The Pioneer 11 1976 Solar Conjunction: A Unique Opportunity to Explore the Heliographic Latitudinal Variations of the Solar Corona

A. L. Berman, J. A. Wackley, S. T. Rockwell, and J. G. Yee
DSN Network Operations Section

The 1976 Pioneer 11 Solar Conjunction provided the opportunity to accumulate a substantial quantity of doppler noise data over a dynamic range of signal closest approach point heliographic latitudes. The observed doppler noise data were fit to the (previously developed) doppler noise model "ISED," and the deviations of the observed doppler noise data from the model were used to construct a (multiplicative) function to describe the effect of heliographic latitude (ϕ_s):

$$f(\phi_s) = 10^{-0.9(\phi_s/90\text{ deg})}$$

This expression was then incorporated (back) into the ISED model to produce a new doppler noise model — "ISEDB."

I. Introduction

In a previous report (Ref. 1), A. Berman and J. Wackley, after extensively analyzing the 1975 solar conjunctions of Pioneer 10, Pioneer 11, and Helios 1, concluded that observed doppler noise (rms phase jitter) was directly proportional to integrated signal path electron density. As a direct consequence of that hypothesis, it was possible to construct a geometrical model for observed doppler noise —ISED (Integrated Solar Electron Density)—as follows:

$$\text{ISED} = A_0\left[\frac{\beta}{(\sin\alpha)^{1.3}}\right] F(\alpha,\beta) + A_1\left[\frac{1}{(\sin\alpha)^5}\right]$$

$$F(\alpha,\beta) = 1 - 0.05\left\{\frac{(\beta - \frac{\pi}{2} + \alpha)^3 - (\alpha - \frac{\pi}{2})^3}{\beta}\right\} - 0.00275\left\{\frac{(\beta - \frac{\pi}{2} + \alpha)^5 - (\alpha - \frac{\pi}{2})^5}{\beta}\right\}$$

α = Sun–Earth–probe angle (SEP), radians

β = Earth–Sun–probe angle (ESP), radians

A best fit of this model to the combined Pioneer 10, Pioneer 11, and Helios 1 1975 solar conjunction doppler noise produced the following fit parameters:

$$A_0 = 9.65 \times 10^{-4}$$

$$A_1 = 5 \times 10^{-10}$$

Additional information concerning the development of the ISED model can be found in Refs. 2, 3, and 4.

If 1975 could be considered as a bountiful year for collection of solar conjunction data, then 1976 must be considered nothing short of a bonanza: six spacecraft undergoing a variety of solar conjunction phases, crowned by a Helios 2 triple occultation. On the face of it, the Pioneer 11 solar conjunction would definitely appear to be least interesting of the lot, as the SEP only reached a minimum of about 12 deg, which in turn results in only weak to moderate solar plasma effects on the spacecraft signal. However, during the period when the SEP was small enough to allow solar plasma effects to be seen in the doppler noise, i.e.,

$$\text{SEP} \lesssim 50 \text{ deg}$$

the heliographic latitude of the signal's closest approach point to the Sun varied slowly from approximately 10 deg to a maximum of approximately 78 deg, and then back to 10 deg again, thus allowing a large amount of noise data to be accumulated over a dynamic range of heliographic latitudes. Figure 1 presents the heliographic latitude of the Pioneer 11 signal closest approach point as a function of day of year (DOY), 1976. A similar curve for Pioneer 10 is included by way of comparison. At this point one is led to consider the following opportunity: why not fit the Pioneer 11 observed doppler noise with the ISED model, and then attempt to correlate the residuals ($\equiv 10 \log_{10} (N_A/N_P)$) with the heliographic latitude? If correlation is evident as expected, the variation with heliographic latitude can be modeled and further incorporated (back) into the ISED model. In this process the Pioneer 11 observed doppler noise formed the bulk of the data base; however, a (much) smaller amount of data was available from the Pioneer 10 (1976) solar conjunction, and hence was additionally included.

II. Calculation of Signal Closest Approach Point Heliographic Latitude

The Sun–Earth–spacecraft geometry referenced to the ecliptic plane is shown in Fig. 2, with the appropriate quantities labeled. One easily obtains the following relationships:

$$X = r_e \sin \alpha$$

$$Z = r_e \cos \alpha$$

$$Y = Z \sin \phi_e$$

$$= r_e \cos \alpha \sin \phi_e$$

$$\phi_s = \sin^{-1} (Y/X)$$

$$= \sin^{-1} (r_e \cos \alpha \sin \phi_e / r_e \sin \alpha)$$

$$= \sin^{-1} (\cot \alpha \sin \phi_e)$$

Now the ecliptic latitude (ϕ_e) is defined in terms of the spacecraft right ascension and declination as follows:

$$\sin \phi_e = -\cos \delta_d \sin \alpha_{ra} \sin \epsilon + \sin \delta_d \cos \epsilon$$

where

α_{ra} = right ascension

δ_d = declination

ϵ = obliquity of ecliptic (23.445 deg)

so that

$$\phi_s = \sin^{-1} [\cot \alpha(-\cos \delta_d \sin \alpha_{ra} \sin \epsilon + \sin \delta_d \cos \epsilon)]$$

