A Markov Model for X-Band Atmospheric Antenna-Noise Temperatures

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A five-state Markov model is suggested for the X-band antenna-noise temperatures based on data collected at Goldstone. The states of the model are determined by changes in the "cloud and rain" condition of the atmosphere so as to take advantage of the correlation observed between the antenna temperatures and changes in rain rates. Then an indication is given of how to obtain the estimates of the parameters of the model from the data.

I. Introduction

This report documents the progress made thus far in constructing a stochastic model for the X-band antenna-noise temperatures using data collected at Goldstone. The model suggested here is a finite-state Markov model (see Ref. 1 for a method of reproducing the data if the temperature increases follow a half-gaussian law).

Figure 1 shows the normalized X-band antenna-noise temperatures and rain rates during a tropical storm in September 1976 (days 253 through 255). This segment of the data is chosen to illustrate the rationale for the model because it is typical of the high temperature fluctuations observed during such a "cloud and rain" condition of the atmosphere. Under such a cloudy and high rain-rate condition, the DSN X-band receiver temperature can be as high as 150 K. Since the DSN receivers are operated at varying elevation angles, these temperature readings are all standardized to the zenith. For example, a temperature reading of 140 K at 30 deg elevation

angle is recorded in Fig. 1 as 70 K at the zenith, the same reading at 60 deg elevation would be recorded as 121 K at the zenith.

Now because of the variation of the antenna-noise temperature with the "cloud and rain" conditions of the atmosphere, the states of the model are allowed to be determined by changes in the water content of the air (in vapor and liquid form). Thus the following are taken to represent the states of the model:

(1) Clear sky with water vapor only, no clouds.

(2) Clouds gathering — dense clouds with no rain.

(3) Dense clouds with light rain.

(4) Dense clouds with medium rain.

(5) Dense clouds with heavy rain.

The diagram of the model suggested is shown in Fig. 2.

The model is described in Section II where the proportion of the time the process spends in each of the states is also given. An indication is given of how to obtain the estimates of the parameters of the model from the data.

II. The Markov Model

Let us designate as base temperature the minimum antenna temperature (in Kelvins) recorded throughout the experiment. Thus in the following, 0 K will refer to the base temperature and 1 K is equivalent to 1 K above the base temperature.

Referring to Fig. 1, let us represent by:

(1) Clear state S_c: water vapor only, no clouds; temperature range 0 to 1 K.

(2) State S_0: clouds gather — dense clouds, no rain; temperature range 1 to 4 K.

(3) State S_1: dense clouds with light rain (0 to 1 mm/h); temperature range 4 to 16 K.

(4) State S_2: dense clouds with medium rain (1 to 5 mm/h); temperature range 16 to 40 K.

(5) State S_3: dense clouds with heavy rain (5 to 10 mm/h); temperature range > 40 K.

The diagram of the transitions between these states is shown in Fig. 2.

In the diagram the direction of the arrows between any pair of states denotes the direction of the transitions between them. Let us write the transition matrix of the process as:

$$M = \begin{array}{c} S_c \\ S_0 \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{pmatrix} S_c & S_0 & S_1 & S_2 & S_3 \\ q_{cc} & q_{co} & 0 & 0 & 0 \\ q_{0c} & q_{00} & q_{01} & q_{02} & q_{03} \\ 0 & q_{10} & q_{11} & q_{12} & q_{13} \\ 0 & q_{20} & q_{21} & q_{22} & q_{23} \\ 0 & q_{30} & q_{31} & q_{32} & q_{33} \end{pmatrix}$$

where we have represented the probability of transition from S_i to S_j by q_{ij}, i.e., $P(S_j|S_i) = q_{ij}$, $ij = c, 0, 1, 2, 3$. Transitions between every pair of states are allowed except between the clear state S_c and each of S_1, S_2, S_3. This is because it is assumed that the atmospheric condition cannot go from clear sky to "dense clouds and rain" in a single step without first passing through "clouds gathering — dense clouds, no rain". Also after each rainfall, clouds linger on (state S_0) for some time before the atmosphere becomes clear (S_c).

Now let

$$p_1 = 1 - q_{00} - \frac{q_{0c}q_{c0}}{1 - q_{cc}} - \frac{q_{10}q_{01}}{1 - q_{11}}$$

$$p_2 = q_{23} + \frac{q_{13}(1 - q_{22})}{q_{12}}$$

$$p_3 = q_{30} + \frac{q_{10}q_{31}}{1 - q_{11}}$$

$$p_4 = q_{20} + \frac{q_{10}q_{21}}{1 - q_{11}}$$

$$C_1 = \frac{q_{01} + q_{21}C_2 + q_{31}C_3}{1 - q_{11}}$$

where

$$C_2 = \frac{p_3 C_3 - p_1}{p_4}$$

$$C_3 = \frac{p_1 p_2 + p_4\left(q_{03} - \dfrac{q_{02}q_{13}}{q_{12}}\right)}{p_2 p_3 + p_4\left[1 - q_{33} + \dfrac{q_{13}q_{32}}{q_{12}}\right]}$$

Then the proportion of times spent in states S_c, S_0, S_1, S_2, S_3 (the stationary distribution) is given by u_c, u_0, u_1, u_2, u_3 where

$$u_0 = \left[1 + C_1 + C_2 + C_3 + \frac{q_{0c}}{1 - q_{cc}}\right]^{-1}$$

$$u_c = \frac{q_{0c}}{1 - q_{cc}}u_0$$

$$u_1 = C_1 u_0$$

$$u_2 = C_2 u_0$$

$$u_3 = C_3 U_0$$

Further, let the sequence of the temperature readings be represented by $\{y_n\}$. Let N_{ij} = the number of times y_n is in state i and y_{n+1} is in state j. Then we can estimate the transition probability q_{ij} by:

$$q_{ij} = \frac{N_{ij}}{\sum\limits_j N_{ij}}$$

III. Conclusion

As constructed, the states of this model represent the main features of the antenna-noise temperature increases shown in Fig. 1. It may however become necessary to modify the number of states when the model is fitted to the data and the appropriate goodness-of-fit test is performed. The results of that phase of the work will be reported in the future.

