Long Frame Sync Words for Binary PSK Telemetry

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Sequences of length 16 to 63, which possess correlation properties suited to frame sync applications for binary phase-shift-keyed (PSK) telemetry, have been found. From a storage viewpoint, these sequences all have the advantage of being generated by a 5- or 6-bit linear feedback shift register.

Consider the problem of determining a suitable frame sync word for a digital communication link employing binary phase-shift-keyed (PSK) modulation. For uncoded data received over the additive white Gaussian noise channel, the optimum procedure for locating the received sync words is based on a bit correlation rule (Ref. 1). In the block-coded case, the optimum sync word search requires a word correlation rule (Ref. 2); however, practical considerations often dictate the use of the suboptimal bit correlation approach.

For example, if format constraints require that the sync word not contain an integral number of block code words, the bit correlation scheme is much less complex than the optimum frame sync rule (Ref. 2, p. 1130). In the case of convolutionally coded telemetry as well, the suboptimal bit correlation sync word search is desirable from a complexity viewpoint.

Suppose the sync word is a K-bit binary sequence of ± 1's, $\mathbf{s} = (s_0, s_1, \cdots, s_{K-1})$, with the nonperiodic (Ref. 3, p. 195) autocorrelation function

$$C_\ell = \sum_{i=0}^{K-1-\ell} s_i s_{i+\ell}\,; \qquad 0 \le \ell \le K \tag{1}$$

In a binary PSK modulation system, because the derived carrier reference in the receiver has an inherent 180 deg phase ambiguity, the detected data stream will be inverted with probability ½ (e.g., Ref. 4). Consequently, when a bit correlation sync rule is used, the probability of false synchronization is minimized by selecting a sync word $\mathbf{s}$ for which $|C_\ell|$ is minimized over the range $1 \le \ell \le K-1$. That is, ideally the autocorrelation function of $\mathbf{s}$ should have the property

$$|C_\ell| \le 1; \qquad 1 \le \ell \le K-1\,. \tag{2}$$

Sequences satisfying this constraint are called Barker codes (Ref. 5): they are optimum sync words for the channels of interest. Unfortunately, no binary sequences of length greater than 13 satisfy the Barker restriction of Eq. (2) (e.g., Ref. 6).

Generally, if sync words s with suitable autocorrelation functions are considered, frame sync performance improves exponentially with increasing length K. For $K > 13$, the Barker constraint must be relaxed somewhat, although it is still desirable that $|C_\ell|$ be near zero for $\ell \neq 0$. To this end, Neuman and Hofman (Ref. 7) have argued that the following criterion can be used to select good frame sync words for arbitrary lengths K. They define a "minimum distance"

$$D_{\min} \equiv \min\{D_\ell : 1 \leq \ell \leq K-1\}, \qquad (3)$$

where the parameter

$$D_\ell \equiv K - \sqrt{\ell} - |C_\ell|. \qquad (4)$$

Note that

$$D_{\min} \leq D_{K-1} = K - 1 - \sqrt{K-1}. \qquad (5)$$

A good K-bit sync word s for the channels of interest is one for which $D_{\min}$ is near the upper bound of Eq. (5). Using computer search techniques, Neuman and Hofman found sequences which maximized $D_{\min}$ over *all* binary K-tuples, for the range $7 \leq K \leq 24$. Massey has demonstrated by simulation that these Neuman-Hofman sequences compare favorably with Barker codes for frame sync applications when $K \leq 13$ (Ref. 1, Tables 1 and 2).

For longer lengths K, the Neuman-Hofman search procedure for a good K-bit sync word involves a prohibitively large number of candidates: even though complimentary and reversed sequences yield identical $D_{\min}$'s, 2^{K-2} binary K-tuples must still be compared. In order to keep the computer search time for longer sync words within practical limits, only a small subset of all possible sequences can be examined; hopefully, the size of this subset should grow linearly, rather than exponentially, with K.

For a given large value of K, it is proposed that the sync word search be confined to the subset of binary K-tuples which are prefixes of pseudo-noise sequences (maximum-length linear recurring sequences; e.g., Ref. 8, p. 75) of length $2^L - 1$, where L satisfies the inequality

$$2^{L-1} < K + 1 \leq 2^L. \qquad (6)$$

Pseudo-noise (PN) sequences have periodic (Ref. 3, p. 195) autocorrelation functions of the form

$$C_\ell = \sum_{i=0}^{K-1} s_i s_{i+\ell} = \begin{cases} K & ; \quad \ell = 0 \\ -1 & ; \quad 1 \leq \ell \leq K-1. \end{cases} \qquad (7)$$

Where the subscript $i + \ell$ is modulo 2. Of course, for frame sync applications, it is the *non-periodic* autocorrelation C_ℓ that is of interest. Nonetheless, PN sequences are sufficiently pseudo-random that the subset defined above contains some excellent sync words, as shown below. Furthermore, a PN sequence of length $2^L - 1$ can be generated by an L-stage linear feedback shift register, which is advantageous from a storage viewpoint.

