_0^\infty NW(h)\, dh \approx 0.26\, NW_s, \mathrm{cm}$$

yielded the following standard deviation:

$$1\sigma = 1\sigma(\Delta R_W - 0.26\, NW_s)$$
$$= 2.3 \text{ cm}$$

which is at least a similar number to that uncertainty obtained from the proposed procedure.

III. Consideration of the Differences in Day and Night Wet Refractivity Profiles

It has been suggested here that there exists a strong (empirical) relationship between integrated wet refractivity and surface wet refractivity:

$$\Delta R_W \approx K \left[\frac{\Delta R_D}{ND_s} \right] NW_s$$

Since there is little dynamic variation in the term $(\Delta R_D / ND_s)$, the above relationship implies an "average" wet profile, say $f_{\text{AVE}}(h)$, such that

$$\Delta R_W = 10^{-6} \int_0^{\infty} NW(h)\, dh$$

$$\approx 10^{-6} \int_0^{\infty} NW_s\, f_{\text{AVE}}(h)\, dh$$

$$\approx \left[10^{-6} \int_0^{\infty} f_{\text{AVE}}(h)\, dh \right] NW_s$$

$$\approx \left[K \left\{ \frac{\Delta R_D}{ND_s} \right\} \right] NW_s$$

The main considerations which detract from an "average" wet refractivity profile are fluctuations in relative humidity:

$$RH(h) \neq (RH)_s$$

and near surface variations in temperature. Whereas the fluctuations in relative humidity appear very random, the variations in surface temperature are much more systematic and can be characterized as either day or night type profiles. A typical appearance of night and day temperature profiles is shown in Fig. 2.

If one assumes an "average" wet refractivity profile, then obviously using a "night" NW_s will give too small a total wet zenith range refraction; conversely, using a "day" NW_s will lead to too large a total wet zenith range refraction. This can be seen in Fig. 3.

The conclusion one expects is that the K (= 0.3224) previously determined would be larger for night only profiles and smaller for day only profiles. This in fact turns out to be the case. The data previously analyzed was separated into day only and night only profiles, and yielded the following results:

Day profiles

$$K = 0.2896$$
$$\sigma(\text{cm}) = 1.1 \text{ cm}$$

Night profiles

$$K = 0.3773$$
$$\sigma(\text{cm}) = 1.9 \text{ cm}$$

This data appears in Table 2 and Fig. 4.

IV. Adjustment of Surface Temperature to Reflect Differences in Day and Night Profiles

In the previous section it was pointed out that systematic diurnal surface temperature variations cause systematic distortions in attempting to fit total wet zenith range refraction to surface wet refractivity. As a final attempt to explore ways to account for this effect the following was tried:

Let

T_{MIN} = lowest previous 24-h temperature

T_{MAX} = highest previous 24-h temperature

Then, to moderate the night and day profile distortions define:

$$T(\text{night profiles}) = \frac{3T_{\text{MIN}} + T_{\text{MAX}}}{4}$$

$$T(\text{day profiles}) = \frac{3T_{\text{MAX}} + T_{\text{MIN}}}{4}$$

Entering these temperatures into the previously described cases yielded

$$K = 0.3281$$
$$\sigma(\text{cm}) = 1.3 \text{ cm}$$

The results can be seen in Table 3 and Fig. 5.

V. Conclusions

From the (admittedly) small data set at hand, there appears to be a strong correlation between total wet zenith range refraction and wet surface refractivity, in the form of

$$R_W = K\left[\frac{\Delta R_D}{ND_s}\right] NW_s$$

$$= K(RH)_s \left[\frac{6677.0}{T_s}\right] \exp\left(\frac{AT_s - B}{T_s - C}\right)$$

Depending on how the equation is used, it appears to potentially yield an uncertainty in total wet zenith range refraction of

$$\sigma(\text{cm}) \approx 1.5 \text{ to } 2.0 \text{ cm}$$

Also, it possesses the advantage of being dependent on surface parameters only, in contrast to previous methods. Table 4 presents a comparison of previous results contrasted to results from the proposed procedure. Obviously, however, a much larger data set would have to be analyzed before the results presented in this report could be verified and accepted.

References

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6. Chao, C. C., "A New Method to Predict Wet Zenith Range Correction From Surface Measurements," in *The Deep Space Network Progress Report*, No. 32-1526, Vol. XIV, pp. 33–41, Jet Propulsion Laboratory, Pasadena, Calif., April 15, 1973.

