Computation of Spacecraft Signal Raypath Trajectories Relative to the Sun

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An updated double-precision computer program has been developed to determine the trajectory of a spacecraft telemetry signal raypath relative to the sun. Using trajectory information available on DPTRAJ save tapes, the program efficiently and accurately computes the desired raypath trajectory and delivers the results in the form of plots, punched cards, and a tabular listing.

I. Introduction

When a spacecraft undergoes a superior conjunction with respect to some body (e.g., the sun or a planet), the signal carrier is affected. In order to predict the occurrence and magnitude of these effects, and to utilize the potential information they may provide, it is essential to determine the trajectory of the signal raypath with respect to the conjunction body. This trajectory information was formerly computed by the program CTS 41 (Ref. 1, pp. 16–19). However, since the program was originally written in 1968, the output of the DPTRAJ program has been expanded to provide data that can be used more efficiently and accurately than is done in CTS 41. Hence, an updated double-precision program, CTS 41B, has been developed.

While the program presented here is specifically designed for solar conjunctions, it could, with slight modification, be applied to planetary conjunctions as well. It has been shown theoretically (Ref. 1, p. 12) that the deviation of an S-band signal trajectory from the geometrical line of sight due to refractive effects in the solar corona is negligible, and experimental observations during previous solar conjunctions have verified that no large refractive effects occur. Hence, no attempt to compensate for refraction has been included, and it should be explicitly understood that CTS 41B refers to the geometrical line of sight rather than the actual signal raypath.

II. Input

The basic input data are obtained in the form of a save tape from the DPTRAJ program, which provides the most accurate possible trajectory and ephemeris information. A save tape is generated by initiating a DPTRAJ run in which the user specifies the data frequency and any conjunction bodies that are desired. The geocentric and heliocentric blocks of data are automatically provided for every DPTRAJ run. For solar conjunctions, the only data required are the Earth–probe range and the Earth–sun range from the geocentric coordinate block and the coordinates of the probe in the heliocentric coordinate block, typically at one-day intervals.

A. Coordinates and Equations

The basic coordinate system to be used is the heliocentric system, defined as follows:

$\hat{X} \equiv$ direction from sun to Earth

$\hat{Z} \equiv$ normal to ecliptic, positive to north

Then the X-Y plane is the ecliptic plane, and the coordinates of Earth and the probe are

$$\text{Earth} = (X_E, 0, 0)$$

$$\text{probe} = (X_P, Y_P, Z_P)$$

where

X_E = Earth–sun range

(X_P, Y_P, Z_P) = coordinates of probe in the heliocentric block

The geometry is illustrated in Fig. 1. Point A $(0, Y_A, Z_A)$ is the point of intersection of the line of sight with the Y-Z plane. In looking from Earth toward the sun, the apparent position of the spacecraft is given by the coordinates of point A. However, the point of closest approach of the signal raypath to the sun is point B (X_B, Y_B, Z_B), and the raypath offset from the sun is the distance $R = \sqrt{X_B^2 + Y_B^2 + Z_B^2}$ from the sun to point B. In most conjunction experiments, the sun–Earth–probe angle (SEP) will be so small that points A and B virtually coincide.

The coordinates of point A can be computed very simply from similar triangles. In the X-Y plane, we thus obtain

$$Y_A = Y_P \frac{X_E}{X_E - X_P}$$

while in the X-Z plane, we find

$$Z_A = Z_P \frac{X_E}{X_E - X_P}$$

The raypath offset R can also be obtained from similar triangles in the sun–Earth probe plane, where

$$R = \frac{X_E \sqrt{Y_P^2 + Z_P^2}}{R_P}$$

where R_P is the Earth–probe range and is given in the block of geocentric coordinates on the DPTRAJ save tape.

Other quantities of interest include the distance from Earth to the point of closest approach R_{EB} and the distance from the probe to the point of closest approach R_{BP}. These distances are

$$R_{EB} = X_E^2 - R^2$$

$$R_{BP} = R_P - R_{EB}$$

A complete listing of the program is provided in the Appendix.