The above derivation assumes that the pole of the Sun is perpendicular to the ecliptic plane; however, in fact, the pole of the Sun is inclined approximately 7.2 deg from the perpendicular to the ecliptic, so that the heliographic latitude ϕ_s as defined will include an error Δ:

$$0 \text{ deg} \leq |\Delta| \lesssim 7.2 \text{ deg}$$

However, because of the already large spread in the observed doppler noise data, it was felt that this inaccuracy in the computation of heliographic latitude would not substantially degrade any correlation present, and hence (the inaccuracy) was not worth eliminating for the purposes of this study.

III. Correlation of Observed Doppler Noise With Heliographic Latitude

Observed doppler noise (pass average, good two-way, 60-second count doppler) from the 1976 solar conjunctions of Pioneer 10 and Pioneer 11 was compared to the ISED model, and the residuals, in "dB" ($\equiv 10 \log_{10} N_A/N_P$) were plotted against the heliographic latitude, as seen in Fig. 3. A very strong correlation is immediately apparent. It was assumed that a multiplicative factor $f(\phi_s)$ could be constructed for ISED such that:

$$f(\phi_s) < 1, \qquad \phi_s > 0$$

$$f(0) = 1$$

The simplest expedient was to fit the (logarithmic) residuals as a linear function of heliographic latitude, or

$$f(\phi_s) = 10^{-A(\phi_s/90\text{ deg})}$$

This function was appended to the ISED formulation and the standard deviation of the combined Pioneer 10 and Pioneer 11 residuals was minimized by choosing:

$$A = 0.9$$

or

$$f(\phi_s) = 10^{-0.9(\phi_s/90\text{ deg})}$$

Since it has been assumed that doppler noise is proportional to (integrated) electron density, this relationship can be used to obtain a rough measure of the ratio of the (signal path integrated) polar coronal electron density to the (signal path integrated) equatorial coronal electron density, as follows:

$$\frac{\text{Polar density}}{\text{Equatorial density}} \approx 10^{-0.9} \approx \frac{1}{8}$$

IV. Comparison With Other Models of Latitudinal Variation in Electron Density

In Ref. 5, C. C. Counselman III indicates that good results were obtained with the term:

$$\cos^2 \phi_s$$

when modeling electron density as a function of heliographic latitude. In Ref. 6, K. Saito presented a complete expression for electron density as follows:

$$N_e(r, \phi_s) = \frac{3.09 \times 10^8}{r^{16}}(1 - 0.5 \sin \phi_s)$$

$$+ \frac{1.58 \times 10^8}{r^6}(1 - 0.95 \sin \phi_s)$$

$$+ \frac{0.0251 \times 10^8}{r^{2.5}}(1 - \sqrt{\sin \phi_s})$$

For this set of Pioneer 10 and Pioneer 11 data, only the lowest order term is significant, so that one is interested only in the term:

$$(1 - \sqrt{\sin \phi_s})$$

A comparison of the three functions is presented in Fig. 4, and Fig. 5 presents the ratios:

$$\frac{\cos^2 \phi_s}{10^{-0.9(\phi_s/90\text{ deg}}} ; \quad \frac{1 - \sqrt{\sin \phi_s}}{10^{-0.9(\phi_s/90\text{ deg}}}$$

An examination of Figs. 4 and 5 reveals that the expression determined from the Pioneer 10 and Pioneer 11 data is a good compromise between the two referenced expressions up until about $\phi_s \approx 60$ deg, but from that point on, both of the referenced expressions fall off far more rapidly than the determined expression. It must be borne in mind, however, that both of the referenced expressions must be considered unrealistically low as ϕ_s approaches 90 deg, since they both are exactly zero when $\phi_s = 90$ deg.

V. The ISEDB Model

Two changes were made to the ISED model to obtain the "ISEDB" model; these are described in this section.

A. Incorporation of Functional Variation With Heliographic Latitude

The previous ISED model is multiplied by the expression determined in this report, or

$$\text{ISEDA} = [\text{ISED}]\,10^{-0.9(\phi_s/90\text{ deg})}$$

B. Incorporation of Small, Non-Plasma-Related Noise Sources

Obviously, a conglomerate noise value due to non-plasma-related sources should be added (rms) to the ISED model to obtain more realistic results under low plasma conditions. The noise value adopted here for this purpose is 0.0015 Hz, so that