Table 1. Surface refractivity vs integrated refractivity ($A = 0.3224$, $\sigma = 0.93\%$)

Case	$100 \times \frac{NW_s}{ND_s}$, %	$A \times \left\{100 \times \frac{NW_s}{ND_s}\right\}$, (%)	$100 \times \frac{\int NW(h)\,dh}{\int ND(h)\,dh}$, (%)	Δ (%)	Δ (cm)
1	4.86	1.57	2.27	+0.70	+1.48
2	3.76	1.21	2.17	+0.96	+2.03
3	5.74	1.85	1.80	−0.05	−0.11
4	5.34	1.72	1.37	−0.35	−0.74
5	4.74	1.53	1.75	+0.22	+0.47
6	7.14	2.30	2.17	−0.13	−0.28
7	24.11	7.77	8.55	+0.78	+1.65
8	31.72	10.23	9.12	−1.11	−2.35
9	7.29	2.35	4.58	+2.23	+4.72
10	9.89	3.19	2.69	−0.50	−1.06

Table 2. Surface refractivity vs integrated refractivity

Case	$100 \times \frac{NW_s}{ND_s}$	$A \times \left\{100 \times \frac{NW_s}{ND_s}\right\}$	$100 \times \frac{\int NW(h)\,dh}{\int ND(h)\,dh}$	Δ (%)	Δ (cm)
		Day profiles (even) $A = 0.2896$ $\sigma = 0.50\%$			
2	3.76	1.09	2.17	+1.08	+2.29
4	5.34	1.55	1.37	−0.18	−0.38
6	7.14	2.07	2.17	+0.10	+0.21
8	31.72	9.19	9.12	−0.07	−0.15
10	9.89	2.86	2.69	−0.17	−0.36
		Night profiles (odd) $A = 0.3773$ $\sigma = 0.89\%$			
1	4.86	1.83	2.27	+0.44	+0.93
3	5.74	2.17	1.80	−0.37	−0.78
5	4.74	1.79	1.75	−0.04	−0.09
7	24.11	9.10	8.55	−0.55	−1.16
9	7.29	2.75	4.58	+1.83	+3.87

Table 3. Surface refractivity vs integrated refractivity
$A = 0.3281 \quad \sigma = 0.63\%$

Case	$100 \times \frac{NW_s{}^a}{ND_s}$	$A \times 100 \times \frac{NW_s{}^a}{ND_s}$	$100 \times \frac{\int NW(h)\,dh}{\int ND(h)\,dh}$	Δ (%)	Δ (cm)
1	6.43	2.11	2.27	+0.16	+0.34
2	2.92	0.96	2.17	+1.21	+2.55
3	7.18	2.36	1.80	−0.56	−1.18
4	4.45	1.46	1.37	−0.09	−0.19
5	6.19	2.03	1.75	−0.28	−0.59
6	5.61	1.84	2.17	−0.33	+0.70
7	26.43	8.67	8.55	−0.12	−0.25
8	29.02	9.52	9.12	−0.40	−0.84
9	9.83	3.23	4.58	+1.35	+2.85
10	7.70	2.52	2.69	+0.17	+0.36

[a]"Modified" surface temperature in calculation of NW_s

Table 4. 1σ Uncertainties in wet zenith range refraction

Data source	Ref. 3	Ref. 6		Proposed Approach		
Model	Berman 1970	Berman 1970	Chao 1973	$K = 0.3224$	$K_D = 0.2896$ $K_N = 0.3773$	Modified surface temperature
Day profiles	1.5 cm	3.5 cm	2.0 cm	–	1.1 cm	–
Night profiles	5.5 cm	4.9 cm	4.1 cm	–	1.9 cm	–
Composite	3.5 cm	4.2 cm	3.0 cm	2.0 cm	1.5 cm	1.3 cm