The symbols used for variables in this description correspond to the binary-coded decimal (BCD) header record names on the DPTRAJ save tape as follows:

$R_P \sim$ REARPR, geocentric block, record 12

$X_E \sim$ REARSU, geocentric block, record 15

$X_P \sim$ XSCSEL, heliocentric block, record 28

$Y_P \sim$ YSCSEL, heliocentric block, record 29

$Z_P \sim$ ZSCSEL, heliocentric block, record 30

B. Output

The values of Y_A, Z_A, R, R_{EB}, and R_{BP} as functions of time are listed in tabular form on the printout at the same frequency as the original DPTRAJ save tape data. In the solar version of the program, the apparent angle between the solar equatorial plane and the ecliptic plane, as seen from Earth, PHIEQ, is also tabulated as a function of time in order to facilitate correlations with standard observations of solar phenomena.

A plot of the coordinates (Y_A, Z_A) shows the trajectory of the line of sight relative to the sun. Figure 2 is an example for the superior conjunction of Mariner 10 in 1974. The trajectory points are plotted at one-day intervals and in units of solar radii. They may be identified by referring to the dates of the first and last points (i.e., day number 146 to 174), which are listed to the lower left of the plot, or by referring to the tabular listing of (Y_A, Z_A) as functions of time. The program automatically supplies three such plots to provide a range of scale and resolution.

A plot of R as a function of time (Fig. 3) shows how the raypath offset varies through the conjunction. The values of Y_A, Z_A, and R as functions of time are also punched on cards.

Reference

1. Stelzried, C. T., *A Faraday Rotation Measurement of a 13-cm Signal in the Solar Corona*, Technical Report 32-1401, Jet Propulsion Laboratory, Pasadena, Calif., July 15, 1970.

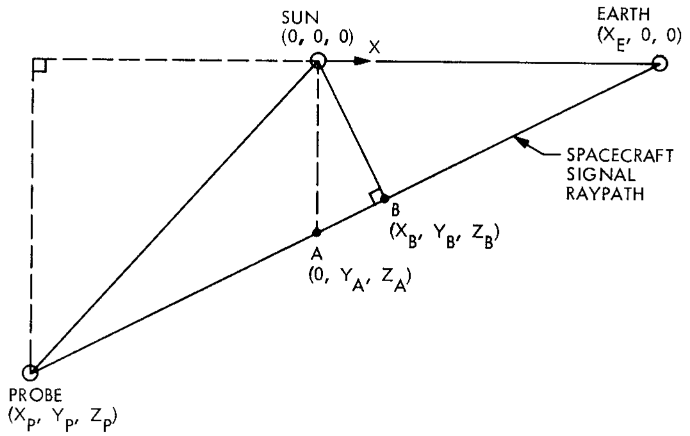

Fig. 1. Diagram of the sun-Earth-probe geometry showing the spacecraft signal raypath

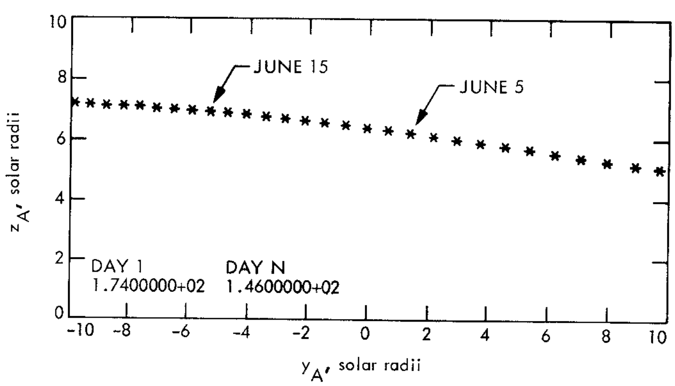

Fig. 2. Mariner 10 probe raypath trajectory shown with increments of one-day relative to a fixed sun-Earth line, 1974

