S/X Band Experiment: Effect of Discontinuities on the Group Delay of a Microwave Transmission Line

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The problem is considered of the effect of reflections from discontinuities at each end of a transmission line on the group delay at microwave frequencies. Previous work is briefly reviewed and a general analysis is made. Graphical data are presented based upon the formulas developed. Experimental results are given which confirm the theory.

I. Introduction

The effects of mismatch on the calculation and measurement of group delay or envelope delay have been investigated in the past because of the importance of delay distortion in transmission systems. In recent experiments with space probes, small variations in the delay of microwave signals have been measured in order to obtain data on planetary atmospheres and the distribution of gaseous matter in space. Small errors in the determination of group delay have significant effects in these applications, and a need has arisen for a more thorough analysis to be carried out.

II. Background

As Lewin et al. (Ref. 1) explained in 1950, the effect of reflections from discontinuities is similar to the multipath problem (Ref. 2). A small portion of the energy traveling down the transmission line is reflected back toward the source and re-reflected to add to the energy transmitted

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to the load. In case of small reflections, one can neglect multiple reflections and consider only the dominant effect. Since the reflected wave travels farther than the main wave, its phase varies faster with frequency variations. Consequently, the resultant phase shift ψ of the transmitted wave does not vary quite linearly with frequency, but has a cyclical variation superimposed on the linear variation. The group delay τ_g is related to the phase shift variation with frequency. At a frequency f_o, the group delay is defined as follows

$$\tau_g = -\frac{1}{2\pi} \cdot \frac{d\psi}{df}\bigg|_{f = f_o} \qquad (1)$$

Hence, the group delay will also undergo cyclical variations due to reflections. Such an effect was observed by Lacy (Ref. 3) in 1961.

Various analyses of this effect have appeared in the literature. In 1961 (Ref. 4), Lacy's analysis was limited to the case of reactive shunt discontinuities on lossless transmission lines and did not yield an explicit expression for group delay. In 1964 (Ref. 5), Cohn and Weinhouse gave a clear explanation of the effect and a simple analysis of the interaction phase error, stopping short of an explicit expression for group delay. In 1969 (Ref. 6), Drazy gave an expression for the error in group delay due to reflections from test port terminations. His expression was valid for small reflections and lossless components but was limited in application. There may be other treatments of this problem in the literature, but the authors are not aware of them.

III. Theory

The following analysis is intended to be more general than previous work and will serve to clarify the assumptions made in calculating and measuring group delay of transmission lines. The analysis will be mainly useful for coaxial transmission line problems, but will be purposely kept general so as to include most uniconductor waveguide applications as well. It is not assumed that the discontinuities must be shunt susceptances, or that the characteristic impedance of the transmission line is identical to the characteristic impedances of the systems on either side. It is not assumed that the reflections from the discontinuities are small or equal. The discontinuities can be lossy or lossless and need not obey the reciprocity condition.

Consider a *uniform* transmission line of length ℓ having discontinuities at each end. A network model is shown in Fig. 1, in which the discontinuities are represented by the 2-ports L and N and the energy is assumed to propagate in the *dominant mode* from port 1 towards port 2.

The group delay τ_g is given by Eq. (1), where ψ is now ψ_{21}, the characteristic phase shift of the model, or the argument of S_{21}, its (transmission) scattering coefficient. If we employ conventional microwave network theory (Ref. 7), we can calculate S_{21} for the three cascaded 2-ports and obtain

$$S_{21} = \frac{\ell_{21}\, n_{21}\, e^{-\gamma \ell}}{1 - \ell_{22}\, n_{11}\, e^{-2\gamma \ell}} \qquad (2)$$

where $[\gamma = \alpha + j\beta]$ is the propagation constant of the transmission line and ℓ_{21}, ℓ_{22}, n_{11} and n_{21} are scattering coefficients of the discontinuities. The attenuation and phase constants of the transmission lines are α and β, respectively. For a general transmission line such as a dominant mode uniconductor waveguide or coaxial line one can let

$$\beta = \frac{2\pi}{\lambda_g} = \frac{2\pi}{\lambda_0}\sqrt{\varepsilon' - \left(\frac{\lambda_0}{\lambda_c}\right)^2} \qquad (3)$$

where λ_g is the waveguide wavelength of a dielectrically filled waveguide, λ_0 is the freespace wavelength, ε' is the relative permittivity of the dielectric material filling the waveguide, and λ_c is the cutoff wavelength. For transverse electric (TE) and transverse magnetic (TM) waves, λ_c is dependent on the physical dimensions of the waveguide opening, but for transverse electromagnetic (TEM) waves λ_c is equal to infinity.

