Effect of Phase Distortion on Effective Signal Power—A Simple Mathematical Model

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A simple mathematical model of the effect of phase distortion on effective signal power is developed. The model is used to determine the approximate degradation of a single-sideband downconverter used for recording the output of an open-loop receiver.

I. Introduction

Recording and playback of open- and closed-loop receiver data produces phase distortion due to additional signal processing both external and internal to the recorder itself. To record the output of an open-loop receiver, for example, a single-sideband (SSB) downconverter (or double-sideband downconverter and bandpass filter) is used to move the spectrum of the receiver output signal to a suitable offset frequency. The recording is then played back through an upconverter into a closed-loop receiver to recover the data. Closed-loop receiver baseband data, on the other hand, is recorded at the output of the video amplifier in the receiver telemetry string and then recovered by playing it back through the balanced modulator of the subcarrier demodulator assembly (SDA). In either case, phase nonlinearities of the signal processing system and recorder itself reduce the effective signal power. The purpose of this article is to develop a simple mathematical model of degradation due to phase distortion that will be useful in predicting system performance.

II. Analysis

The output signal of an open-loop or closed-loop receiver can be written as

$$v(t) = (2P_T)^{1/2} \cos [\omega_{os}t + \theta S(t)\, d(t)] \tag{1}$$

where

P_T = total power

$\omega_{os} = 2\pi f_{os}$ = offset frequency (f_{os} = 10 MHz for a closed-loop receiver)

θ = modulation index

$d(t) = \pm 1$ = data

$S(t) = \pm 1$ = square-wave subcarrier

Using phasors or trigonometric identities we may rewrite (1)

$$v(t) = \underbrace{(2P_T)^{1/2} \cos\theta \cos \omega_{os} t}_{\text{carrier}} - \underbrace{(2P_T)^{1/2} \sin\theta\, S(t)\, d(t) \sin \omega_{os} t}_{\text{sidebands}} \tag{2}$$

Since only the relative amplitudes of the sidebands will enter into the analysis, we need only consider the term (normalized to unity power)

$$f'(t) = \sqrt{2} S(t)\, d(t) \sin \omega_{os} t \tag{3}$$

Expanding the square-wave subcarrier $S(t)$ in a Fourier series and substituting into (3) yields

$$f'(t) = \sqrt{2}\, d(t) \left[\sum_{n=1}^{\infty} a_n \cos n\omega_{sc} t \right] \sin \omega_{os} t$$

where the reference is chosen so $S(t)$ is even,

$$a_n = \begin{cases} \dfrac{4}{\pi(2n-1)} & n \text{ odd} \\ 0 & n \text{ even} \end{cases} \tag{4}$$

and $\omega_{sc} = 2\pi f_{sc}$ = fundamental frequency of the square-wave subcarrier.

In any actual system, bandwidth limitations will limit the number of square-wave subcarrier harmonics. Thus the waveform of concern is

$$f(t) = \sqrt{2}\, d(t) \left[\sum_{n=1}^{N} a_n \cos n\omega_{sc} t \right] \sin \omega_{os} t \tag{5}$$

Expanding (5) yields

$$f(t) = \frac{d(t)}{\sqrt{2}} \sum_{n=1}^{N} [\underbrace{a_n \sin(\omega_{os} + n\omega_{sc})t}_{n\text{th upper sideband}} + \underbrace{a_n \sin(\omega_{os} - n\omega_{sc})t}_{n\text{th lower sideband}}] \tag{6}$$

Consider the nth upper sideband

$$nSB = \frac{d(t)\, a_n}{\sqrt{2}} \sin(\omega_{os} + n\omega_{sc})t$$

and suppose that it is shifted in phase by θ_n^U due to the signal processing system. If the data $d(t)$ is at a low frequency, the spectrum it produces about $\omega_{os} + n\omega_{sc}$ will be narrow band. This means that the spectral components of $d(t)$ are all shifted in phase by approximately θ_n^U. Thus, after the phase shift, the nth upper sideband may be expressed as

$$nSB' = \frac{d(t)\, a_n}{\sqrt{2}} \sin[(\omega_{os} + n\omega_{sc})t + \theta_n^U]$$

Following this procedure for each sideband of (6) yields

$$f''(t) = \frac{d(t)}{\sqrt{2}} \sum_{n=1}^{N} \{a_n \sin[(\omega_{os} + n\omega_{sc})t + \theta_n^U] + a_n \sin[(\omega_{os} - n\omega_{sc})t + \theta_n^L]\} \tag{7}$$

Now suppose a perfect carrier and subcarrier reference (with the exception of the θ_n^U and θ_n^L) is available. Multiplying (7) by a unity power carrier and passing the result through a low-pass filter yields

$$[f''(t)\sqrt{2} \sin \omega_{os} t]_{L.P.} = \frac{d(t)}{2} \sum_{n=1}^{N} [a_n \cos(n\omega_{sc} t + \theta_n^U) + a_n \cos(n\omega_{sc} t - \theta_n^L)] \tag{8}$$

