A New Algorithm for Predicting the Apparent Polarization Angle of Linearly Polarized Spacecraft

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With the advent of nonecliptic spacecraft orbits and the inclusion of a NASA X-Y mounted antenna within the DSN it has become apparent that the current polarization angle prediction formulas may be insufficient for future needs. This article presents a new formulation for predicting the polarization angle which properly accommodates these new features in a concise, straightforward form.

I. Introduction

Although the Deep Space Stations (DSSs) are designed to receive a signal which is right circularly polarized, it is sometimes desirable to have the spacecraft transmit a linearly polarized signal (e.g., to study the Faraday rotation effect). While the DSS can track such spacecraft, the resultant polarization mismatch causes a loss of about one half of the available signal power. To prevent this loss of signal power, a rectangular waveguide is so constructed that when it is oriented at a forty-five degree angle to the linearly polarized input signal, the output signal is right circularly polarized with (theoretically at least) no loss of signal power. This waveguide, analogous to the "quarter wave plate" used in optics, is known as a polarizer.

To gain the full advantage of this effect, it is crucial to orient the polarizer at the proper angle with respect to the input signal. Indeed, if the error in the orientation approaches ninety degrees, the output will be left circularly polarized causing a loss of virtually all of the available signal power! It is, therefore, important to accurately predict the required polarizer setting as measured with respect to the microwave feed at each DSS. This is currently done by the polarization angle prediction program using the method developed by Stelzreid in Ref. 1.

At the time the current method was developed, only azimuth-elevation (az-el) and hour angle-declination (HA-dec) antenna mounts were used within the DSN. Furthermore, all of the spacecraft that the DSN commonly tracked had orbits that were virtually coincident with the ecliptic. With the advent of nonecliptic spacecraft trajectories and the inclusion of a NASA X-Y mounted antenna within the DSN, it became apparent that a new method for predicting the polarization angle was needed.

II. Derivation of the New Formulation

At this point it is useful to introduce a new term: the antenna horizon. This is the great circle on the celestial sphere that lies in the X-Y plane of the local Cartesian coordinate frame. The local coordinate frame is carefully chosen, according to the type of antenna mount in use, as follows:

(1) *Az-el mount.* For an az-el mount, the standard coordinate frame is chosen. In this frame, the Z-axis passes through the zenith; the Y-axis is perpendicular to the Z-axis and intersects the local meridian circle. Finally, the X-axis is perpendicular to both the Y and Z axes and satisfies the right-hand rule.

(2) *HA-Dec mount.* To develop the coordinate frame for a HA-dec mount, the frame for an az-el mount is constructed and then rotated about the X-axis so as to cause the Z-axis to pass through the North Celestial Pole.

(3) *NASA X-Y mount.* To develop the frame for the NASA X-Y mount, once again the frame for an az-el mount is constructed. This is then continually rotated about the X-axis so that the spacecraft always lies in the X-Y plane of the coordinate frame (i.e., the Z component of the station-spacecraft vector is always zero in this frame).

Now consider the (antenna position dependent) great circle that is perpendicular to the antenna horizon and contains the point at which the antenna is pointing. As the antenna tracks a moving object on the celestial sphere, this circle moves with the antenna and maintains a constant orientation with respect to the microwave feed. This circle is the reference against which the polarizer angle is measured; a polarizer set to convert a signal that is linearly polarized parallel to the plane containing this circle is said to be set at zero degrees.

Referring to Fig. 1, A is the reference circle at some moment in time; B is the great circle perpendicular to the spacecraft's orbit and passing through its position on the celestial sphere. If the signal is polarized perpendicular to the orbit (as is usually the case), the polarization angle measured at the station is the angle ρ at which the circles A and B intersect.

By inspection of Fig. 1,

$$\rho = \pi/2 - \tau$$

and from the cosine law for spherical triangles

$$\begin{aligned}\tau &= \cos^{-1}(-\cos\phi\cos\pi/2 + \sin\phi\sin\pi/2\cos\widehat{QR}) \\ &= \cos^{-1}(\sin\phi\cos\widehat{QR}) \\ &= \cos^{-1}(\sin(\pi-\phi)\cos\widehat{QR}) \\ &= \cos^{-1}(\sin\xi\cos\widehat{QR})\end{aligned}$$

Defining

$$\beta = \widehat{Q0'} \qquad \text{and}$$
$$\sigma = \widehat{0'R}$$

gives

$$\widehat{QR} = \beta + \sigma$$

and

$$\tau = \cos^{-1}(\sin\xi\cos(\beta+\sigma)).$$

Since $0'$ is the point where the X-axis intersects the equator,

$$\sigma = \begin{cases} \pi/2 - \text{HA}, & \text{if HA/Dec mount} \\ 3\pi/2 - \text{Az}, & \text{if Az/El mount} \\ \pi/2 + \text{Y}, & \text{If NASA X-Y mount} \end{cases}$$

From the sine law

$$\beta = \sin^{-1}(\sin\widehat{00'}\sin\iota/\sin\xi)$$

and by the cosine law

$$\xi = \cos^{-1}(-\cos\iota\cos\phi' + \sin\iota\sin\phi'\cos 00')$$

where

$$\phi' = \begin{cases} \text{zero} & \text{if HA/Dec mount} \\ \text{station colatitude} & \text{if Az/El mount} \\ \text{X-station latitude} & \text{if NASA X-Y mount} \end{cases}$$

By inspection

$$\widehat{00'} = \text{Sidereal Time (S.T.)} - \Omega - \pi/2$$

where Ω is the right ascension of the orbit's ascending node on the equator.