We can let

$$h = |\ell_{22}\, n_{11}|\, e^{-2\alpha\ell}$$

$$\theta = 2\beta\ell - \arg(\ell_{22}) - \arg(n_{11})$$

$$\phi = \beta\ell - \arg(\ell_{21}) - \arg(n_{21})$$

Then we can write

$$S_{21} = \frac{|\ell_{21}\, n_{21}|\, e^{-\alpha\ell} \cdot e^{-j\phi}}{1 - h\, e^{-j\theta}} \qquad (4)$$

The characteristic phase shift ψ_{21} is then

$$\psi_{21} = \arg(S_{21}) = -\,[\phi + \arg(1 - h\, e^{-j\theta})] \qquad (5)$$

The group delay is obtained by multiplying Eq. (5) by -1 and differentiating the result with respect to angular frequency ($2\pi f$), and is

$$\tau_g = \tau_{g\ell} + \tau_{gn} + \tau_{go} + \Delta\tau \qquad (6)$$

where

$$\tau_{g\ell} = -\frac{1}{2\pi}\frac{d}{df}\arg(\ell_{21})$$

$$\tau_{gn} = -\frac{1}{2\pi}\frac{d}{df}\arg(n_{21})$$

and if v_g is the group velocity in the dielectric filled transmission line having phase constant given by Eq. (3) and c is the velocity of electromagnetic waves *in vacuo,*

$$\tau_{go} = \frac{1}{2\pi}\left(\frac{d\beta}{df}\right)\ell = \frac{\ell}{v_g} = \frac{\ell}{c}\left(\frac{\lambda_g}{\lambda_0}\right)\epsilon' \qquad (7)$$

Furthermore,

$$\Delta\tau = \frac{1}{2\pi}\frac{h(\cos\theta - h)\dfrac{d\theta}{df} + \left(\dfrac{dh}{df}\right)\sin\theta}{1 - 2h\cos\theta + h^2} \qquad (8)$$

in which

$$\frac{1}{2\pi}\frac{d\theta}{df} = 2\,\tau_{go} - \frac{1}{2\pi}\frac{d}{df}\left[\arg(\ell_{22}) + \arg(n_{11})\right] \qquad (9)$$

and

$$\frac{1}{2\pi}\frac{dh}{df} = \frac{|\ell_{22}\,n_{11}|}{2\pi}e^{-2\alpha\ell}$$

$$\times\left[\frac{1}{|\ell_{22}|}\frac{d|\ell_{22}|}{df} + \frac{1}{|n_{11}|}\frac{d|n_{11}|}{df} - 2\ell\frac{d\alpha}{df}\right] \qquad (10)$$

We can see in Eqs. (6) to (10) how a change with frequency of the attenuation or phase shift of the line or the reflection coefficients of the discontinuities, for example, might affect the group delay. In practical cases where the delays of the individual discontinuities are small, and we are interested in a relatively small bandwidth at microwave frequencies, we can neglect a number of terms that would contribute an insignificant amount to the final result. Then Eq. (6) reduces to

$$\tau_g = \tau_{go}\left[1 + \frac{2h(\cos\theta - h)}{1 - 2h\cos\theta + h^2}\right] = \tau_{go} + \Delta\tau \qquad (11)$$

