Viking Doppler Noise Used to Determine the Radial Dependence of Electron Density in the Extended Corona

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The common form for radial dependence of electron density in the extended corona is:

$$N_e\,(r) \sim \frac{B}{r^{2+\xi}}$$

By assuming proportionality between doppler noise and integrated signal path electron density, Viking doppler noise can be used to solve for a numerical value of ξ. This process yields:

$$\xi = 0.30$$

I. Introduction

The general form of electron density for a spherically symmetric, static corona is:

$$N_e\,(r) = \frac{A}{r^6} + \frac{B}{r^{2+\xi}}$$

where

r = heliocentric distance

The value of ξ is either assumed from theory ($\xi = 0$) or experimentally determined; the range of ξ is usually considered to be:

$$0 \lesssim \xi \lesssim 0.5$$

Examples of values of ξ adopted or determined by various investigators are:

$\xi = 0.0$ Hollweg, Ducher (Refs. 1, 2)

$= 0.05$ Anderson (Ref. 3)

= 0.3 Blackwell, Muhleman (Ref. 4)

= 0.4 Muhleman (Ref. 4)

= 0.5 Saito, Van DeHulst (Ref. 5, 6)

Using the "ISEDC" doppler noise model (Ref. 7) and a large data base of pass average Viking S-band two-way doppler noise, ξ has been solved as follows:

$$\xi = 0.30$$

Somewhat conveniently, this determined value for ξ agrees rather well with the value adopted from the (earlier) work of D. Muhleman:

$$\xi = 0.3$$

II. The Data Base

871 points of "pass-average" Viking S-band doppler noise were accumulated during the following time period:

$$168 \leqslant \text{DOY (1976)} \leqslant 355$$

and for the following range of Sun-Earth-Probe (SEP) angles (in degrees):

$$00.58 \leqslant \text{SEP} \leqslant 54.13$$

The data were collected for all DSSs and for all (Viking) spacecraft and converted (via the ISEDC model) to "equivalent" 60-second sample interval data.

III. Reestimation of ξ

The ISEDC model follows:

$$\text{ISEDC, Hz} = \left[\left(\left\{ A_0 \left[\frac{\beta}{(\sin \alpha)^{1.3}} \right] F(\alpha, \beta) + A_1 \left[\frac{1}{(\sin \alpha)^5} \right] \right\} 10^{-A_8 (|\phi_s|/90)} \right)^2 \times (K(f_{dn}))^2 \left(\left\{ \frac{60}{\tau} \right\}^{0.3} \right)^2 + (D'_0)^2 + \left(D'_1 \left\{ \frac{60}{\tau} \right\}^{D'_2} \right)^2 \right]^{1/2}$$

where:

f_{dn} = downlink frequency (S- or X-band)

$$K(f_{dn}) = \begin{cases} 1.0 & f_{dn} = \text{S-band} \\ 0.7 & f_{dn} = \text{X-band} \end{cases}$$

τ = doppler sample interval, s

$$D'_0 = \begin{cases} 0 & \text{Pioneer} \\ 0.0014 & \text{Helios} \\ 0.0015 & \text{Viking} \end{cases}$$

$$D'_1 = \begin{cases} 0.0029 & \text{Pioneer} \\ 0.0019 & \text{Helios} \\ 0 & \text{Viking} \end{cases}$$

$$D'_2 = \begin{cases} 0.9 & \text{Pioneer} \\ 0.8 & \text{Helios} \\ - & \text{Viking} \end{cases}$$

$$F(\alpha, \beta) = 1 - 0.05 \left\{ \frac{(\beta - \pi/2 + \alpha)^3 - (\alpha - \pi/2)^3}{\beta} \right\} - 0.00275 \left\{ \frac{(\beta - \pi/2 + \alpha)^5 - (\alpha - \pi/2)^5}{\beta} \right\}$$

α = Sun-Earth-Probe (SEP) angle, radians

β = Earth-Sun-Probe (ESP) angle, radians

and

ϕ_s = heliographic latitude, degrees

$= \sin^{-1} [\cot \alpha (-\cos \delta_d \sin \alpha_{ra} \sin \epsilon + \sin \delta_d \cos \epsilon)]$

α_{ra} = right ascension

δ_d = declination

ϵ = obliquity of the ecliptic (23.445 deg)

with:

$A_0 = 0.1182 \times 10^{-2}$

$A_1 = 5 \times 10^{-10}$

$A_8 = 0$

Residuals (in "db") between observed and predicted noise are formed as follows:

$$\Delta (\text{"db"}) = 10 \log_{10} \left\{ \frac{\text{observed noise}}{\text{ISEDC noise}} \right\}$$

These residuals are then used to produce a standard deviation for a particular data set:

$$\sigma \equiv \left\{ \frac{1}{N} \sum_{i=1}^{N} \Delta_i^2 \right\}^{1/2}$$

To solve for ξ, one makes the following assumptions:

(1) Doppler noise is proportional to integrated signal path electron density.

(2) The best fit of the model to the data will yield the best estimate of ξ.

