Computation of Gravity RMS for HA-DEC Antennas

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The gravity load distortion rms (half path length) values are calculated for the hour angle-declination angle (HA-dec) antenna by computing the positions of the three principal axes fixed on the reflector structure, but rotating in declination and hour angle. For a symmetric structure, the rms values are equal to the results from the standard square root of the sum of the squares equation. For unsymmetric structures, the required modifications to the equation are described. Contour level plots for a sample 27.4-m HA-dec antenna show the variations of the rms with respect to selected surface panel setting positions. Also plotted are the X, Y, and Z load components for the panels set at zenith look.

I. Introduction

To compute the gravity loading distortions of a ground radio antenna structure, a model is created that is input to the NASTRAN or IDEAS program and the displacements of the surface panel mounting points are best fitted with a paraboloid and the rms of the residuals becomes an index useful for determining the RF performance.

A practical structural model will usually be described by using a rectangular or cylindrical coordinate system fixed on the reflector with the three principal axes (1) "X" parallel to the elevation axis, (2) "Y" normal to X and (3) "Z" normal to the X-Y plane and coincident with the centerline of the reflector structure.

For the azimuth-elevation (az-el) axes configuration, the angle between the gravity vector and the Y and Z axes is either the elevation angle or its complement. However, for the hour angle-declination angle (HA-dec) axes configuration, the angle between the gravity vector and the principal axes of the structural model becomes a function of the declination, hour, and latitude angles.

The standard coordinate conversion equations from the HA-dec system to the az-el system are used to solve for the components of the gravity vectors along the principal axes of the structural model.

Gravity loadings applied in the directions of the three principal axes are the basic loadings of the structural model. The associated three rms values can then be combined by superposition with the geometric relationships to determine the resultant rms for a particular pointing direction of the antenna.

The rms is computed in this report, as described above and detailed below, by superposition. However, an alternative rms value can be computed for any particular pointing direction with the rms program (Ref. 1) by adding three component displacements for the three basic loadings into a single displacement vector before the best fit by the paraboloid is made. Some comparisons using the results from both methods are presented together with the explanations of the accuracies of the superposition method.

The sample antenna is a modified reflector structure of the Caltech 27.4-m (90-ft) HA-dec antenna.

II. Calculations

The motion of the RF boresight of a HA-dec antenna may be described by the equatorial coordinates system. The two axes of rotation are centered at point "O" of Fig. 1. The polar axis is OP and the rotation about this axis is measured in hour angle "t" from the meridian circle. The rotation of the declination axis is measured as declination angle "δ" from the equator OUQR. The resultant direction of the RF boresight is along the radial line OM, which intersects the spherical surface at M.

The computer model is assumed to use the coordinate system described in the second paragraph. The Z axis of the reflector structure is coincident with radial line OM and the RF boresight direction line. The azimuth measuring vertical plane $OZMH$ contains these lines and the vertical gravity vector **MG**. As shown in Fig. 2, the X and Y principal axes of the reflector are rotated with respect to this azimuth measuring plane as the Y axis is coincident with the hour angle great circle. This angle of rotation measured at point M is β, the angle between the hour angle circle and the azimuth measuring circle ZMH.

Spherical triangle PZM may then be used to compute angle β. That is, given

$$A = \text{azimuth angle}$$

$$PZ = 90\text{-deg} - \text{polar angle } \phi$$

$$t = \text{hour angle}$$

$$PM = 90\text{-deg} - \text{declination angle } \delta$$

then compute

$$ZD = \text{zenith distance}$$

from

$$\text{Cos}\,(ZD) = \sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(t)$$

which is a standard astronomical formula.