One can see that $\Delta\tau$ is a function of θ which varies with frequency, so that $\Delta\tau$ varies between the following limits

$$-\frac{2\,\tau_{go}\,h}{1+h} \leq \Delta\tau \leq \frac{2\,\tau_{go}\,h}{1-h} \qquad (12)$$

or from substitutions

$$\frac{-2\left[\dfrac{\ell}{c}\left(\dfrac{\lambda_g}{\lambda_0}\right)\epsilon'\right]\left[|\ell_{22}|\,|n_{11}|\,\mathcal{A}\right]}{1 + |\ell_{22}|\,|n_{11}|\,\mathcal{A}} \leq \Delta\tau$$

$$\leq \frac{2\left[\dfrac{\ell}{c}\left(\dfrac{\lambda_g}{\lambda_0}\right)\epsilon'\right]\left[|\ell_{22}|\,|n_{11}|\,\mathcal{A}\right]}{1 - |\ell_{22}|\,|n_{11}|\,\mathcal{A}} \qquad (13)$$

where $\mathcal{A} = e^{-2\alpha\ell}$ is the attenuation (power) ratio of the length of transmission line which has an attenuation A = $-10\log_{10}\mathcal{A}$. The reader is reminded that in the expression given above, λ_g is for a general case single-mode dielectric filled waveguide as may be seen from Eq. (3). For an air-dielectric medium we may let $\epsilon' = 1$, and for coaxial line TEM mode, we set $\lambda_c = \infty$.

IV. Graphical Data

A graph of the limits of $\Delta\tau$ is given in Fig. 2. It is assumed for simplicity that $|\ell_{22}| = |n_{11}|$, and that τ_{go} = 100 ns. If $|\ell_{22}| \neq |n_{11}|$, one can assume that the line has equivalent identical discontinuities at each end where the equivalent discontinuity has a reflection coefficient magnitude equal to

$$\sqrt{|\ell_{22}|\,|n_{11}|}$$

For a transmission line having the same line attenuation but a τ_{go} different from 100 ns, one multiplies the result obtained from the graph by the ratio of the actual τ_{go} in nanoseconds to 100.

As an example of the use of Fig. 2, assume that a transmission line has a delay of 20 ns and a line attenuation of 5 dB prior to adding discontinuities of $|\ell_{22}| = 0.4$ and $|n_{11}| = 0.1$. Then an equivalent discontinuity placed at each end of this line would have a reflection coefficient magnitude of $\sqrt{(0.4)\,(0.1)} = 0.2$. From Fig. 2, one finds that if the reflection coefficient magnitude of each discontinuity is 0.2 and the line attenuation is 5 dB for a 100-ns line, the limits of cyclical variation are ± 2.5 ns. Then the limits of cyclical variation for the above 20-ns line example are $\pm(20/100)\,(2.5)$ ns.

V. Experimental Results

Experimental results were obtained to confirm the theory for the case of fairly large reflections. Metal disks having a diameter of 6.12 mm (0.241 in.) and a thickness of 0.152 mm (0.006 in.) were attached to the center conductors of short sections of 7-mm coaxial line to form the discontinuities L and N in Fig. 1. The discontinuity assemblies may be seen in Fig. 3. Over a frequency range of 8.365 to 8.465 GHz, a value of 0.42 for $|\ell_{22}|$ and $|n_{11}|$ of the disk assemblies was calculated using computer programs developed for 2-port standards (Ref. 8). Over the same frequency range a value of 0.43 was measured using an automatic network analyzer.

The 30-ns coaxial line delay standard shown in Fig. 4 was connected between the discontinuities. Measurements of τ_g and of $-20 \log_{10} |S_{21}|$ using an automatic network analyzer are shown in Figs. 5 and 6, respectively. The group delay and the measured attenuation equal to $-10 \log_{10} e^{-2a\ell}$ of the transmission line plus the short sections of 7-mm coaxial line with disks removed are also shown. A comparison of calculated and measured results is shown in Table 1. The tabulated results show only limits near the center of the frequency range but are typical of the results over the frequency range of 8.365 to 8.465 GHz. The calculated limits do not include the additional effects of reflections from the coaxial line connectors. Closer agreement was obtained by measuring the $|\ell_{22}|$ and $|n_{11}|$ taking into account the coaxial connectors.

Good agreement between theory and experiment was also obtained when additional measurements were made with different disks, different transmission line lengths, and at an additional frequency range of 2.235 to 2.355 GHz. The details are not given here but will be included in a subsequent issue of this publication.

VI. Conclusions

No attempt was made to experimentally confirm all aspects of the general theory. However, for the special case considered [identical discontinuities at each end, and negligible frequency dependence of α, $|\ell_{22}|$, $|n_{11}|$, $\arg(\ell_{21})$, $\arg(n_{21})$, $\arg(\ell_{22})$, and $\arg(n_{11})$ over the bandwidth of interest], adequate confirmation of the theory was obtained.