The terms in ISEDC which are a function of ξ are:

$$\frac{\beta}{(\sin\alpha)^{1.3}}, \qquad F(\alpha, \beta)$$

After the fashion of Ref. 8, $(\cos w)^\xi$ is expanded as follows:

$$(\cos w)^\xi \cong 1 + \frac{w^2}{2!}(-\xi) + \frac{w^4}{4!}(3\xi^2 - 2\xi)$$

and $F(\alpha, \beta)$ becomes:

$$F(\alpha, \beta, \xi) = 1 - (\xi/6)\left\{\frac{(\beta - \pi/2 + \alpha)^3 - (\alpha - \pi/2)^3}{\beta}\right\}$$

$$+ (\xi/120)(3\xi - 2)\left\{\frac{(\beta - \pi/2 + \alpha)^5 - (\alpha - \pi/2)^5}{\beta}\right\}$$

one thus replaces:

$$\frac{\beta}{(\sin\alpha)^{1.3}}$$

with

$$\frac{\beta}{(\sin\alpha)^{1+\xi}}$$

and

$$F(\alpha, \beta)$$

with

$$F(\alpha, \beta, \xi)$$

These modifications were made and computer runs were initiated to obtain the conditions:

$$\frac{\partial \sigma}{\partial A_0} = 0$$

$$\frac{\partial \sigma}{\partial \xi} = 0$$

Figure 1 presents the σ achieved over the entire data base for various A_0 and ξ; Figure 2 presents the lowest σ achieved for each ξ. As is readily apparent from examination of Figure 2, $\sigma(\xi)$ has a well defined minimum at:

$$\xi = 0.30$$

with a resolution of ≈ 0.005; hence, a form is assumed for electron density in the extended corona of:

$$N_e(r) \sim \frac{B}{r^{2.30}}$$

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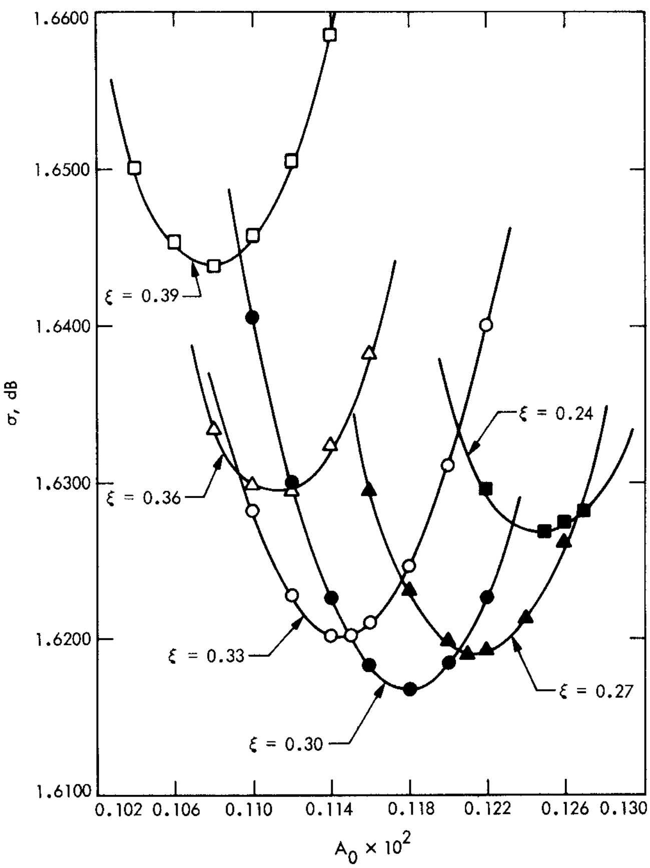

Fig. 1. Viking doppler noise fit vs A_0 and ξ

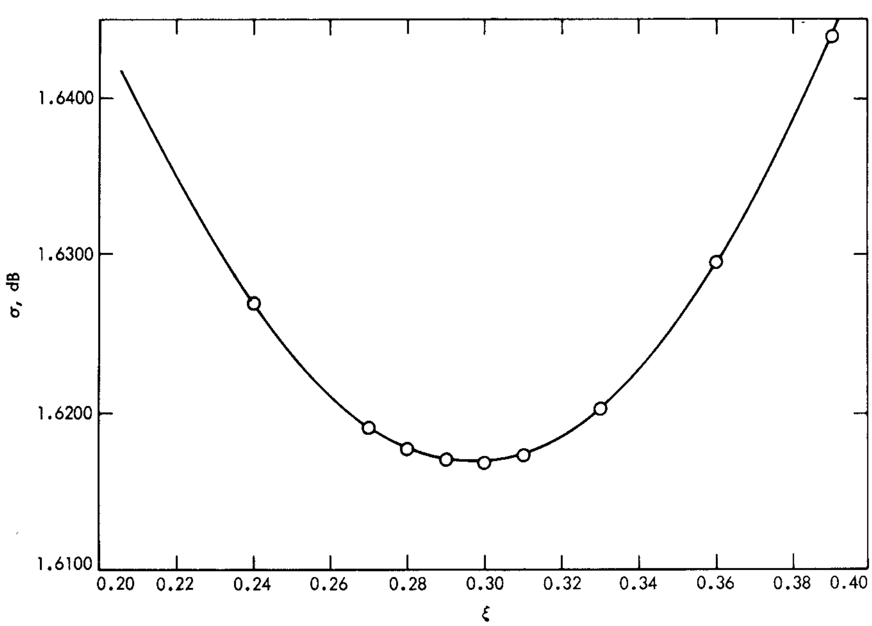

Fig. 2. Viking doppler noise best fit vs the parameter ξ (with $\partial\sigma/\partial A_0 = 0$)