Reference

1. E. C. Posner and F. J. Ziegler. "A Technique for Generating Correlated X-Band Weather Degradation Statistics," in *The Deep Space Network Progress Report 42-35*, pp. 180-195, Jet Propulsion Laboratory, Pasadena, Calif., October 15, 1976.
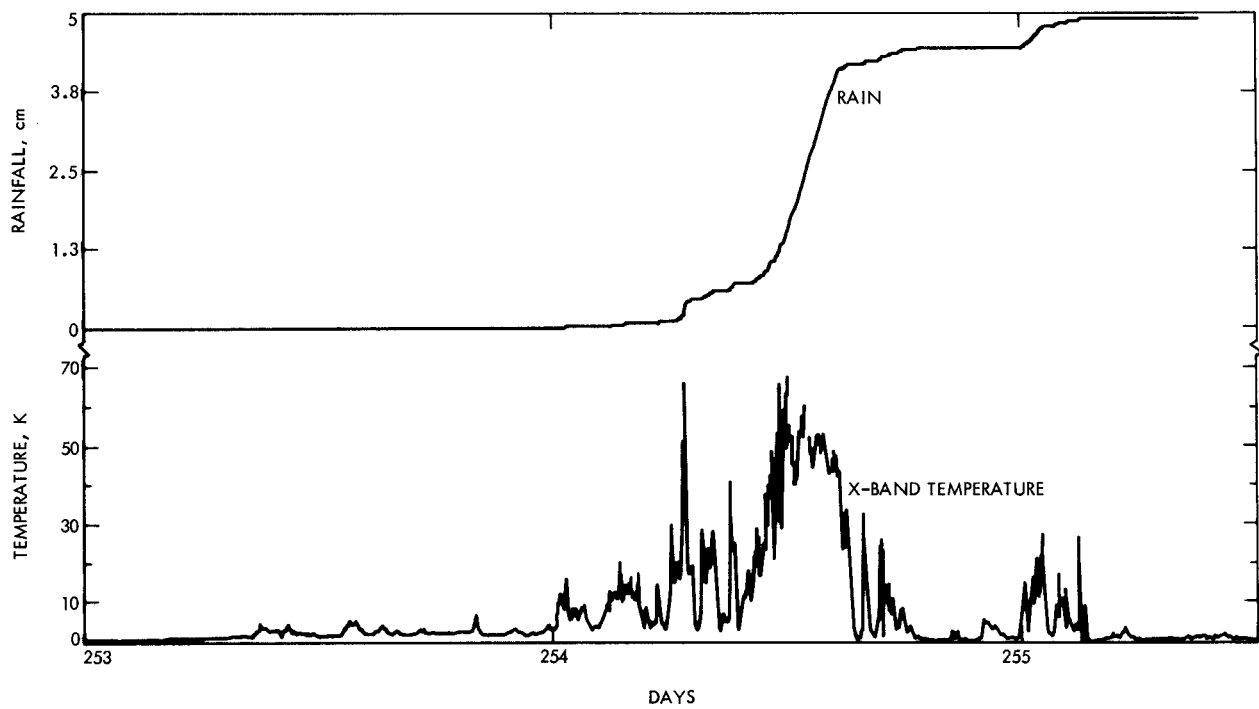
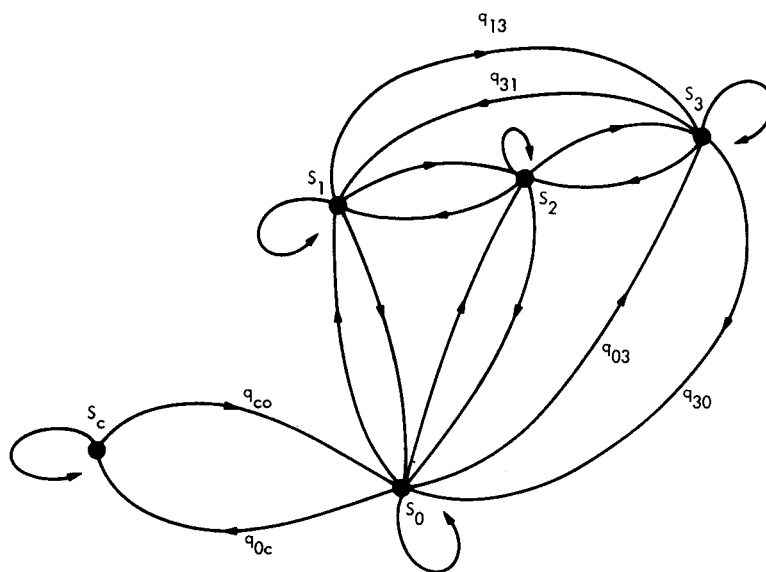**Fig. 1. Rainfall and X-band zenith noise temperatures during tropical storm (Sept. 1976)**

Fig. 2. Transition diagram of the Markov model