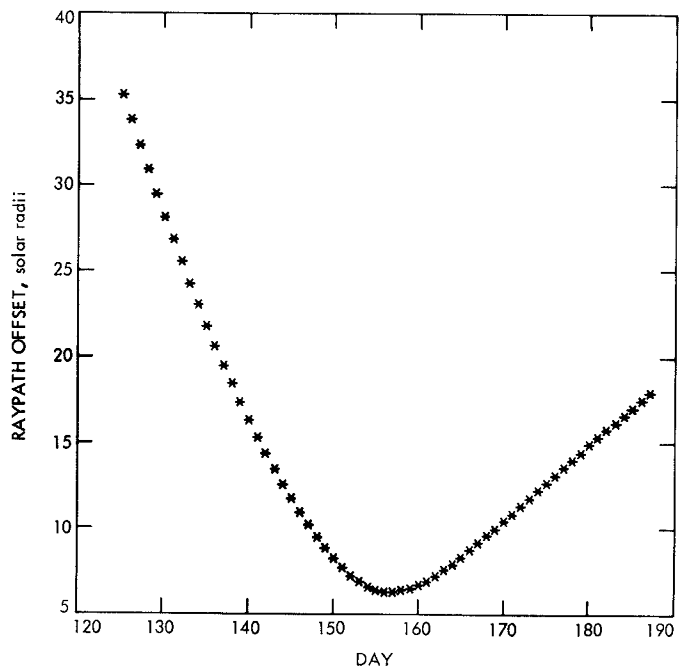

Fig. 3. Mariner 10 probe raypath offset vs 1974 day of year

Appendix

Spacecraft Signal Raypath Trajectory Computer Program, CTS 41B Listing

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TJC*AA.CTS41B
          INTEGER NDAY(366),IREC(750)
C*****START=STACTB,ABS=ACTSB
          REAL DDA(366)
          DOUBLE PRECISION REC(375),X(366),Y(366),Z(366),
         1RE(366),RS,P,A,R(366),RPR(366),REB(366),RBP(366),
         1SEP(366),SPE(366),DSPE(366),RTD
          EQUIVALENCE (IREC(1),REC(1))
          DIMENSION TI(2),X1(366),Z1(366),R1(366),D(366),P(366)
          DATA /RS/.69598D6/TPI/6.2831853/PI/3.14159265/
          DATA /RTD/57.2957795131D0/
          READ(5,4000)XXXX
     4000 FORMAT(A6)
C*****XXXX IS A DUMMY AND NOT USED
          READ(5,4005)NSTART,NFIN,NYEAR
     4005 FORMAT()
          READ(5,4000)XXXX
          READ(5,4000)XXXX
          READ(5,4000)XXXX
          READ(5,4005)NH,NP,NT
          READ(5,4010)TI
     4010 FORMAT(2A6)
          IF(NP.EQ.1)CALL PLOT
          WRITE(6,1015)TI
     1015 FORMAT(1H1,//,1X,'PROBE PATH (',2A6,1H))
          IF(NT.NE.2)GO TO 3
          WRITE(6,4015)
     4015 FORMAT('0DAY',9X,'XSCSEL',9X,'YSCSEL',9X,'ZSCSEL',9X,'REARSU')