The analysis presented applies both to transmission lines operating in the TEM mode or to single-mode propagation in waveguides in general. The graphical data presented can be useful for: (1) estimating the limits on the variation of group delay with frequency or (2) determining how much reduction of discontinuities is necessary in order to achieve a given accuracy in predicting group delay.

Acknowledgment

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Table 1. Comparison of calculated and measured upper and lower limits of group delay and loss of a 30-ns line with a discontinuity assembly at each end

Parameter	Calculated value	Measured value[a]	Difference
Maximum delay	34.60 ns	34.38 ns at 8.407 GHz	0.22 ns
Minimum delay	26.56 ns	26.70 ns at 8.415 GHz	−0.14 ns
Maximum loss	6.51 dB	6.42 dB at 8.415 GHz	0.09 dB
Minimum loss	5.36 dB	5.25 dB at 8.407 GHz	0.11 dB

[a]The frequency interval between successive upper and lower limits is $1/(4\tau_{g0})$

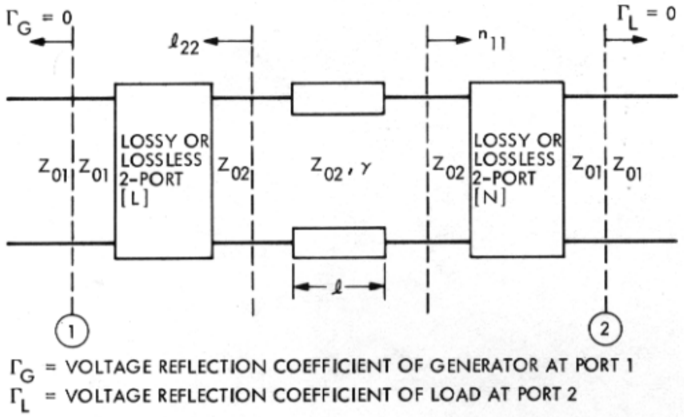

Fig. 1. Network model of transmission line and discontinuities

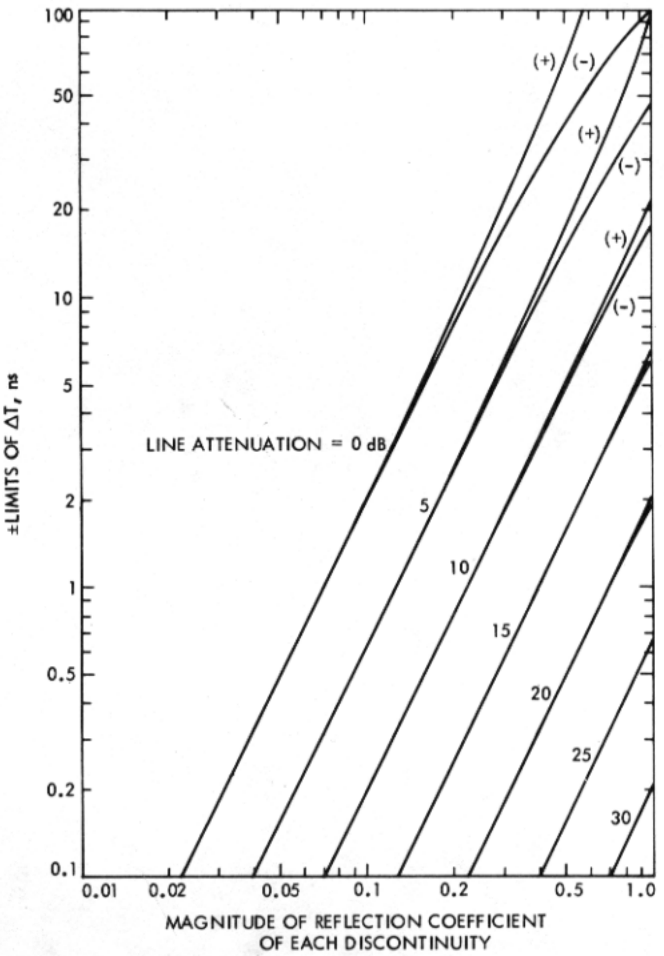

Fig. 2. Calculated limits of cyclical variation of group delay of a 100-ns transmission line with identical discontinuities at each end

