Toward a Mathematical Model of Solar Radiation for Engineering Analysis of Solar Energy Systems

C. L. Hamilton
TDA Planning Office

M. S. Reid
Communications Elements Research Section

This report presents some first thoughts on mathematical models of solar radiation suitable for use in engineering analysis of solar energy systems. Included is a discussion of the currently most-used insolation model and what improvements might be made in it to better suit it for use in designing energy systems. An approach to constructing an upgraded model is sketched.

I. Introduction

After many years of study and experimentation on the subject, and in the face of continuing uncertainty over supply and price of conventional energy sources, serious consideration is now being given to the question of utilizing solar-powered energy systems on a relatively large scale. A necessary precursor to construction of well-designed, efficient, and economically viable solar energy systems is engineering analysis not only of the systems themselves but also of the solar radiation that will drive them. This report presents some first thoughts on mathematical models of insolation characteristics suitable for use in analysis of solar energy systems. Included is a discussion of the currently most-used insolation model and what improvements might be made in it to better suit it for use in designing solar energy systems. An approach to constructing an upgraded model is sketched.

II. Where an Insolation Model Fits

Before construction of any energy system is undertaken, there must be reasonable assurance that it will meet the demand it was planned to satisfy, and that it will do so with a low enough life-cycle cost to make the project economically attractive. System performance models can be used first to judge design alternatives against each other and against criteria for performance and cost and then to alter the design of the most promising systems to improve performance and/or lower costs. The function of an insolation model can be better understood by looking at some of the essential features of a solar energy system model.

Solar-powered systems can cover a range of applications, from space and water heating in a single structure to central station generation of electricity, and can vary widely in complexity. A generalized solar–thermal system will be made up of solar collectors, possibly some sort of storage subsystem, and a subsystem to convert thermal energy to the desired form. Depending on the application, each of the subsystems might be quite simple or very complex; for illustrative purposes it will be sufficient to think in terms of the general groups. In a gross sense, the energy output of a solar-powered system is determined over a given time period by the amount of solar radiation collected by the system and the overall system efficiency. The efficiency with which the system operates depends in turn on characteristics of the included subsystems and the parameters on which their individual performances will depend.

Of the solar energy hitting a collector, a fraction, depending on the sun's position relative to the collector surface and the collector's own geometry and optical properties, fails to get to the absorbing surface. A portion of the absorbed energy is lost via heat leaks and reradiation; the amount is determined by collector properties and the temperature at which it operates along with other factors like ambient temperature and perhaps wind speed. The remaining energy is transferred from the collector as sensible heat in a fluid at a temperature depending on fluid characteristics, the temperature of fluid entering the collector, and the collector temperature. The temperature of fluid entering the collector depends on how much heat is removed from it by other subsystems, such as that devoted to energy conversion. The amount of heat required by the conversion subsystem is governed by the load it is to satisfy, by its own internal properties, by the temperature of the heat supplied to it, and by the temperature of the sink to which it rejects heat (if it must). The characteristics of the storage subsystem exert an effect on both collector and conversion components. All of these influences are reflected in a system model that is made up of an interrelated set of mathematical models representing the performance of each component. Because each piece depends strongly on factors that vary significantly with time, the resulting model should reflect the important dynamics. Inaccuracies associated with the various component models will propagate and compound during analysis of the system, of course. This means that each of them must represent the performance of the associated subsystem with greater accuracy than is required of the whole system model.

Given that a system model can be developed that will allow calculation of system output as a function of insolation and other weather parameters, where are we then? A viable energy system, solar or otherwise, must be capable of supplying the output expected of it over the course of its useful lifetime. Conventional systems can be designed with the appropriate capacity and then provided with the amount of fuel necessary to do their job. Fuel for a solar-powered system, sunlight, is completely outside the control of man. Design of a solar energy system—the relative sizing and performance specifications of components—must be done then on the basis not only of its intended output but also on the basis of the energy input that can be expected during its lifetime. The question arises—how does one supply appropriate values for the prime driving function, solar radiation, to allow an estimate to be made of system performance over the span of 10 to 15 future years? That performance analysis must be accurate enough to permit design of a system that meets output criteria and cost criteria in a situation where compensation for even moderate uncertainty by oversizing components can be prohibitively costly.