To demonstrate the merit of the shortened sync word search procedure described above, it was used to find good sync words of lengths 16 to 63. For a given K in the range $16 \leq K \leq 31$, $D_{\min}$ was maximized over all possible K-bit prefixes of 31-bit PN sequences ($L = 5$ according to Eq. (6)). Neglecting reversed PN sequences, there were only 3 distinct 5-stage linear feedback shift register configurations to consider (Ref. 9, Appendix C): these are completely described by the linear recursion formulas in Table 1A. Each configuration generates any of 31 different PN sequences (which are cyclic permutations of one another), depending upon which 5-bit initializing sequence $s' = (s_0, s_1, s_2, s_3, s_4)$ is used (excluding the all-1 sequence). Thus, for each K in the stated range, 93 candidate sync words were compared. The best sync words found are listed in Table 2: for a given length K, the sync word s consists of the *first* K bits generated by the indicated shift register configuration, initialized by the designated sequence s′.

Similarly, for $32 \leq K \leq 63$, $D_{\min}$ was maximized over K-bit prefixes of 63-bit PN sequences. Here, there were 3 distinct 6-stage linear feedback shift register arrangements of interest (Ref. 9, Appendix C), described by the recursion formulas of Table 1B. So 189 possible sync words were compared for each K in this range, and the best are listed in Table 2. Also, out of curiosity, the K-bit prefixes of these 63-bit PN sequences were tested with respect to $D_{\min}$ as possible sync words for $16 \leq K \leq 31$. As indicated in Table 2, for 4 values of K in this range, a sync word derived from a 63-bit PN sequence has a larger $D_{\min}$ than the best 31-bit derivative. In fact, for $K = 17$ the sync word derived from a 63-bit PN sequence has a $D_{\min}$ which equals the upper bound of Eq. (5).

To demonstrate how close the $D_{\min}$'s of the sync words in Table 2 approach the upper bound of Eq. (5), they are plotted in Fig. 1.

References

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Table 1. Linear recursion formulas for generating 31-bit PN sequences from 5-stage linear feedback shift registers (the s_i's are ± 1)

Configuration	Formula
5A	$s_i = s_{i-3} \cdot s_{i-5}$
5B	$s_i = s_{i-1} \cdot s_{i-2} \cdot s_{i-3} \cdot s_{i-5}$
5C	$s_i = s_{i-1} \cdot s_{i-3} \cdot s_{i-4} \cdot s_{i-5}$

Table 2. Recursion formulas for generating 63-bit PN sequences from 6-stage shift registers

Configuration	Formula
6A	$s_i = s_{i-5} \cdot s_{i-6}$
6B	$s_i = s_{i-1} \cdot s_{i-4} \cdot s_{i-5} \cdot s_{i-6}$
6C	$s_i = s_{i-1} \cdot s_{i-3} \cdot s_{i-4} \cdot s_{i-6}$

Table 3. Listing of best K-bit frame sync words derived from 31- and 63-bit PN sequences.

K	Upper bound on D_{min} $K-1-\sqrt{K-1}$	D_{min}	$\|C_\ell\|_{max}$ $(1 \le \ell \le K-1)$	Configuration	Initializing sequence, s′
16	11.13	10.26	3	5A	1,−1,−1,−1,1
16	11.13	10.26	3	5B	−1,−1,1,1,−1
17	12.00	10.84	4	5A	1,−1,−1,−1,1
17	12.00	12.00	4	6B	1,1,−1,−1,−1,−1
18	12.88	11.55	4	5A	1,1,−1,1,1
18	12.88	11.55	4	5C	1,−1,1,1,1
18	12.88	12.00	4	6B	1,1,1,−1,−1,−1
19	13.76	12.88	4	5C	1,−1,1,1,1
20	14.64	13.68	3	5C	1,−1,1,1,1
21	15.53	14.59	5	5B	−1,−1,−1,−1,−1
21	15.53	14.64	4	6B	1,−1,−1,−1,−1,−1
22	16.42	15.53	4	5B	−1,−1,−1,−1,−1
23	17.31	16.00	4	5B	−1,−1,1,1,−1
24	18.20	16.64	5	5A	−1,1,1,−1,−1
24	18.20	16.64	5	5C	−1,−1,1,1,1
25	19.10	17.76	5	5A	1,−1,1,1,1
26	20.00	18.76	5	5C	−1,−1,1,1,1
26	20.00	18.88	5	6A	1,−1,−1,−1,−1,−1
27	20.90	19.55	5	5A	1,1,−1,−1,−1
28	21.80	20.20	5	5B	−1,1,−1,−1,−1
29	22.71	21.53	5	5A	1,−1,−1,−1,−1
30	23.61	22.26	4	5B	1,−1,1,−1,−1
31	24.52	23.31	4	5C	1,1,1,−1,−1
32	25.43	22.90	6	6B	−1,1,−1,−1,1,−1
33	26.34	23.80	5	6A	1,−1,−1,1,1,−1
34	27.26	24.88	6	6C	−1,1,1,1,−1,1
35	28.17	25.80	5	6A	−1,1,−1,−1,1,1
36	29.08	26.90	4	6A	−1,−1,1,−1,−1,1
37	30.00	27.43	7	6A	1,−1,1,1,−1,1
37	30.00	27.43	7	6B	1,1,−1,−1,1,1
38	30.92	28.42	5	6A	−1,1,1,1,−1,−1
39	31.84	29.13	7	6A	−1,1,−1,−1,−1,1
40	32.76	30.34	6	6C	−1,−1,−1,1,1,1
41	33.68	30.90	7	6A	−1,1,−1,−1,−1,1
41	33.68	30.90	7	6C	−1,1,1,1,1,−1
42	34.60	31.80	8	6C	−1,1,1,1,1,−1
43	35.52	32.64	7	6A	1,1,1,−1,−1,1
44	36.44	33.80	7	6C	1,1,1,−1,1,1
45	37.37	34.34	8	6C	1,−1,−1,1,−1,1
46	38.29	35.17	8	6C	1,−1,1,−1,−1,1
47	39.22	36.00	7	6C	−1,−1,−1,1,−1,1
48	40.14	36.92	7	6B	1,−1,1,1,1,−1
49	41.07	38.00	7	6C	1,1,−1,1,1,−1
50	42.00	38.90	7	6A	−1,1,−1,1,−1,−1
51	42.93	40.00	7	6C	1,1,1,1,−1,1
52	43.86	40.92	7	6A	1,−1,1,1,1,1
53	44.79	41.68	8	6C	−1,1,−1,−1,1,1