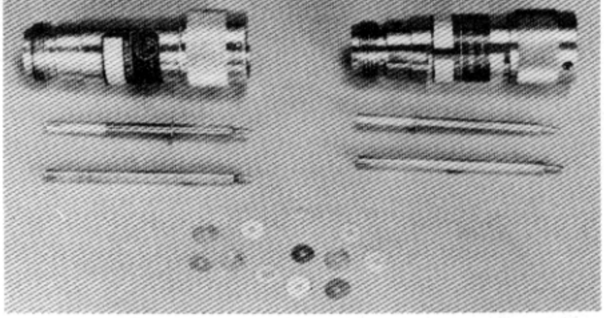

Fig. 3. 7-mm coaxial line discontinuity assemblies

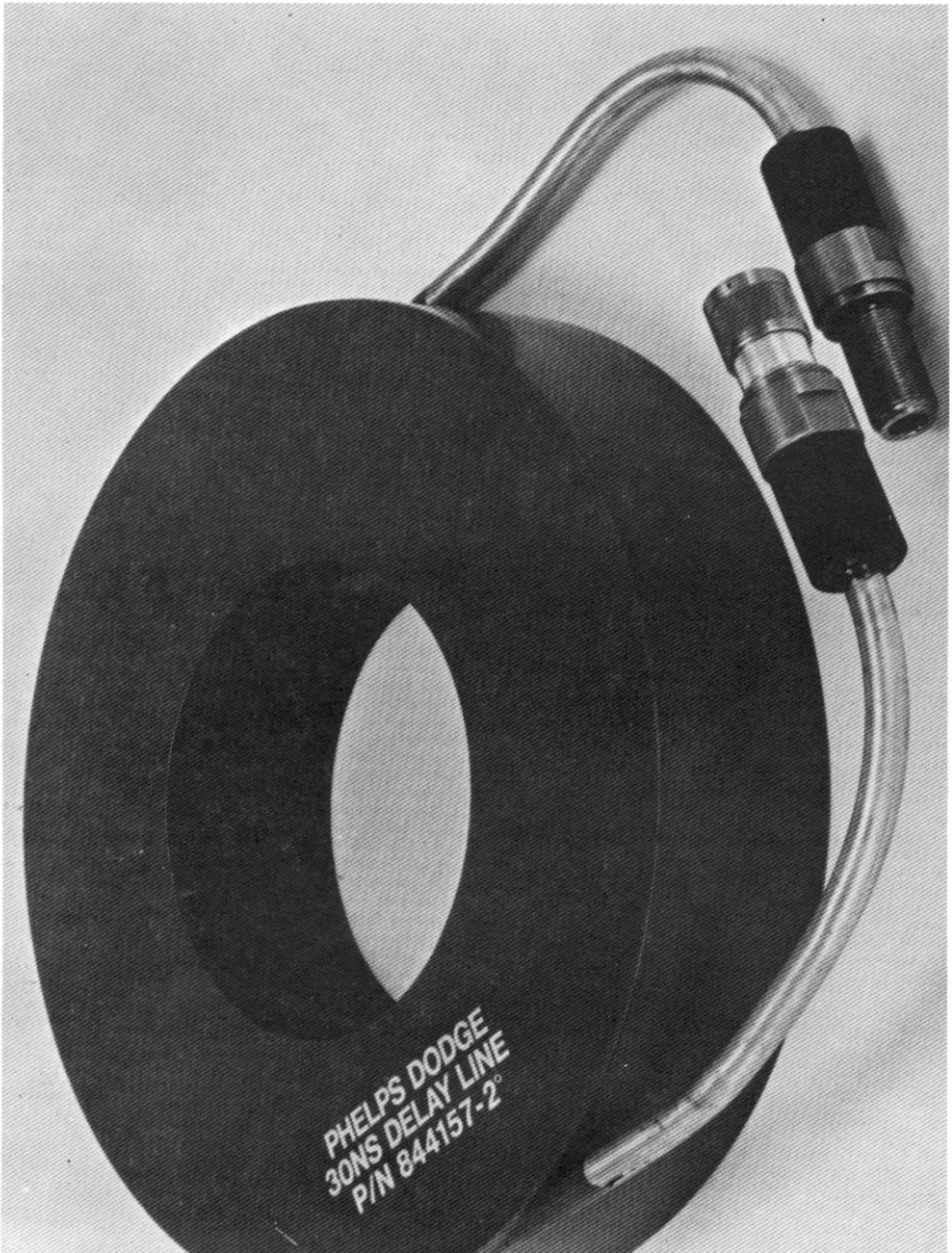

Fig. 4. Coaxial line delay standard of 30 ns