Using half angle relations of the sides and angles of a spherical triangle to minimize sign anomalies, the following equations were used:

$$1/2[A + \beta] = \tan^{-1}\left[\frac{\cos\left(\dfrac{PM - PZ}{2}\right)\cos\dfrac{t}{2}}{\cos\left(\dfrac{PM + PZ}{2}\right)\sin\dfrac{t}{2}}\right] \tag{1}$$

$$1/2[A + \beta] = \tan^{-1}\left[\frac{\sin\left(\dfrac{PM - PZ}{2}\right)\cos\dfrac{t}{2}}{\sin\left(\dfrac{PM + PZ}{2}\right)\sin\dfrac{t}{2}}\right] \tag{2}$$

β and A may be resolved from the above equations by computing the right sides of the equations. Then

$$\beta = \text{right sides } ((1) - (2))$$

$$A = \text{right sides } ((1) + (2))$$

The antisymmetric gravity vector component MJ acts at angle β to the X and Y axes of the reflector. Therefore the resultant component factors with respect to the unit gravity vector for the two X_c and Y_c axes are:

$$X_c = -\sin(ZD)\sin(\beta) - X_c' \tag{3}$$

$$Y_c = -\sin(ZD)\cos(\beta) - Y_c' \tag{4}$$

For the Z component, since gravity is on or working at any position and our primary interest is that due to the change in the direction of the **Z** gravity vector, it follows that

$$Z_c = 1.0 - \cos(ZD) - Z_c' \tag{5}$$

1.0 is the unit gravity vector value at zenith look where ZD-zenith distance is equal to 0.

X'_c, Y'_c and Z'_c are the component values at the surface panels setting position. The *rms* for a particular pointing direction may then be computed by

$$rms = \sqrt{(rms_x)^2(X_c)^2 + (rms_y)^2(Y_c)^2 + (rms_z)^2(Z_c)^2} \tag{6}$$

When the antenna structure is symmetric about the three axes, the above equation may be used. When the structure is not symmetrical, the method of resolution described in the Appendix will be necessary. The Appendix also illustrates the use of vector analysis to compute the load components.

III. Results

A computer program was coded to solve for the load components and for the rms distortions for a range of declinations and hour angles. A JPL library subroutine was used to output contour level plots of the results.

First, the surface panels of the antenna were assumed to be set to a perfect paraboloid at zenith look, with hour angle = 0 and declination angle = 37 deg (the polar angle = 37 deg). Then the surface panels setting position was changed to 0 deg declination 0 deg hour angle and to −10 deg declination. The rms were computed and output on contour level plots (Figs. 3, 4, and 5 respectively).

The X, Y, and Z load components for only the zenith look panels setting cases are plotted where the X'_c Y'_c and Z'_c components are equal to 0 (Figs. 6, 7, and 8, respectively).

For other surface panels setting angles, the particular load components at the setting angle may be subtracted from the curves for the zenith look case to obtain the resulting changes from the setting position.

Table 1 lists some computed rms values for using the superpositioning method and the displacement adding and best fitting rms program.

IV. Summary

The rms values in this report are only for the surface panel mounting points of the reflector structure. Errors of the surface panels, for setting, etc., must be added to obtain the rms values the RF uses.

For DSN use, the celestial targets are close to the ecliptic plane. Inspection of the rms values along the 0 deg declination angle vs. hour angles shows that the minimum degradation due to gravity loads results when the surface panels are set with the reflector close to 0 deg declination angle.

For the northern hemisphere locations, in the next decade, the far out planets will be below the ecliptic. For this condition, better performance under gravity loads can be expected with the initial setting of −10 deg or lower elevation angles.

It is interesting to note that the setting of the surface panels at 0 deg declination minimizes the Y-component changes. Referring to Fig. 7, the Y-component does not change throughout the hour angle change; it starts with about 0.6 and remains at this figure. For surface panels set at 0 deg declination, the gravity distortions contribution by the Y component is zero for targets along the ecliptic.