Experimental measurements of solar radiation intensity could be used to drive a system model. Such measurements are scarce, limited to a few locations, and more often than not of questionable accuracy. Only rarely, in fact, have the needed aspects of incident radiation been measured. Empirical data suffer from a more fundamental deficiency, however. Using radiation measurements as input for a well-conceived system analysis may give a good estimate of how the system would have performed during the time the data were taken, but that estimate would only be good for the period in which the measurements were recorded. To arrive at the desired system performance it would be necessary, in addition, to simulate the system's behavior over that whole period, a procedure that could be unnecessarily costly and time-consuming.

What is needed is a representation of insolation characteristics that depicts those aspects of both its long-term and short-term behavior on which system performance depends, expressed in terms of a one-year description. That one-year description may never match insolation behavior for a particular measured year, but would be extrapolatable to match closely all important aspects of insolation integrated over a long time. The representation would, in short, be the output of a mathematical model describing solar radiation. Such a model, along with a suitable data base, would allow average or representative future behavior to be predicted, along with estimates of the frequency and magnitude of deviations from that average. As noted above, the accuracy of the outputs from this model must be greater than the accuracy required of the outputs of the composite system model.

III. The ASHRAE Model

An insolation model currently enjoying wide use is that developed by the American Society of Heating, Refrigeration and Air-Conditioning Engineers (ASHRAE). It was not created for the purpose of analyzing performance in solar-powered energy systems. Rather, its intended application was in estimating heat load on buildings for the purpose of specifying heating-cooling systems for installation there. The form of the ASHRAE model was dictated by its purpose—this is the case with all mathematical models. In general, the effect of insolation on cooling system requirements is felt on clear sunny days; they specify the conditions with which a cooling system must cope. Only clear days are modeled by the ASHRAE equations.

Some discussion of what happens to sunlight on its way to Earth's surface will aid in dissecting the ASHRAE model. Energy emitted by a point on the sun arrives at the edge of our atmosphere in parallel rays. Its intensity at that point depends on Earth's distance from the sun, and varies slightly with time of year. As the solar radiation passes through the atmosphere, its direct normal intensity (intensity on the plane perpendicular to the ray bundle's direction) is attenuated. Some of the energy is absorbed by molecules of atmospheric constituents and some is figuratively knocked out of the bundle of parallel rays by molecular and particulate scattering. The degree of attenuation from these effects is a function of the distance the radiation has to traverse in the atmosphere, and the concentration of absorbing and scattering species contained there. The ASHRAE model uses the following equation to mimic these influences under clear day conditions:

$$I_{DN} = NAe^{-B/\sin\beta} \tag{1}$$

I_{DN} is the direct normal intensity of radiation at Earth's surface. N is a clearness number that varies up or down slightly from a value of 1, depending on geographical location and season, and reflects the inevitable variation in clarity of what is considered to be a clear day. The parameter A is classified as apparent radiation at atmosphere's edge; it has a different value for each month and includes the combined influence of the sun's distance from Earth and some atmospheric attenuation. The value of B, the atmospheric extinction coefficient, also varies monthly, reflecting the concentration of absorbing and scattering species. Sets of values for both A and B were determined by empirical curve fitting. That is, they were the values that, when inserted into Eq. (1), produced values of I_{DN} that best matched data actually measured over a long period at a site with a defined clearness number of 1. Finally, $1/\sin\beta$ (where β is the sun's elevation angle) approximates the distance that the parallel bundle of rays travels in the atmosphere, which varies with time of day and time of year.