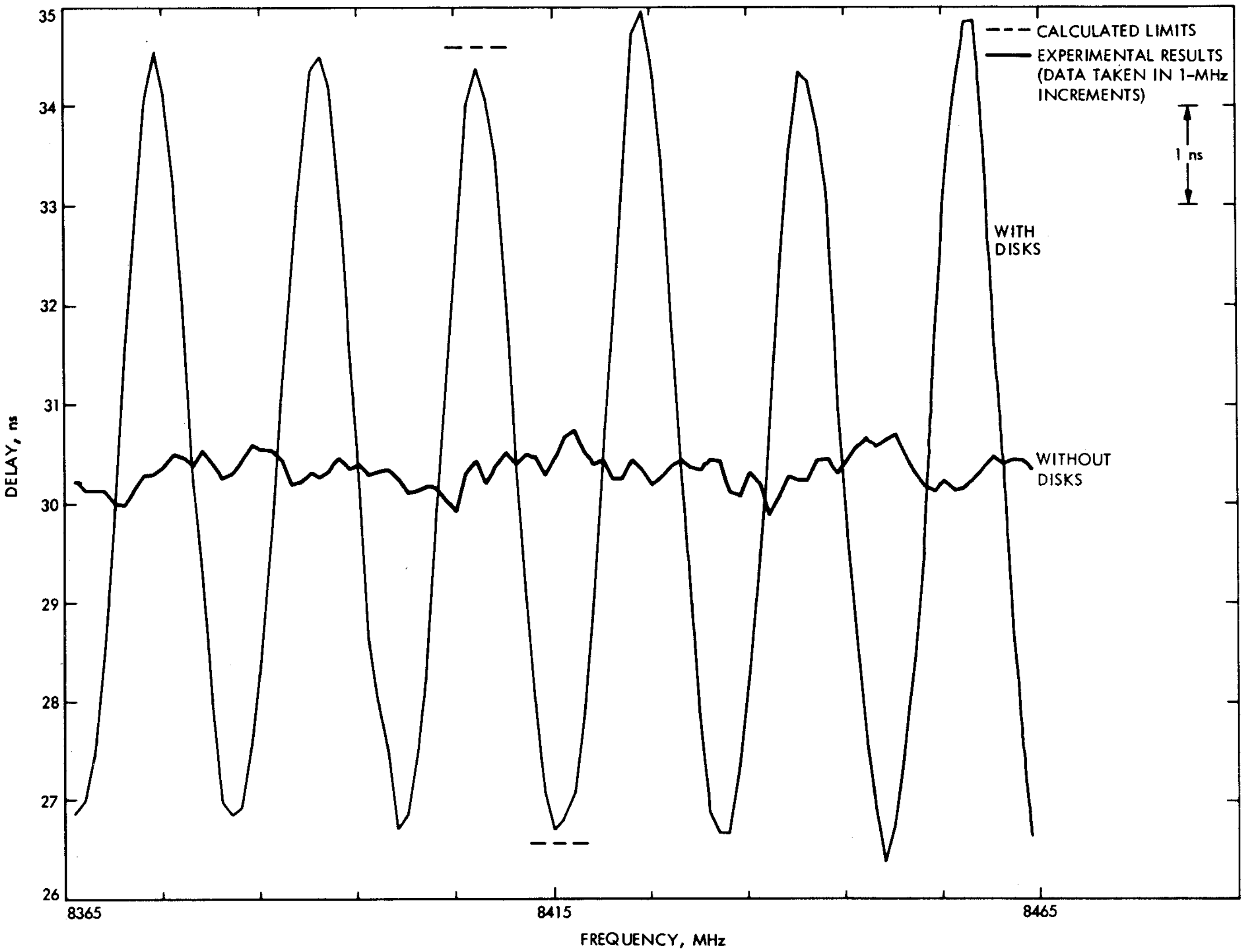

Fig. 5. Measured group delay of a 30-ns transmission line plus short end sections of 7-mm lines with and without 6.12-mm (0.241 in.) diam disks