Appendix

General Formula for RMS Computation

In the foregoing, relationships have been given to express the loading components in the direction of the reflector coordinate axes, and an equation has been supplied for computation of the rms in terms of the three rms values computed independently for the loadings along each of the three reflector axes. The equation given, however, is correct only for the case of a reflector structure that is symmetric about its own X-Z and Y-Z planes. In the general case where this symmetry does not occur, the equation must be modified to account for coupling of the loading. In Ref. 2 relationships have been given to compute the rms for a general, unsymmetric az-el antenna. That development will be extended to the HA-dec type of axes arrangement.

We will consider a moving set of X, Y, and Z axes attached to the reflector as described previously and a set of reference axes fixed on the ground. For the reference set of axes let X_g and Y_g establish the horizon plane

where

X_g points east

Y_g points north

and let

Z_g point to the zenith

If we consider a set of unit vectors $\{\mathbf{e}_r\}$ aligned with the reflector's X, Y, and Z axes, and set of unit vectors $\{\mathbf{e}_g\}$ aligned with the X_g, Y_g, and Z_g axes, then the transformation from the reflector to the ground axes can be made by

$$\{\mathbf{e}_r\} = [T]\{\mathbf{e}_g\}$$

The components of the transformation (orthogonal) $[T]$, in terms of the latitude, ϕ, the declination, δ, and the hour angle t, can be shown to be

$$[T] = \begin{matrix} \cos t & -\sin t \sin \phi & \sin t \cos \phi \\ \sin \delta \sin t & \cos \delta \cos \phi + \sin \delta \sin \phi \cos t & \cos \delta \sin \phi - \sin \delta \cos \phi \cos t \\ -\cos \delta \sin t & \sin \delta \cos \phi - \cos \delta \sin \phi \cos t & \sin \delta \sin \phi + \cos \delta \cos \phi \cos t \end{matrix}$$

The components of the third row of the transformation are the components of a unit vector in the direction of the reflector Z axis, which is the pointing vector. Consequently, in azimuth-elevation coordinates, the elevation angle α, (the complement of the zenith distance) is given by

$$\sin \alpha = T_{3,3} = \sin \delta \sin \phi + \cos \delta \cos \phi \cos t$$

Similarly, the azimuth angle, A, can be found from T as

$$\cos A = T_{3,2}/\cos \alpha = (\sin \delta \cos \phi - \cos \delta \sin \phi \cos t)/\cos \alpha$$

As an alternative to computing the gravity load components according to relationships given previously, we can take the vector scalar product of a unit vector in the direction of gravity loading (components 0.0, 0.0, −1.0) with each of the three components of $\{\mathbf{e}_r\}$ in turn to obtain the projections $\underline{X}$, $\underline{Y}$, $\underline{Z}$ on the reflector X, Y, and Z axes. As the result, we find:

$$\underline{X} = -T_{1,3} = -\sin t \cos \phi$$

$$\underline{Y} = -T_{2,3} = \sin \delta \cos \phi \cos t - \cos \delta \sin \phi$$

$$\underline{Z} = -T_{3,3} = -\sin \alpha$$

The panel setting position is defined by the elevation rigging angle γ. When the setting angle has declination $\underline{\delta}$ and zero hour angle, then γ can be found from

$$\gamma = 90 - \phi + \underline{\delta}$$

At the setting position, the unit loading vector has the following projections, $\underline{X}_s$, $\underline{Y}_s$, $\underline{Z}_s$, on the reflector axes

$$\underline{X}_s = 0.0$$

$$\underline{Y}_s = -\cos\gamma$$

$$\underline{Z}_s = -\sin\gamma$$

Consequently the net projections of the unit loading vector are

$$\xi = \underline{X} - \underline{X}_s = -\sin t \cos\phi$$

$$\eta = \underline{Y} - \underline{Y}_s = \cos\gamma - \cos\delta\sin\phi + \sin\delta\cos\phi\cos t$$

$$\zeta = \underline{Z} - \underline{Z}_s = \sin\gamma - \sin\alpha$$

From the linearity of the antenna structure response to gravity loading it follows that the displacements of the structure at any orientation (δ, t) are a linear combination of the separate displacements caused by gravity loading in the directions of the X, Y, and Z axes. That is, let