A unit area of surface at ground level will receive direct radiation, I_D at a rate corresponding to the direct normal intensity modified by the cosine of α, the angle between the direction of incoming rays and the direction perpendicular to the surface.

$$I_D = I_{DN} \cos \alpha \tag{2}$$

In addition to the energy arriving in a direct line from the sun, the surface in question will receive radiation from two other sources. Some of the scattered rays will, after bouncing about in the atmosphere, reach ground level and the receiving surface, coming from all directions. Light that has been reflected from the surroundings will also be picked up. These two effects are treated in the terms

$$I_{DS} = CI_{DN}F_{SS} \tag{3}$$

and

$$I_{DG} = rI_G F_{SG} \tag{4}$$

where I_{DS} is intensity of diffuse radiation coming from the sky, C is an empirically determined factor showing monthly variation, and F_{SS} is a geometrical factor relating to the amount of sky in a position to radiate to the surface. I_{DG} represents radiation reflected onto the surface from the ground around it; I_G is the total radiation intensity falling on the ground (determined as for any surface); r is ground reflectance, and F_{SG} is another geometrical factor. Analogous terms dealing with reflection from other surfaces might be required if the surroundings warranted. To sum up, the total radiation intensity received by a surface near ground level, according to the ASHRAE model, is expressed as

$$I_T = I_{DN} \cos \alpha + CI_{DN}F_{SS} + rI_G F_{SG} \tag{5}$$

When the quantities included are properly evaluated, this model provides good approximations for total radiation intensity as a function of time during clear weather. This is one aspect of several needed for accurate analysis of solar system performance.

IV. Interaction of Solar Radiation and Collectors

There are many ways of collecting solar energy. As far as their dependence on the characteristics of insolation is concerned, they may be classified in terms of the degree of concentration they involve. While solar energy is intrinsically of high quality, it arrives at Earth's surface widely distributed and must be reconcentrated to be put to useful work. Flat plate collectors use large areas of absorbing material to intercept the radiation as it falls unaltered on the collecting surface. The resulting energy is removed as heat by a fluid circulating over the surface. A flat plate collector can use all the radiation that hits it, but at high operating temperatures heat losses from the large area of hot surface limit its efficiency. Collector designs that concentrate the radiation before it strikes the absorbing surface seek to reduce these heat losses by cutting down on the surface area of hot material, allowing higher efficiency. In effect, the concentrating collectors focus the light on a small absorbing area, from which heat is removed by a circulating fluid. Concentration ratio is a measure of the area over which radiation is captured relative to the area on which it is focused; the higher the concentration ratio the more precise focusing is required.

Only direct radiation is useful to a concentrating collector. Diffuse radiation cannot be focused. The ASHRAE model allows estimation of direct radiation on clear days, where it comprises a large fraction of the total incident light. Energy systems must also work on days that are not entirely clear. Then the proportion of diffuse radiation is much larger, and concentrating collectors will experience their own degradation of output. In comparing systems, one must decide whether large areas of possibly less expensive nonfocusing collectors, with high heat losses but capable of using all the components of incident light, are more or less effective than perhaps smaller arrays of more expensive focusing collectors that will attain high temperatures more efficiently but can't use all the light. This comparison cannot be made without knowing the availability of both direct and diffuse radiation as a function of time for all kinds of days.

The ASHRAE equations embody a semi-empirical, deterministic model. By dealing exclusively with one kind of day, a type that is practically eventless except for the rise and fall of the sun, they can be successfully applied. A requirement for dealing with all days demands a model with probabilistic components as well as deterministic ones. An ideal model will reflect the occurrence and density of clouds and haze, and will mimic their effects on both direct and indirect radiation. For general application to all collectors, another phenomenon must be considered. That is the circumsolar radiation. This is caused mainly by scattering of the sun's rays by Earth's atmosphere, and possibly also by refraction to a smaller extent, and is always present. On clear days, the effect is small and is limited to a narrow angular diameter about the sun's disk. On hazy days the turbidity of the atmosphere increases the circumsolar radiation at the expense of the direct component. It also increases the angular extent of circumsolar radiation, which then merges with the diffuse radiation. The result is that concentrating collectors cannot focus the sun's disk sharply. Loss from atmospheric defocusing of the sun's image becomes more severe as concentration ratio increases. Neither this effect nor the frequency and nature of unclearness can be modeled deterministically at this time. Random variables must be employed to estimate their influence.