$$\text{ISEDB} = [(\text{ISEDA})^2 + (0.0015)^2]^{1/2}$$

or

$$\text{ISEDB} = \left[\left(\left\{ A_0 \left[\frac{\beta}{(\sin\alpha)^{1.3}}\right] F(\alpha,\beta) + A_1 \left[\frac{1}{(\sin\alpha)^5}\right]\right\} 10^{-A_8(\phi_s/90\ \text{deg})}\right)^2 + (0.0015)^2\right]^{1/2}$$

where

$$F(\alpha,\beta) = 1 - 0.05\left\{\frac{\left(\beta - \frac{\pi}{2} + \alpha\right)^3 - \left(\alpha - \frac{\pi}{2}\right)^3}{\beta}\right\} - 0.00275\left\{\frac{\left(\beta - \frac{\pi}{2} + \alpha\right)^5 - \left(\alpha - \frac{\pi}{2}\right)^5}{\beta}\right\}$$

and

$$A_0 = 9.65 \times 10^{-4}$$

$$A_1 = 5 \times 10^{-10}$$

$$A_8 = 9 \times 10^{-1}$$

The observed doppler noise accumulated during the Pioneer 10 and Pioneer 11 1976 solar conjunctions can be seen and compared to the ISED and ISEDB formulations in Figs. 6 and 7.

VI. Pioneer 11/Saturn Encounter Doppler Noise Prediction

To illustrate the effect of the correction for heliographic latitude, the ISED and ISEDB models have been computed for the Saturn encounter period, and are presented in Fig. 8. For instance, for the planned encounter day (DOY 244, 1979) and for the minimum SEP point (DOY 254, 1979), the reduction due to heliographic latitude is as follows:

DOY, 1979	ISED, Hz	ISEDB, Hz	Reduction, %
244	0.031	0.024	23
254	0.221	0.048	78

VII. Summary

The 1976 solar conjunctions of Pioneer 10 and particularly of Pioneer 11 allowed a large data base of observed doppler noise to be accumulated over a dynamic range of signal closest approach point heliographic latitudes. These data were processed with a previously developed doppler noise model (ISED), and the residuals are shown to correlate strongly with heliographic latitude. An expression for the heliographic latitude effect is constructed as follows:

$$10^{-0.9(\phi_s/90\ \text{deg})}$$

and this term is then (multiplicatively) applied to the ISED model to produce a new model — ISEDB. Finally, the substantial effect of the heliographic latitude effect is illustrated by comparing ISED to ISEDB during the Pioneer 11/Saturn encounter period.

References

1. Berman, A. L., and Wackley, J. A., "Doppler Noise Considered as a Function of the Signal Path Integration of Electron Density," in *The Deep Space Network Progress Report 42-33*, Jet Propulsion Laboratory, Pasadena, Calif., June 15, 1976.

2. Berman, A. L., "Analysis of Solar Effects Upon Observed Doppler Data Noise During the Helios 1 Second Solar Conjunction," in *The Deep Space Network Progress Report 42-32*, Jet Propulsion Laboratory, Pasadena, Calif., Apr. 15, 1976.

3. Berman, A. L., and Rockwell, S. T., "Correlation of Doppler Noise During Solar Conjunctions With Fluctuations in Solar Activity," in *The Deep Space Network Progress Report 42-30*, Jet Propulsion Laboratory, Pasadena, Calif., Dec. 15, 1975.

4. Berman, A. L., and Rockwell, S. T., "Analysis and Prediction of Doppler Noise During Solar Conjunctions," in *The Deep Space Network Progress Report 42-30*, Jet Propulsion Laboratory, Pasadena, Calif., Dec. 15, 1975.

5. Counselman, C. C., III, and Rankin, J. M., "Density of The Solar Corona From Occultations of NP0532," *Astrophys. J.*, Vol. 175, Aug. 1, 1972.

6. Saito, K., "A Non-Spherical Axisymmetric Model of The Solar K Corona of The Minimum Type," *Ann. Tokyo Astron. Observ.*, University of Tokyo, Second Series, Vol. XII, No. 2, Mitaka, Tokyo, 1970.