$$\{\mathbf{u}(\delta, t)\} = \text{the displacement vector at } \delta, t$$

and

$$\{\mathbf{u}_x\}, \{\mathbf{u}_y\}, \{\mathbf{u}_z\} = \begin{array}{l}\text{displacement vectors for gravity}\\ \text{applied in the X, Y, Z, directions}\\ \text{respectively}\end{array}$$

then we have

$$\{\mathbf{u}(\delta, t)\} = \xi\{\mathbf{u}_x\} + \eta\{\mathbf{u}_y\} + \zeta\{\mathbf{u}_z\}$$

Since the pathlength deviations of the reflector surface from a paraboloid are linear functions of the displacements and the geometry of the surface, the foregoing equation for superposition of displacements also applies to pathlength deviations.

Furthermore, as shown in Ref. 2, the pathlength deviations from the best-fitting paraboloid are also a linear function of the displacements. Therefore it follows also that pathlength deviations from the best-fit paraboloid at any reflector attitude can be obtained by superposition of the deviations for the three sets of gravity loadings applied independently in the X, Y, and Z directions.

Therefore, if we let

> SSX, SSY, SSZ be the mean square half pathlength deviations from the best fitting paraboloids for gravity loadings in the X, Y, Z directions respectively

and

> SXY, SXZ, SYZ be the mean inner product of the half pathlength deviation vectors for X and Y, X and Z, Y and Z, gravity loading respectively,

then the mean square pathlength deviation SS at any reflector orientation is given by

$$SS = \xi^2 SSX + \eta^2 SSY + \zeta^2 SSZ + 2\xi\eta SXY + 2\xi\zeta SXZ + 2\eta\zeta SYZ$$

To make this equation represent the mean square pathlength deviation for an az-el antenna, set

$$\xi = 0$$

$$\eta = \cos\gamma - \cos\alpha$$

For a reflector structure that is symmetric about the X-Z plane, SYZ and SXY are zero. For a reflector that is symmetric about the Y-Z plane, SXZ and SXY are zero.

Finally, take the square root of the mean square to obtain the rms half pathlength deviation.

References

1. Katow, M. S., and Schmele, L. W., "Antenna Structures: Evaluation Techniques of Reflector Distortions." *Space Programs Summary 37-40*, Vol. IV, pp. 176–184. Jet Propulsion Laboratory, Pasadena, California, Sept. 30, 1968.

2. Levy, R., "A Method for Selecting Antenna Rigging Angles to Improve Performance." *Space Programs Summary 37-65*, Vol. II, pp. 72–76. Jet Propulsion Laboratory, Pasadena, California.

Table 1. RMS vs antenna position for 27.4-m (90-ft) HA-dec antenna[a]

Panels set position		Antenna attitude		Reflector principal axes load components			Reflector distortion – rms, mm[b]	
Dec, deg	HA, deg	Dec, deg	HA, deg	X_c	Y_c	Z_c	Superposition	Rms program
37	0	−53	0	0.	−1.0	1.0	1.29	1.29
0	0	−53	0	0.	−0.39819	0.79864	0.87	0.87
−10	0	−53	0	0.	−0.26865	0.68200	0.73	0.73
37	0	0	90	−0.79864	−0.60182	1.0	1.23	1.23
0	0	0	90	−0.79864	−0.	0.79864	0.95	0.95
37	0	−10	50	−0.61179	−0.68181	0.59895	0.90	0.90
0	0	−10	50	−0.61179	−0.08000	0.39759	0.56	0.56
−10	0	−10	50	−0.61179	−0.04954	0.28095	0.48	0.48