Then, finally,

$$\xi = \cos^{-1}(-\cos\iota\cos\phi' + \sin\iota\sin\phi'\sin(\mathrm{S.T.} - \Omega))$$

$$\beta = \sin^{-1}(-\cos(\mathrm{S.T.} - \Omega)\sin\iota/\sin\xi)$$

and

$$\rho = \sin^{-1}(\sin\xi\cos(\beta + \sigma))$$

with ϕ' and σ found from Table 1.

For spacecraft whose orbits are coincident with the ecliptic,

$$\Omega = 0 \qquad \text{and}$$

$$\iota = \epsilon$$

where ϵ is the obliquity of the ecliptic.

The above formulas then become

$$\xi = \cos^{-1}(-\cos\epsilon\cos\phi' + \sin\epsilon\sin\phi'\sin\mathrm{S.T.})$$

$$\beta = \sin^{-1}(-\cos\mathrm{S.T.}\sin\epsilon/\sin\xi)$$

$$\rho = \sin^{-1}(\sin\xi\cos(\beta + \sigma)).$$

It should be noted that although Ω is not one of the standard orbital parameters, it is easily computed.

Referring to Fig. 2,

$$\Omega = \sin^{-1}(\sin\iota_e\sin\Omega_e/\sin(\pi - \iota))$$

$$= \sin^{-1}(\sin\iota_e\sin\Omega_e/\sin\iota)$$

and

$$\iota = \cos^{-1}(\cos\iota_e\cos\epsilon - \sin\iota_e\sin\epsilon\cos\Omega_e)$$

where

Ω_e is the longitude of the ascending node

ι_e is the inclination of the orbit to the ecliptic.

III. Conclusion

By comparing this result with the method currently in use (Ref. 1), it can be seen that the new formulation is functionally simpler. Furthermore, it will accommodate nonecliptic spacecraft and the new (to the DSN) NASA X-Y mount. Finally, since the formulas used are the same for all DSN antenna mounts, the development of a well-structured computer program for predicting the polarization angle will be facilitated.

Reference

1. Stelzreid, C. T., and Abreu, A., "Received Signal Polarization of the Pioneer VI Spacecraft During the 1968 Superior Conjunction," Space Programs Summary 37-59, Vol. II, Jet Propulsion Laboratory, Pasadena, Calif., Sept. 30, 1969.

Table 1. The σ and ϕ' for DSN antenna mounts

Mount type	σ	ϕ'
HA/Dec	$\pi/2 - \mathrm{HA}$	0
Az/El	$3\pi/2 - \mathrm{Az}$	Station colatitude
NASA X-Y	$\pi/2 + \mathrm{Y}$	X − station latitude

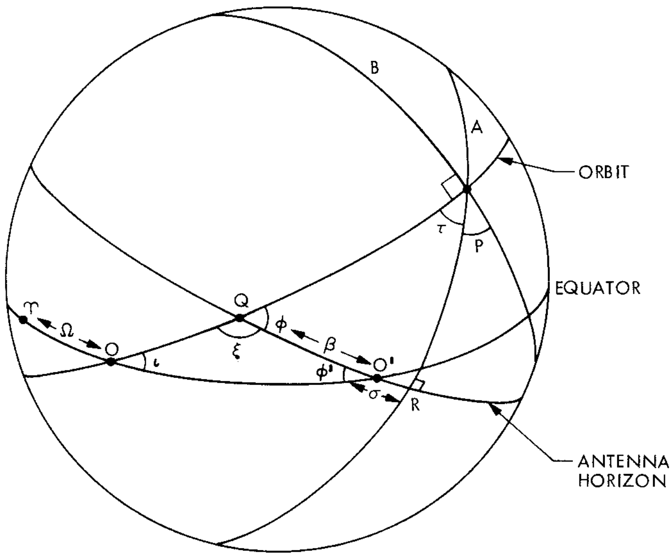

Fig. 1. Relation of polarization angle to antenna horizon and spacecraft orbit

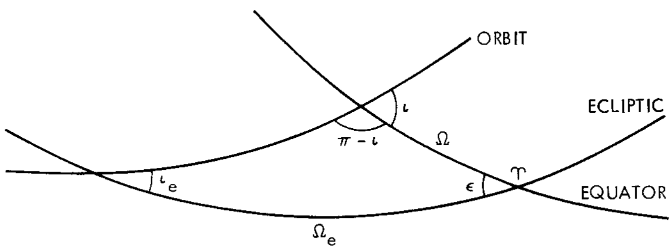

Fig. 2. Ecliptic and equatorial intersections with spacecraft orbit