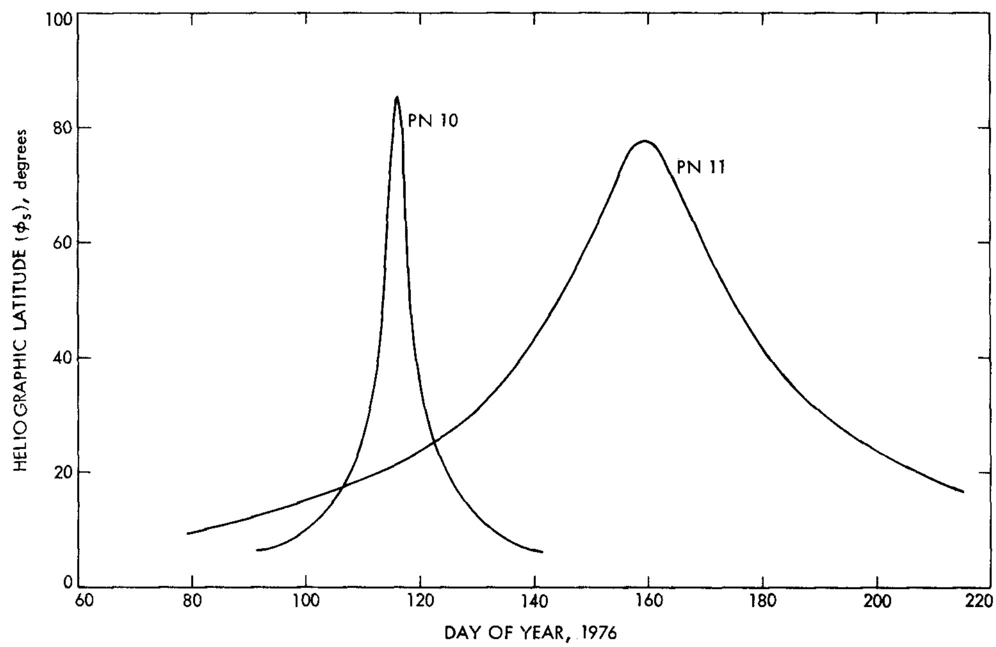

Fig. 1. Pioneer 10 and 11 signal closest approach point heliographic latitude

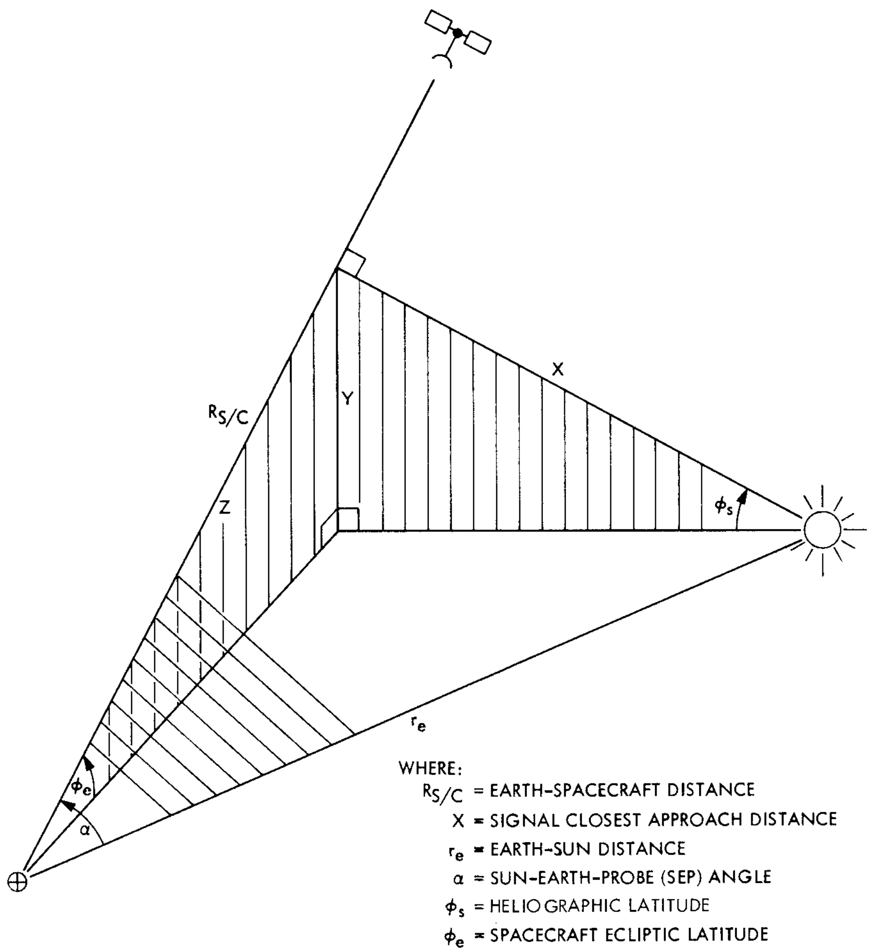