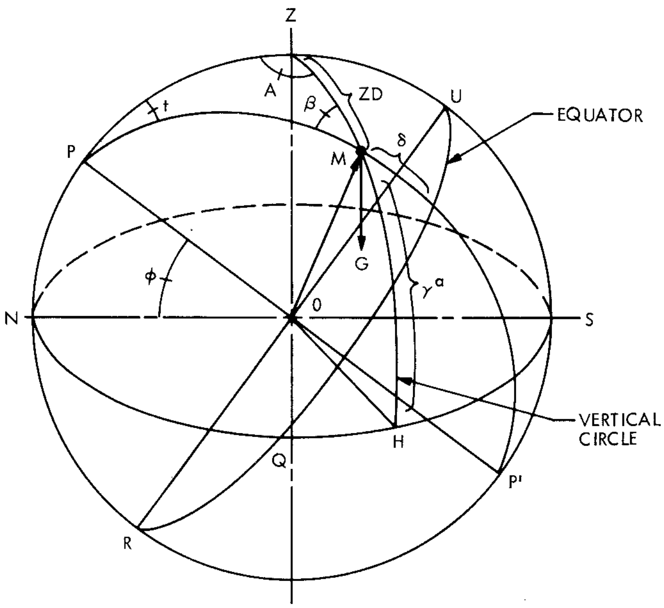
[a]Gravity – off/on – rms values

rms_x = 0.626237 mm (0.024655 in.)

rms_y = 0.787527 mm (0.031005 in.)

rms_z = 1.018235 mm (0.040088 in.)

[b]Rms, mm = ½ pathlength errors.



[a]GAMMA IS DEFINED IN THE APPENDIX

Fig. 1. Topocentric coordinates

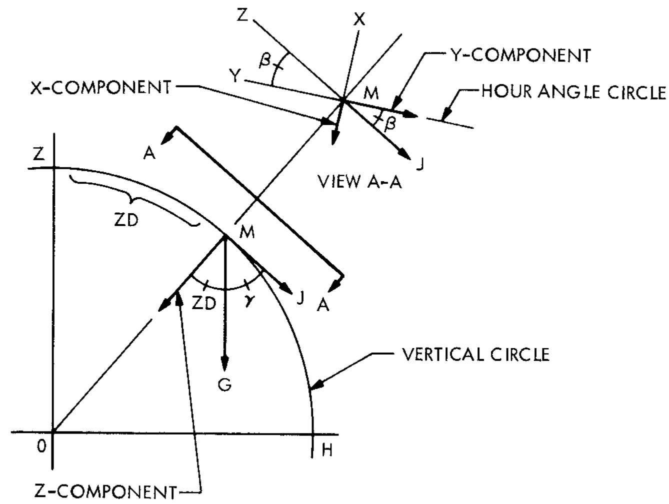


Fig. 2. Azimuth measuring plane

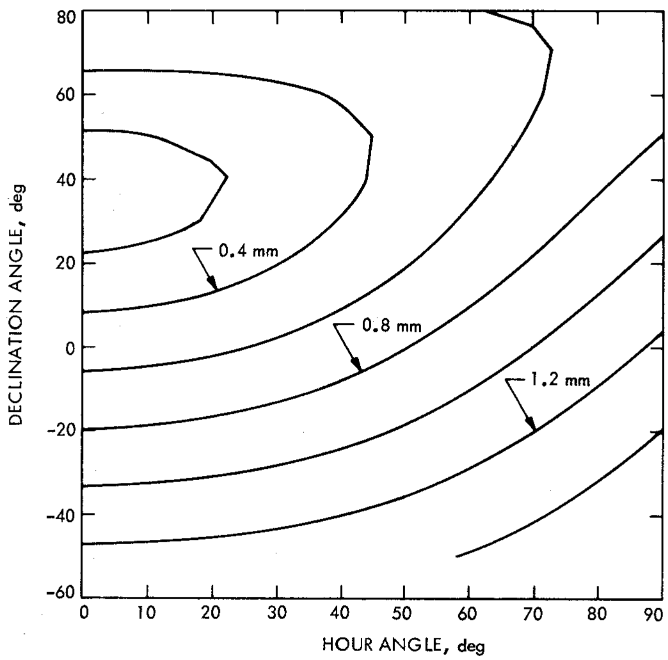


Fig. 3. RMS, mm (panels set at zenith look)