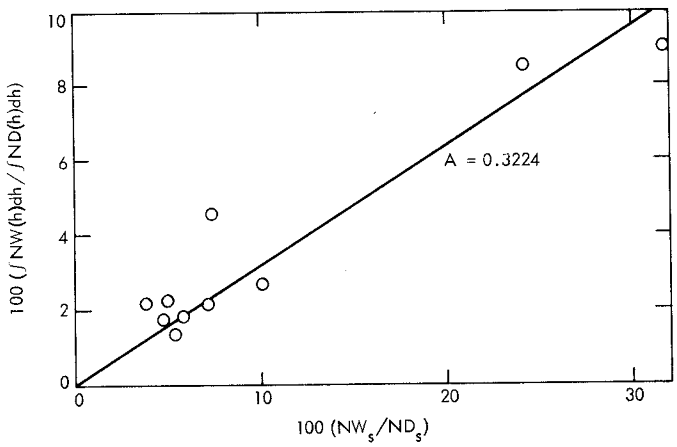

Fig. 1. Integrated refractivity vs surface refractivity, $A = 0.3224$

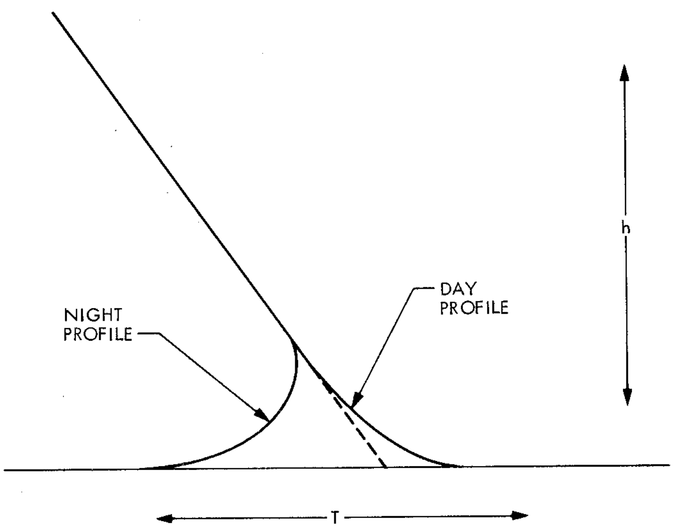

Fig. 2. Typical night and day temperature profiles

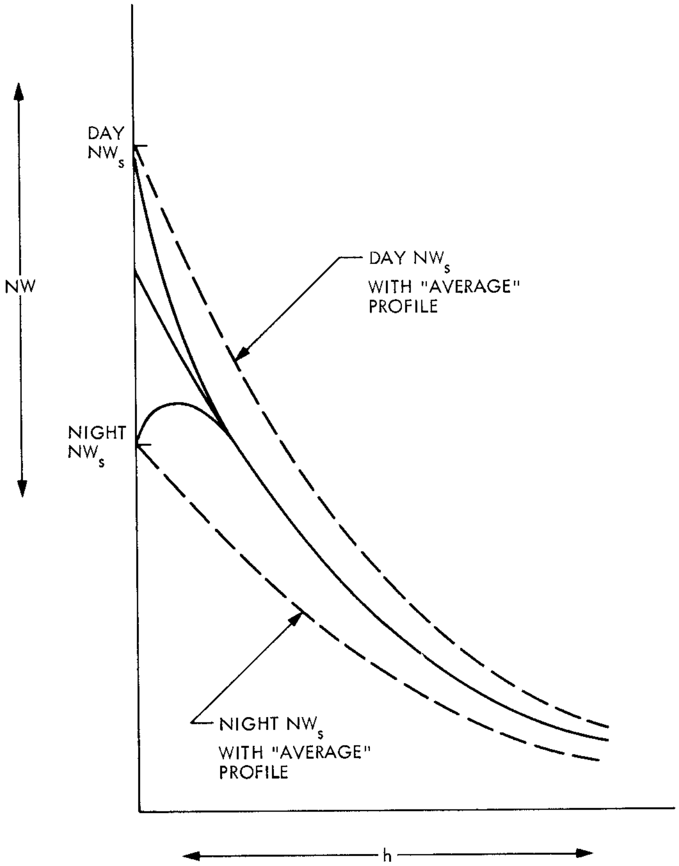

Fig. 3. Typical night and day refractivity profiles

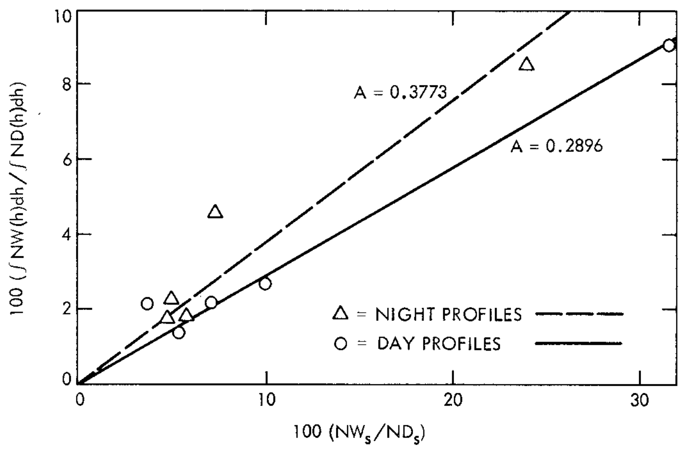

Fig. 4. Integrated refractivity vs surface refractivity, *A* = 0.3773 and *A* = 0.2896

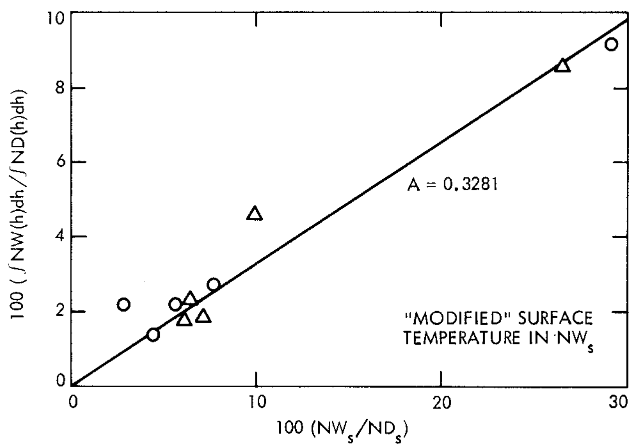

Fig. 5. Integrated refractivity vs surface refractivity, *A* = 0.3281