Table 3 (contd)

K	Upper bound on D_{min} $K-1-\sqrt{K-1}$	D_{min}	$\lvert C_\ell \rvert_{max}$ $(1 \leq \ell \leq K-1)$	Configuration	Initializing sequence, s′
54	45.72	42.92	7	6C	−1,1,−1,1,−1,−1
55	46.65	43.76	7	6C	1,−1,1,1,1,1
56	47.58	44.76	6	6A	1,1,1,1,−1,1
57	48.52	45.61	6	6C	−1,−1,1,−1,−1,−1
58	49.45	46.00	7	6C	1,1,1,1,−1,−1
59	50.38	47.22	7	6A	−1,−1,1,−1,−1,−1
60	51.32	48.00	8	6C	1,1,1,−1,−1,1
61	52.25	49.43	6	6A	1,1,1,1,−1,1
62	53.19	50.34	6	6B	−1,1,1,1,−1,−1
63	54.13	51.52	6	6B	1,−1,1,1,−1,1

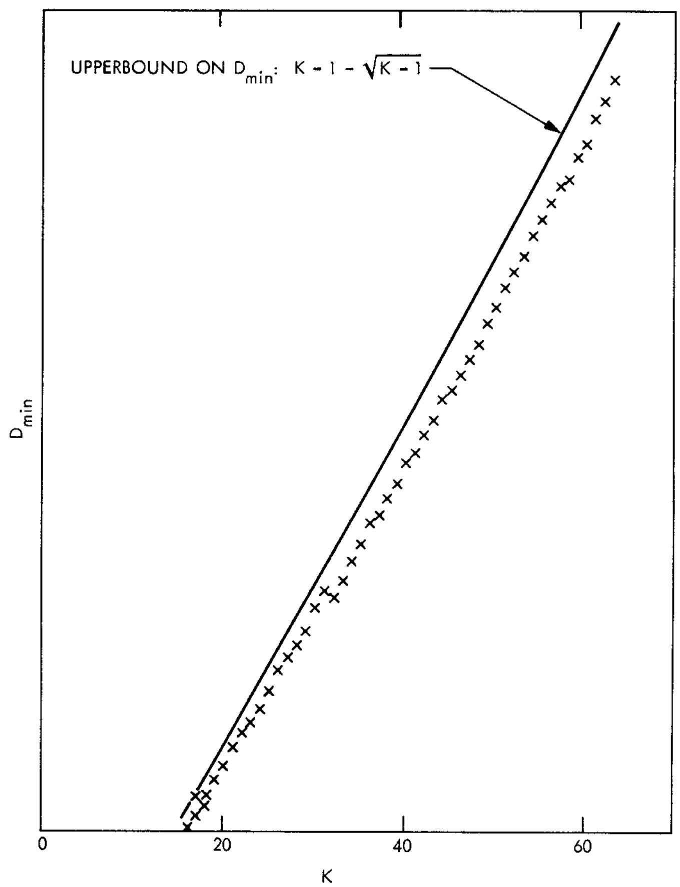

Fig. 1. Measure of correlation properties of K-bit sync words in Table 3 using Neuman-Hofman distance parameter D_{min}