Multiplying by a perfect square-wave subcarrier, then multiplying the result by $d(t)$ and averaging over a symbol time of $d(t)$ yields the effective power available (relative to unity)

$$S = \frac{1}{4} \sum_{n=1}^{N} \{a_n^2 \cos\theta_n^U + a_n^2 \cos\theta_n^L\} \tag{9}$$

Note that θ_n^U and θ_n^L are thus considered deviations of the phase of the nth upper and lower sidebands, respectively, from a linear relationship (discussed further in the example of the next section) centered at ω_{os}. Any linear phase shift would contribute only a time delay and would be taken care of by shifting the phase of the reference square-wave subcarrier. If $\theta_n^U = \theta_n^L = 0$ for $n = 1, \cdots, N$, (9) reduces to

$$S = \frac{1}{2} \sum_{n=1}^{N} a_n^2 \tag{10}$$

which is of course the relative power available if all subcarrier sidebands above the Nth are completely filtered out.

The degradation of signal power in dB is given by

$$\rho = 10 \log_{10} S \tag{11}$$

III. Example—Single-Sideband Downconverter Loss Due to Phase Shift

A plot of phase shift vs frequency for a single-sideband downconverter used for open-loop receiver recording is shown in Fig. 1. Suppose system filtering limits the subcarrier harmonics to 5 with a subcarrier fundamental $f_{sc} = 32.8$ kHz and a data rate of 64 symbols/sec. Consider an offset frequency $f_{os} = \omega_{os}/2\pi$ of 174 kHz. A straight line must be drawn intersecting the curve at this frequency to form the reference for measurement of θ_n^U and θ_n^L, $n = 1, \cdots, N$. The line should be drawn to correspond to the delay introduced by the subcarrier demodulator assembly subcarrier tracking loop. Since this delay will vary in a complicated way depending on the specific phase distortion, only an approximate lower bound to the degradation in signal power will be attempted. This can be done by adjusting the slope of the line intersecting the curve at f_{os} until the degradation appears to be minimized. The values of θ_n^U and θ_n^U for $n = 1, 3, 5$ (even harmonics are absent anyway) are given in Table 1, using straight line references A, B, and C.

Substituting the values of Table 1 into (9) yields

$$\rho_A = -0.48 \text{ dB}$$
$$\rho_B = -0.45 \text{ dB} \qquad (12)$$
$$\rho_C = -0.45 \text{ dB}$$

where ρ_A is the degradation using reference line A, etc. Actual measurements at CTA 21 yielded a downconverter/upconverter degradation of 0.7 dB.

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Table 1. θ_n^L and θ_n^U for n = 1, 3, 5 for reference lines A, B, and C of Fig. 1

	A, deg	B, deg	C, deg
θ_5^L	138	124	97
θ_3^L	32	24	8
θ_1^L	6	3	−3
θ_1^U	0	44	8
θ_3^U	−2	8	24
θ_5^U	2	17	44

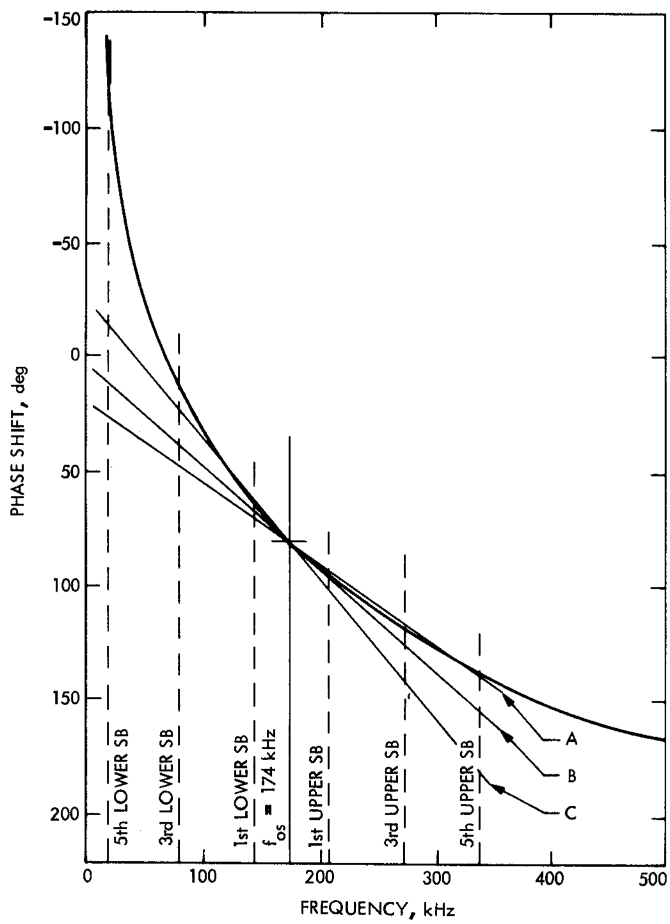

Fig. 1. Single-sideband downconverter phase response