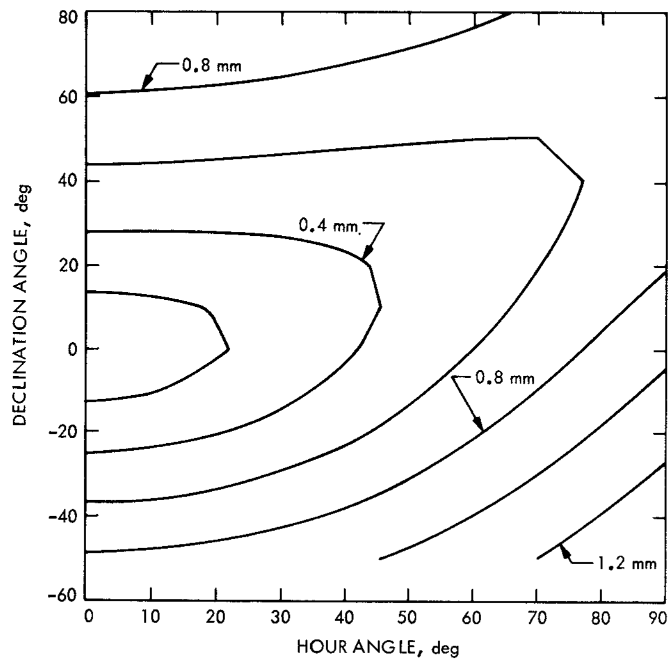


Fig. 4. RMS, mm (panels set at 0 deg declination)

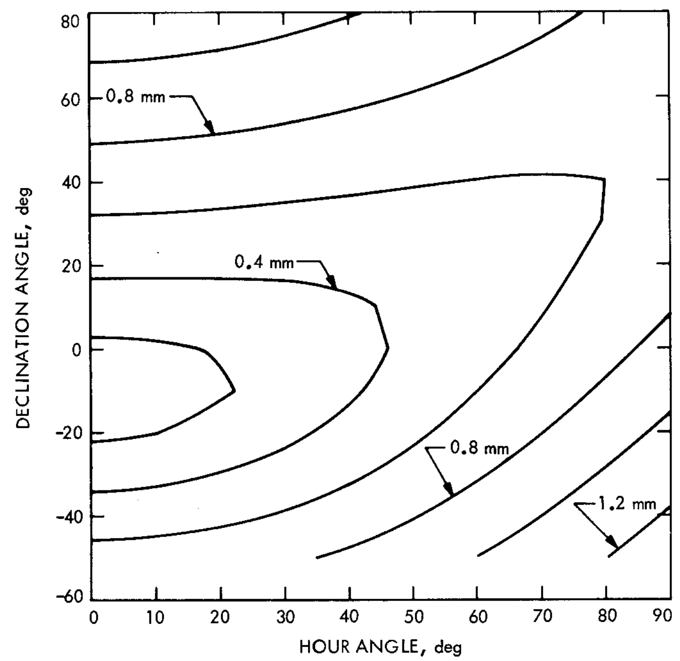


Fig. 5. RMS, mm (panels set at −10 deg declination)