Fig. 2. Signal closest approach heliographic latitude

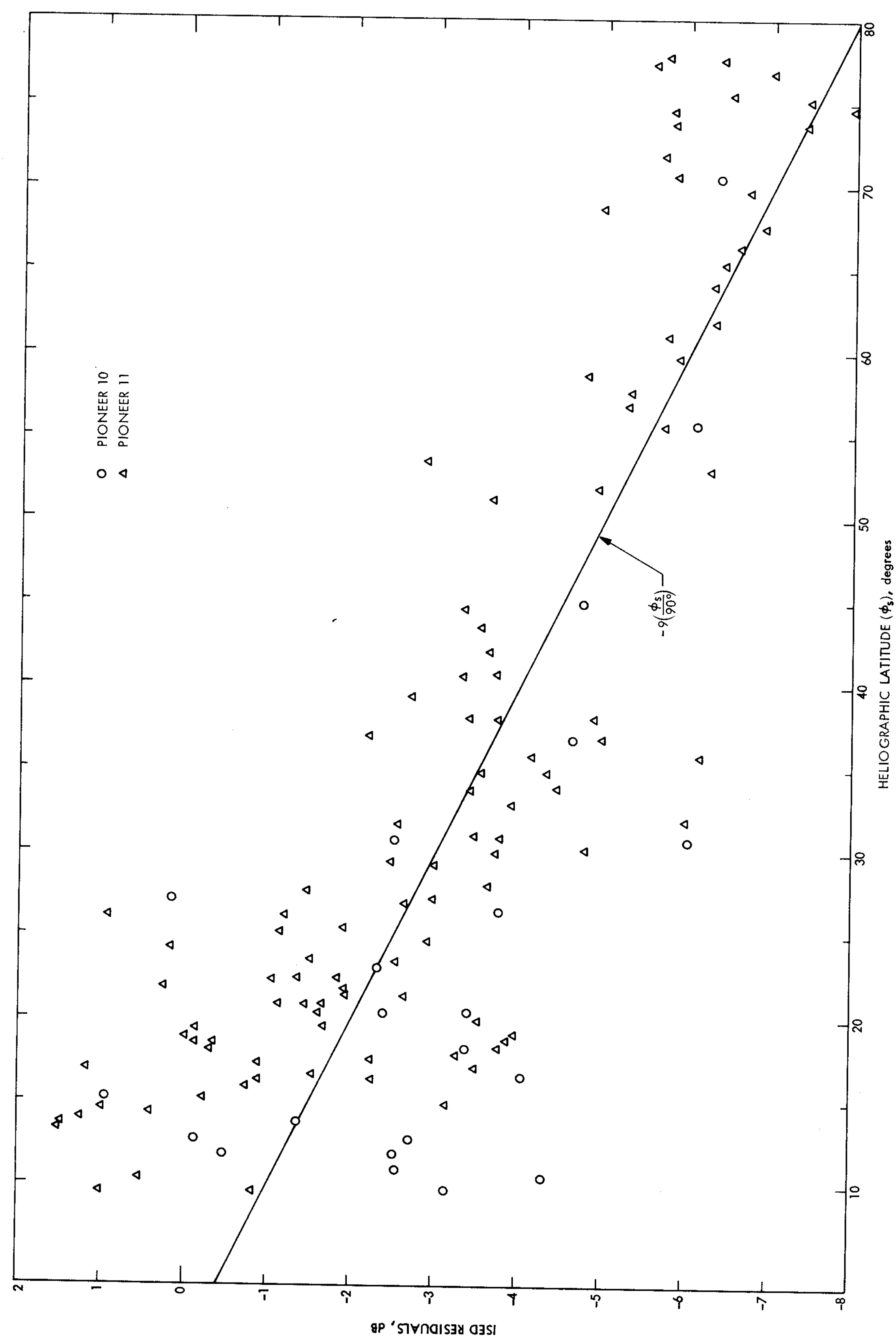

Fig. 3. Pioneer 10 and 11 ISED residuals vs heliographic latitude