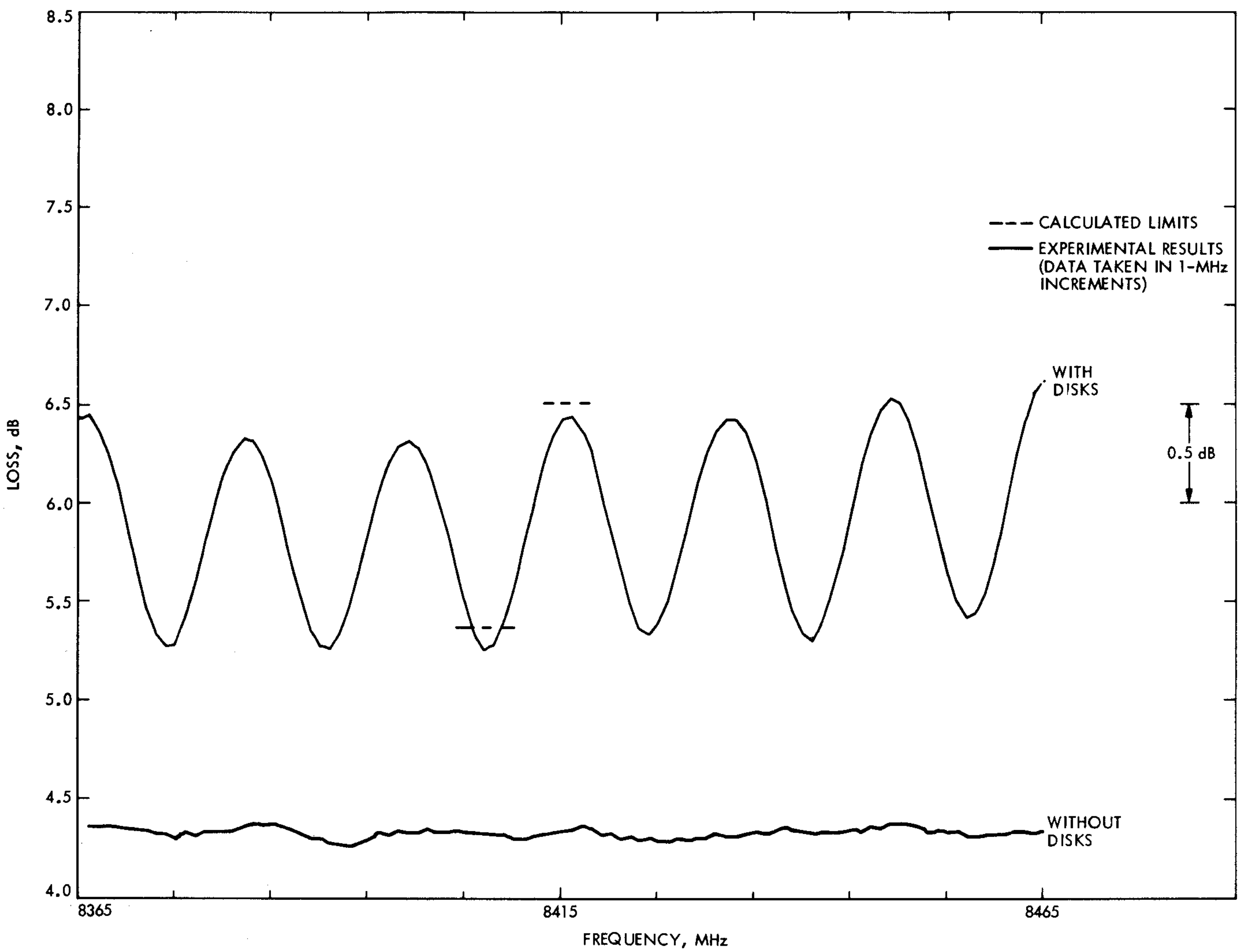

Fig. 6. Measured $-20 \log_{10} |S_{21}|$ of a 30-ns transmission line plus short end sections of 7-mm lines with and without 6.12-mm (0.241 in.) diam disks