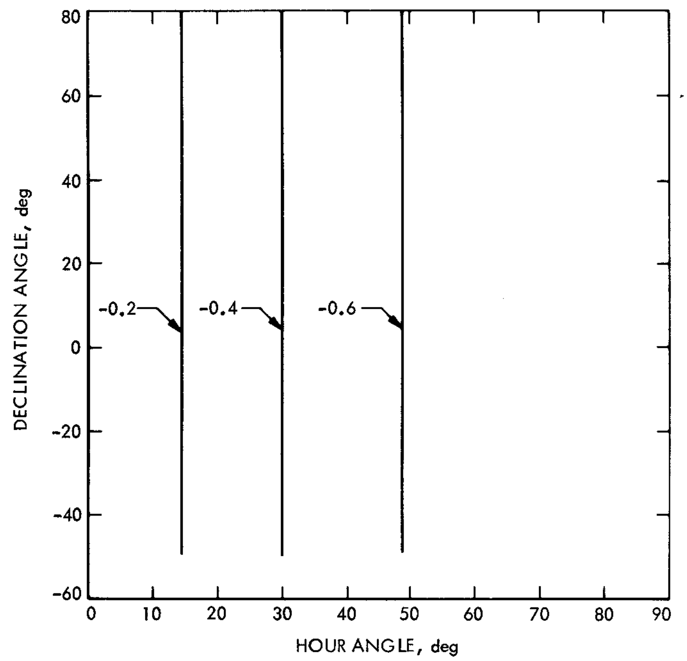


Fig. 6. X_c loading component

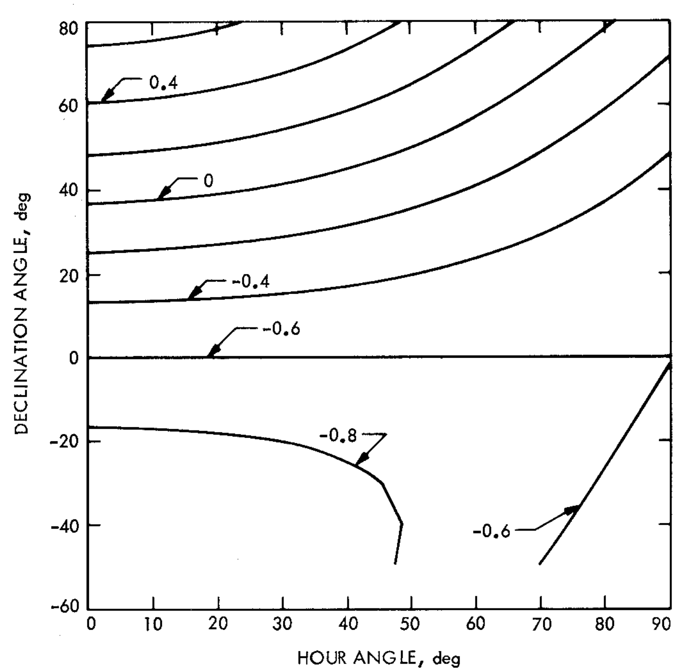


Fig. 7. Y_c loading component

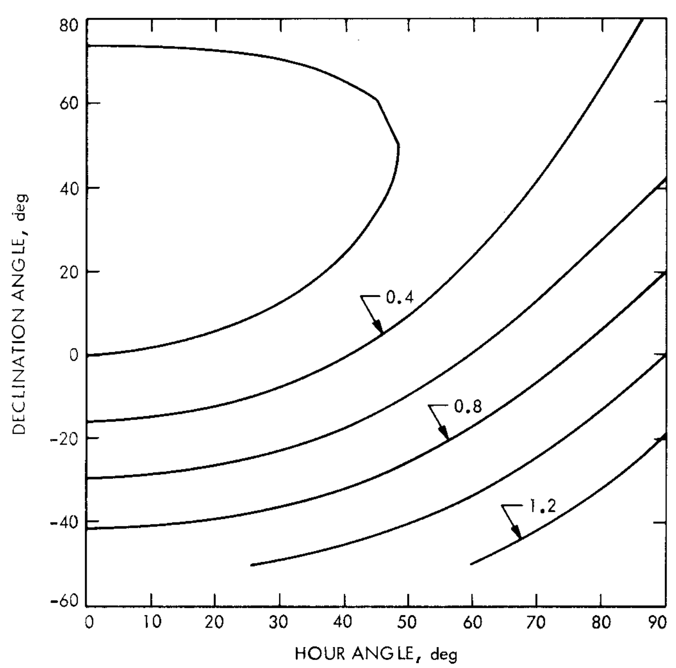


Fig. 8. Z_c loading component