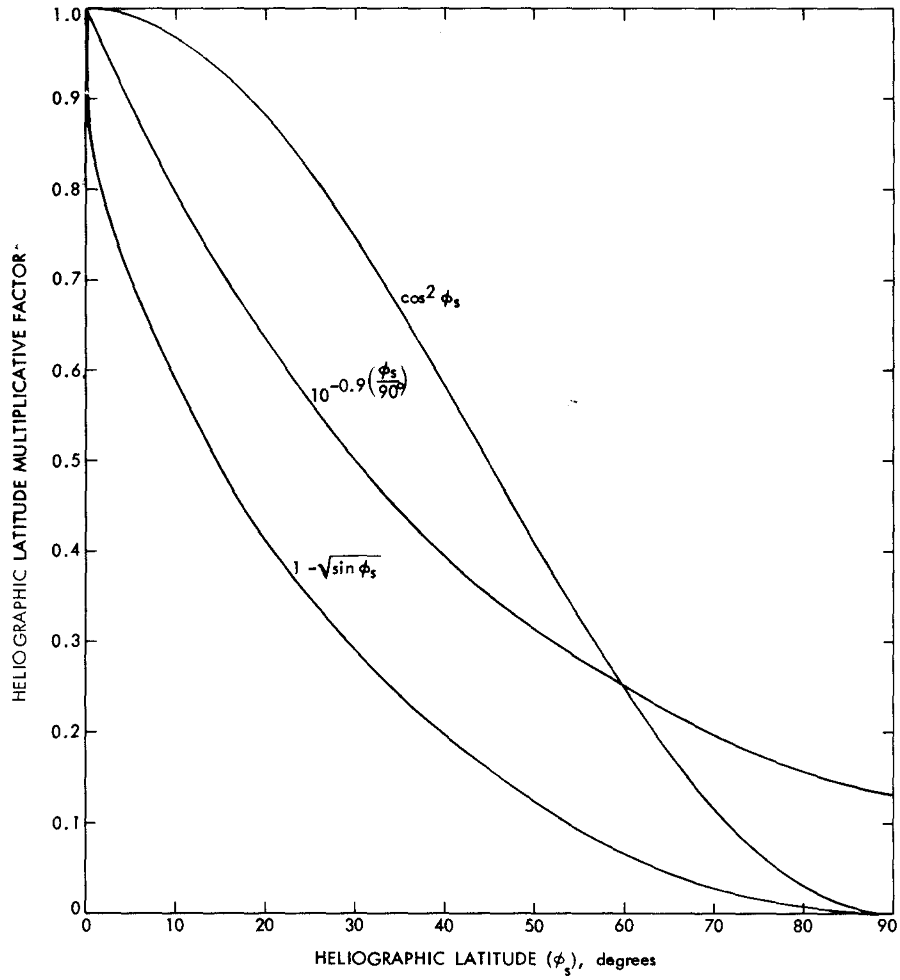

Fig. 4. Comparison of heliographic latitude factors

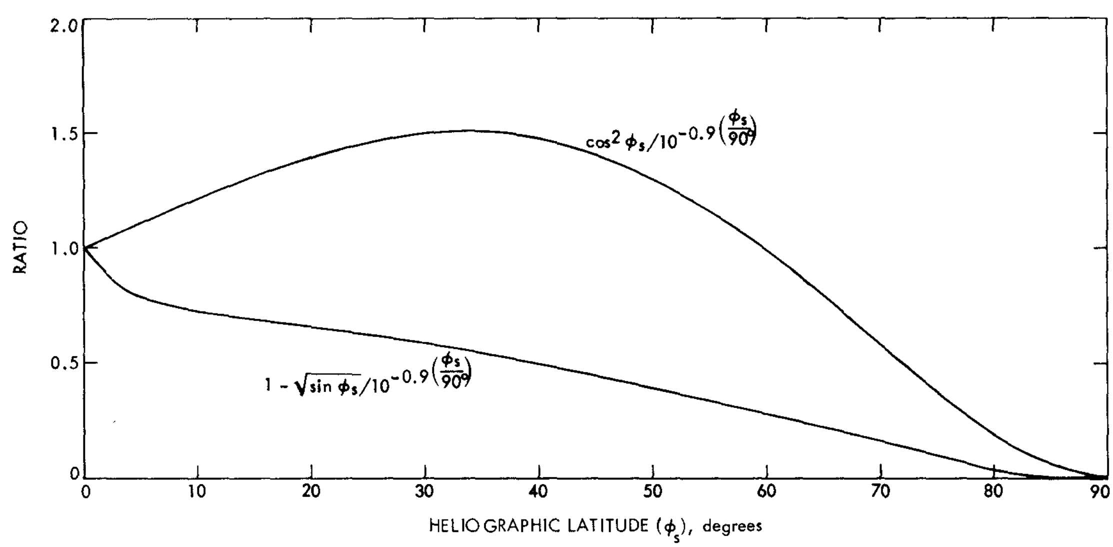

