Proportionality Between Doppler Noise and Integrated Signal Path Electron Density Validated by Differenced S-X Range

A. L. Berman
TDA Engineering Office

Previous articles have hypothesized that doppler rms phase jitter (noise) is proportional to integrated signal path electron density. In this article, observations of Viking differenced S-band/X-band (S-X) range are shown to correlate strongly with Viking doppler noise. A ratio of proportionality between downlink S-band plasma-induced range error and two-way doppler noise is calculated as:

$$\frac{\text{Downlink S-band plasma range error}}{\text{2-way S-band doppler noise}} \approx 7 \frac{\text{meters}}{\text{mHz}}$$

A new parameter (similar to the parameter ϵ which defines the ratio of "local" electron density fluctuations to mean electron density) is defined as a function of observed data sample interval (τ):

$$\epsilon'(\tau) = 7.9 \times 10^{-4} \left[\frac{\tau}{60}\right]^{0.7}$$

where the time-scale of the observations is 15τ. This parameter is interpreted to yield the ratio of "net" observed phase (or electron density) fluctuations to integrated electron density (in rms meters/meter). Using this parameter and the "thin phase-changing screen" approximation, a value for the scale size L is calculated:

$$L = a \left[\frac{7.9 \times 10^{-4}}{\epsilon}\right]^2 \left[\frac{\tau}{60}\right]^{1.4}$$

To be consistent with Doppler noise observations, it is seen necessary for L to be proportional to closest approach distance a, and a strong function of the observed data sample interval, and hence the time-scale of the observations.

I. Introduction

Previous articles have presented empirical evidence which suggests that doppler noise (rms phase jitter) is proportional to integrated signal path electron density. Strong as this hypothesis may have been, it was strictly circumstantial in that independent measurements of total columnar electron content were not available for comparison to doppler noise data. With the advent of the first Viking Solar Conjunction, differenced S-X range (S-band range minus X-band range) provided a readily accessible measurement of total columnar electron content for comparison to doppler noise. This report utilizes preliminary Viking differenced S-X range (provided by T. Komarek, Ref. 1) and Viking doppler noise to demonstrate a strong correlation between S-band plasma range error and S-band plasma induced doppler noise. Using these two data types, values have been calculated which relate S-band two-way doppler noise to S-band downlink plasma range error and total columnar electron content. Using the relationship between doppler noise and differenced S-X range, a new parameter is defined which is interpreted as yielding the ratio of "net" observed phase (and hence electron density) fluctuations to total columnar electron density. Utilizing this new parameter, various observations are made in reference to the "thin phase-changing screen" theory.

II. Comparison of Differenced S-X Range to Doppler Noise

Figure 1 presents differenced S-X range from Ref. 1 and Fig. 2 presents Viking two-way S-band doppler noise; in addition, the range values from Fig. 1 have been plotted on Fig. 2. Inspection of Fig. 2 discloses an immediate and very strong similarity in the range and noise signatures between DOY 261 and DOY 306, 1976. The conversion value which (arbitrarily) causes the range data to overlie the doppler noise data in Fig. 2 was calculated to be approximately:

$$\frac{23 \text{ RU}}{\text{mHz}}$$

The differenced S-X range in RU is converted to downlink S-band plasma range error in meters as follows:

$$\text{S-band } \Delta R, \text{ meters} = \frac{1}{3.53}\left\{\frac{1}{1-(3/11)^2}\right\} \text{S-X } \Delta R, \text{ RU}$$

$$= \left\{\frac{1.08}{3.53}\right\} \text{S-X } \Delta R, \text{ RU}$$

$$= 0.306 \text{ S-X } \Delta R, \text{ RU}$$

and

$$\text{one-way S-band noise} = \frac{1}{\sqrt{2}} \text{ two-way S-band noise}$$

Therefore, the conversion between meters and mHz is:

$$\frac{23\ \{1.08/3.53\} \text{ meters}}{\{1/\sqrt{2}\} \text{ mHz}} = 10 \frac{\text{meters}}{\text{mHz}}$$

From Ref. 2 one has the conversion between columnar electrons and meters of range error for downlink S-band:

$$\frac{1.31 \times 10^{17} \text{ electrons/m}^2}{\text{meter}}$$

which yields

$$\frac{1.31 \times 10^{18} \text{ electrons/m}^2}{\text{mHz}}$$

Table 1 provides a variety of additional conversion factors between plasma range error and plasma induced doppler noise.