          WRITE(6,4020)
     4020 FORMAT(4X,4(13X,'KM'))
        3 REWIND 4
          READ(4)NWRDS
          READ(4)NWRDS
          MSAVE=0
          N=0
        5 READ(4,END=1010)NWRDS,(IREC(J),J=1,NWRDS)
          M1=IREC(5)/1000000
          M2=IREC(5)-M1*1000000
          M3=M2/10000
          M2=M2-M3*10000
          M=JD(M1,M3,M2)-JD(M1,1,1)+1
          IF(M1.LT.NYEAR)GO TO 5
          IF(M.LT.NSTART.AND.N.EQ.0)GO TO 5
          IF(M.EQ.MSAVE)GO TO 5
          MSAVE=M
          N=N+1
          NDAY(N)=M
          DDA(N)=NDAY(1)+N-1
          J1=JD(M1,3,6)
          J2=JD(M1,9,8)
          JJ=JD(M1,M3,M2)
          IF(M.GE.66.AND.M.LE.251)P(N)=7.25*SIN(PI*(J1-JJ)/186.)
          IF(M.LT.66.OR.M.GT.251)P(N)=7.25*SIN(PI*(JJ-J2)/179.)
          X(N)=REC(28)
          Y(N)=REC(29)
          Z(N)=REC(30)
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      RPR(N)=REC(12)
      RE(N)=REC(15)
      IF(NT.EQ.2)WRITE(6,1005)NDAY(N),X(N),Y(N),Z(N),RE(N)
 1005 FORMAT(I4,4D15.8)
      IF(M.NE.NFIN)GO TO 5
 1010 IF(NT.NE.2)GO TO 4030
      WRITE(6,1015)TI
      WRITE(6,1020)
 1020 FORMAT('0DAY',9X,'REARPR',12X,'REB',12X,'RBP',10X,'PHIEQ')
      WRITE(6,4025)
 4025 FORMAT(4X,3(13X,'KM'),12X,'DEG')
 4030 DO 10 I=1,N
      R(I)=RE(I)*DSQRT(Y(I)**2+Z(I)**2)/RPR(I)
      A=RE(I)/(RE(I)-X(I))
      X(I)=Y(I)*A
      X1(I)=X(I)/RS
      Z(I)=Z(I)*A
      Z1(I)=Z(I)/RS
      R1(I)=R(I)/RS
      D(I)=NDAY(I)
      REB(I)=DSQRT(RE(I)**2-R(I)**2)
      RBP(I)=RPR(I)-REB(I)
      SEP(I)=DASIN(R(I)/RE(I))*RTD
      SPE(I)=DATAN(R(I)/RBP(I))*RTD
      IF(I.GE.3)DSPE(I-1)=(SPE(I)-SPE(I-2))/2.D0
      IF(NH.EQ.1)PUNCH 4055,NDAY(I),X1(I),Z1(I),R1(I)
 4055 FORMAT(I3,3F15.8)
   10 IF(NT.EQ.2)WRITE(6,1005)NDAY(I),RPR(I),REB(I),RBP(I),R(I)
      DSPE(1)=0
      DSPE(N)=0
      IF(NT.NE.2)GO TO 1023
      WRITE(6,1015)TI
      WRITE(6,4060)
 4060 FORMAT('0DAY',12X,'SEP',12X,'SPE',11X,'DSPE')
      WRITE(6,4065)
 4065 FORMAT(4X,2(12X,'DEG'),8X,7HDEG/DAY)
      DO 11 I=1,N
   11 WRITE(6,1005)NDAY(I),SEP(I),SPE(I),DSPE(I)
 1023 IF(NT.EQ.0)GO TO 25
      WRITE(6,1015)TI
      WRITE(6,1025)
 1025 FORMAT('0DAY',13X,'YA',13X,'ZA',14X,'R')
      WRITE(6,1027)
 1027 FORMAT(4X,3(13X,'KM'))
      DO 12 I=1,N
   12 WRITE(6,1005)NDAY(I),X(I),Z(I),R(I)
      WRITE(6,1015)TI
      WRITE(6,1025)
      WRITE(6,1029)
 1029 FORMAT(4X,3(4X,'SOLAR RADII'))
      DO 15 I=1,N
   15 WRITE(6,1030)NDAY(I),X1(I),Z1(I),R1(I)
 1030 FORMAT(I4,4E15.8)
   25 IF(NP.NE.1)STOP
      A3=TI(1)
      A4=TI(2)
      CALL EPLOT(X1,Z1,N,A3,A4,D)
      CALL EPLT(DDA,R1,N)
      STOP
      END
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