Fig. 5. Ratio comparison of heliographic latitude factors

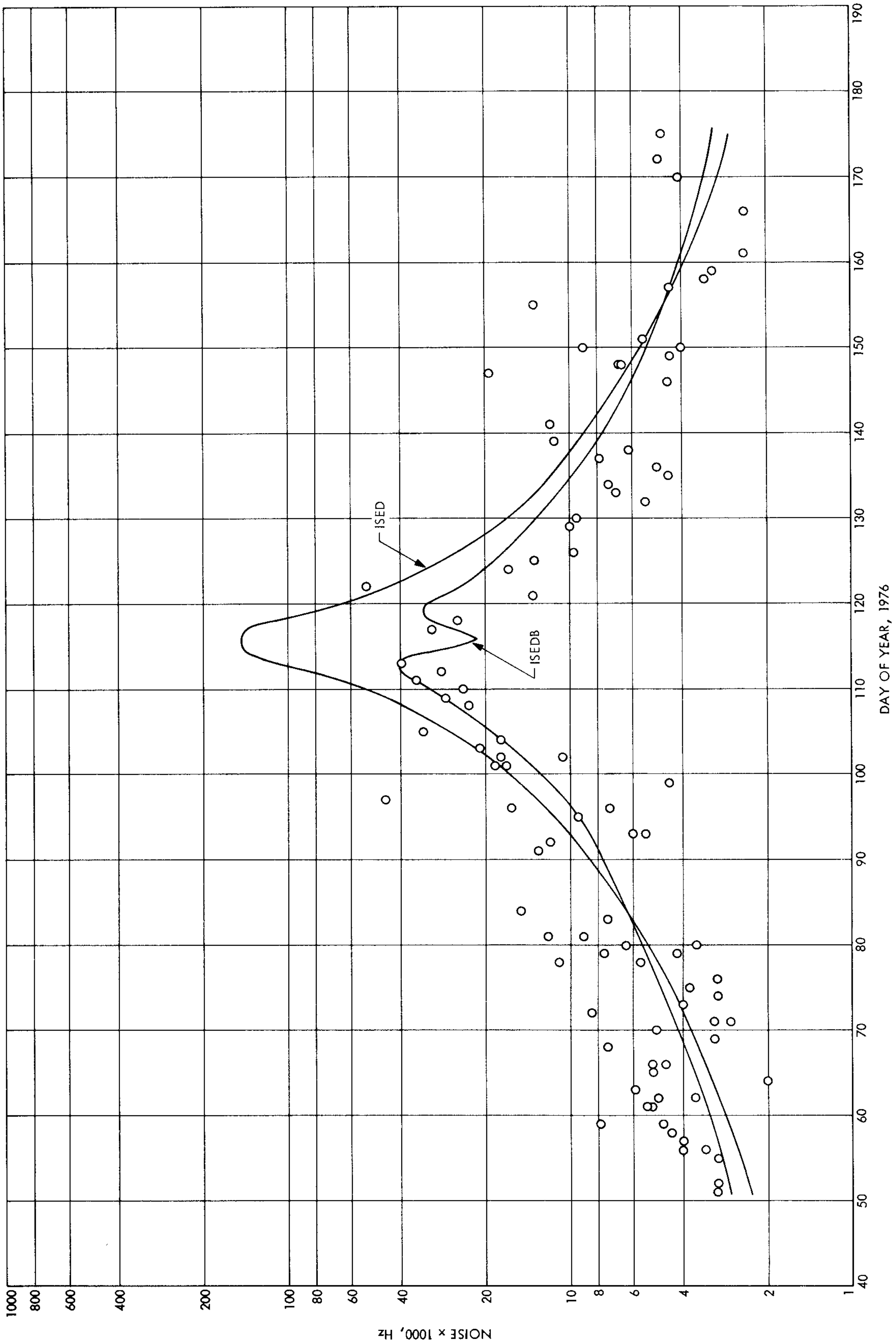

Fig. 6. Pioneer 10 actual noise vs day of year

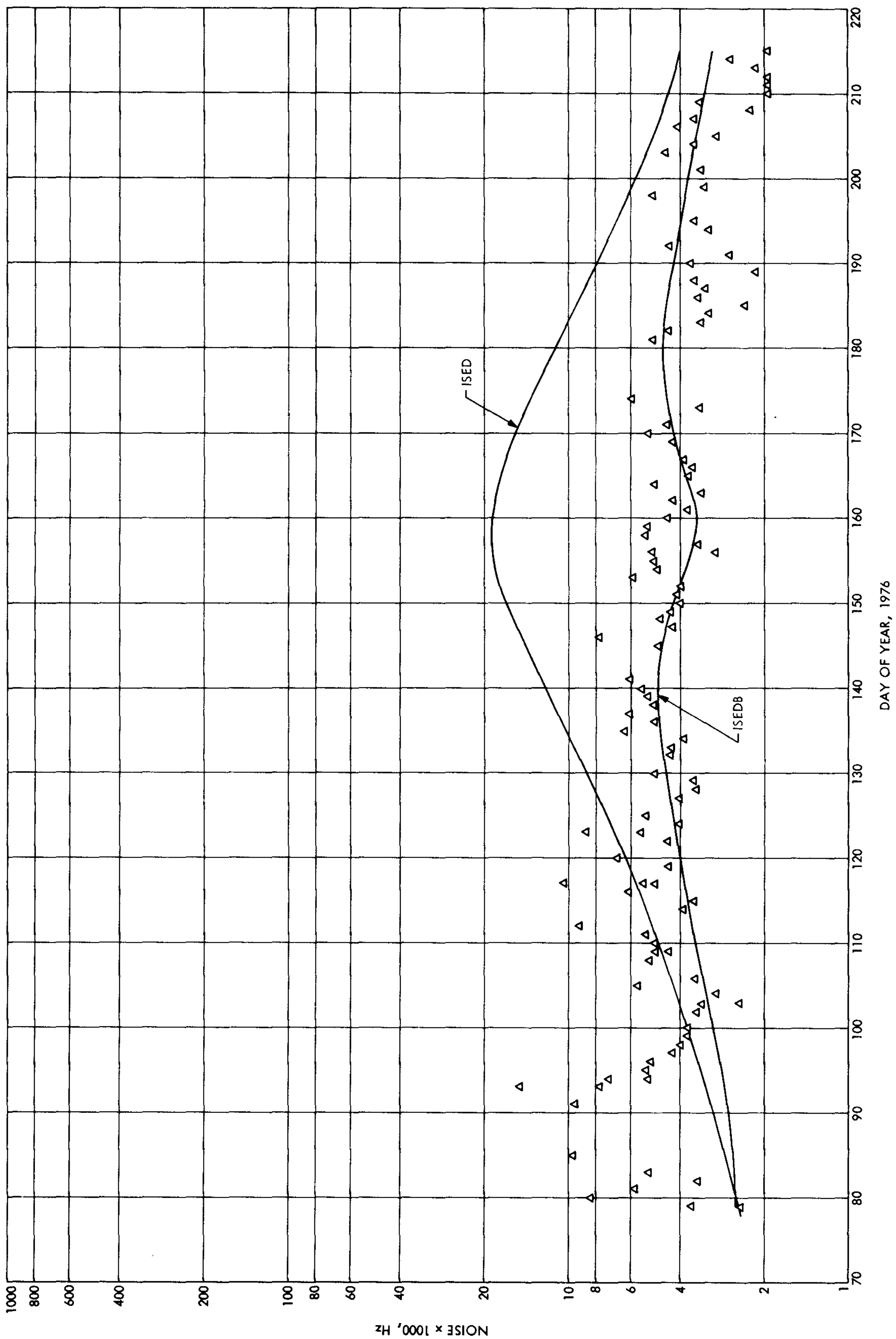