Since the ASHRAE model performs well in predicting radiation on clear days, it forms a sensible starting point for first attempts at constructing a generalized model. We will concentrate on the terms described by Eqs. (1) and (3), regarding the clear-day expressions for direct and diffuse radiation as being a baseline condition that is modified by the random effect of the weather. Modifications would occur via the insertion of a pair of random variables (call them M and m), one in each equation.

$$I_{DN} = MNAe^{-B/\sin\beta} \tag{6}$$

$$I_{DS} = mCI_{DN}F_{SS} \tag{7}$$

Any terms for reflected radiation (Eq. 4) that might be required to model a situation will automatically be modified, since they would be derived from modified estimates of total radiation on the reflecting surface. On a clear day, both M and m would carry values of 1, and the original ASHRAE equations would stand. As "unclearness" increases, the value of M would vary on a short time scale —say, hourly. An additional variable could be inserted into Eq. (6) to model the circumsolar radiation, giving in the end

$$I_D = DMNAe^{-B/\sin\beta} \tag{8}$$

An estimate of how the variable D might depend on concentration ratio has been made, but must be verified. Equations (6) and (8) form a solar radiation model that should be much more suitable than either experimental measurements or the ASHRAE model for supporting engineering analysis of solar energy systems. The improved model will be more representative of solar behavior than a set of measurements, and will deal somehow with all the aspects of radiation that are important to a collector.

The question arises—where do values of M, m, and D come from? Their basis is a series of simultaneous measurements made as accurately as possible in one location over as long a period of time as practicable. The parameters collected would include total insolation intensity and direct radiation as measured by a set of devices with a number of different concentration ratios. Data would be taken at small time intervals, on the order of a few minutes. Diffuse radiation intensity could be derived by differencing measurements of total intensity and direct radiation determined without concentration. These data and Eqs. (6) and (8) would be used to calculate simultaneous values for the three random variables. From the calculated values, a joint probability density function for M and m would be determined, as well as a functional relationship between D and concentration ratio. The derived probability density functions are used, while model calculations are being carried out, to generate values of the random variables which will lead to a time series of calculated intensities with the same statistical properties as the original measurements.

For probability density functions to give the most representative results, they should be based on a very large number of measurements. It is often the case that their form can be determined from a more limited set of data, after which they can be upgraded by small adjustments in the parameters in which they are expressed as more and more data become available. Modification of the density functions for application to another location may be possible, using a limited number of insolation measurements at the new site and correlation with other weather data that might be more abundant. Verification of such a transfer, and establishment of the conditions under which it would be valid, would require careful measurements for comparison with predicted values.

A program for gathering meteorological and solar data using absolute calibration standards has been underway at the Goldstone Space Communications Complex since June 1974 (Refs. 1 and 2). These measurements provide an archive of solar data calibrated to the international standard. In addition, they provide a data base that has allowed a start on development of a solar model such as the one sketched, which will be the subject of a forthcoming publication. It must be kept in mind, however, that the approach to an accurate insolation model described in these paragraphs is only a beginning. This probabilistic model is still very much simplified, and future effort might profitably be spent investigating those factors, now determined empirically, to more precisely identify and separate their deterministic and probabilistic components.

References

1. Reid, M. S., Gardner, R. A., and Parham, O. B., "The Goldstone Solar Energy Instrumentation Project: Description, Instrumentation and Preliminary Results," in *The Deep Space Network Progress Report 42-26*, pp. 133-144, Jet Propulsion Laboratory, Pasadena, Calif., April 15, 1975.

2. Reid, M. S., and Gardner, R. A., "A Versatile Data Acquisition System for Gold-Stone," in *The Deep Space Network Progress Report 42-30*, pp. 132-143, Jet Propulsion Laboratory, Pasadena, Calif., Dec. 15, 1975.

NASA—JPL—Coml., L.A., Calif.