Fig. 7. Pioneer 11 actual noise vs day of year

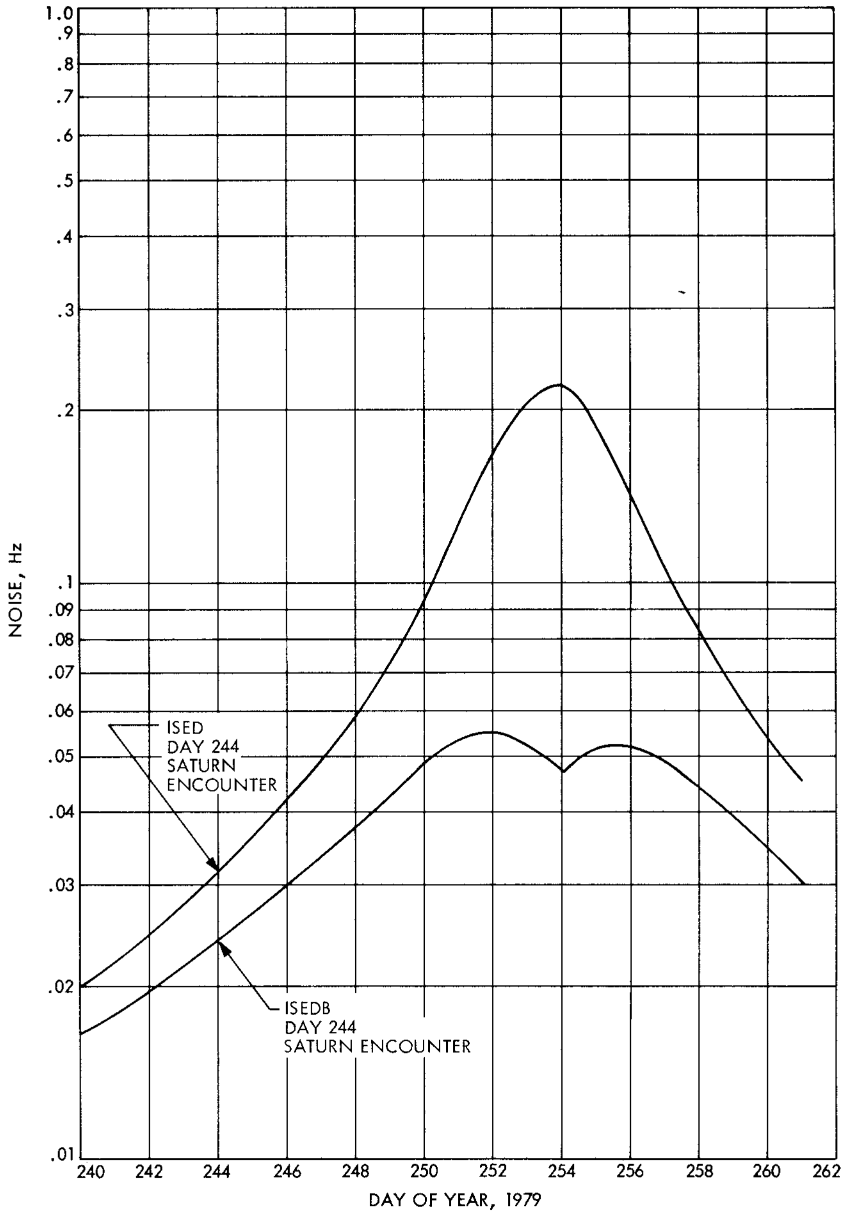

Fig. 8. Saturn encounter predicted doppler noise