As a further example of the proportionality between plasma range error and doppler noise, a range acquisition performed by DSS 14 on DOY 331 while tracking Viking Orbiter 1 (at an SEP $\approx$ 0.58 deg; information provided by A. Zygielbaum (Ref. 3)) produced an S-X range error[1] of:

$$61\ \mu\text{s} \times 300 \frac{\text{meters}}{\mu\text{s}} = 18 \text{ km}$$

During the same pass, a two-way pass average doppler noise value of 3533 mHz was recorded, or:

$$3533 \text{ mHz} \times 6.5 \frac{\text{meters}}{\text{mHz}} = 23 \text{ km}$$

This agreement is quite good, particularly if one considers that the constant of proportionality between plasma range error and doppler noise (23 RU/mHz) was determined at approximately 35 mHz, or at a plasma level two orders of magnitude different (less) than the plasma level in this example.

[1] This value is a preliminary estimate and the range acquisition(s) is still subject to final determination of validity.

III. Calculation of a New Electron Density Fluctuation Parameter: $\epsilon'(\tau)$

From Ref. 4 one has the following expression for rms (temporal) phase error:

$$\Delta\phi\ (\text{rad}) = 2.8 \times 10^{-15}\ \{\lambda\} n \Delta R$$

where:

λ = wavelength, meters

= 0.131

n = electron density fluctuation

ΔR = distance along the signal path

if one defines:

$N_e(R, t)$ = instantaneous electron density, electron/m^3

then the mean electron density becomes:

$$N_e(R) = \frac{1}{t}\int_0^t N_e(R, t)\, dt$$

and the (rms) electron density fluctuation is:

$$n(R) = \left[\frac{1}{t}\int_0^t \{N_e(R, t) - N_e(R)\}^2\, dt\right]^{1/2}$$

The ratio of "local" electron density fluctuation to mean electron density is frequently defined as:

$$\epsilon \equiv \frac{n(R)}{N_e(R)}$$

With respect to doppler noise observations, there are two difficulties with the parameter ϵ. First, this ratio is frequently treated as a constant in the literature, whereas it should more correctly be considered as a function of the pertinent time-scale, as electron density fluctuations are well known to span a wide range of time-scales, from fractions of a second to hours, days, weeks. Second, there is no way of explicitly knowing[2] the value of ϵ from received doppler noise. Instead, a new parameter:

$$\epsilon'(\tau)$$

where

$$\tau = \text{data sample interval, seconds}$$

and[3]

$$15\tau = \text{time-scale}$$

will be defined, and which will be expected to yield the ratio of net observed columnar electron density fluctuation to (total) columnar electron density (in rms meters/meter).

Suppose, then, by extrapolation of the previously detailed phase error expression from Cronyn (Ref. 4), that observed phase fluctuations can be obtained by taking the rms value of local electron density fluctuation along the signal path times the signal path distance, but also times a ratio which expresses the proportionality between net observed electron density fluctuation and local electron density fluctuation. In terms of $\epsilon'(\tau)$, this ratio becomes:

$$\left\{\frac{\epsilon'(\tau)}{\epsilon}\right\}$$

$\epsilon'(\tau)$ is anticipated as a function of time-scale but *not* of geometry, and as previously mentioned, ϵ should also be considered as a function of time-scale. RMS phase fluctuation (observed) is then *defined* as:

$$\phi_{rms} \equiv 2.8 \times 10^{-15}\ [\lambda] \left\{\frac{1}{R}\int_0^R \{n\}^2\, dR\right\}^{1/2} R\left[\frac{\epsilon'(\tau)}{\epsilon}\right]$$

[2] Deducing the value of ϵ from Earth observations of scintillation, phase fluctuation, etc, has evoked much theoretical analysis during the past decade, but no definitive, widely accepted solutions have been produced.

[3] The doppler noise utilized in this study is the output from the Network Operations Control Center (NOCC) Psuedo Residual program. Pass average doppler noise is abstracted from a "running" standard deviation calculated about a least squares linear curve fit to the most recent 15 samples. Hence the time scale is considered to be 15 times the data sample interval, or 15τ.

By using this definition for observed phase fluctuations *in conjunction with* the observed relationship between phase fluctuations (doppler noise) and integrated signal path electron density (S-X range) from Section II, information will be obtained about $\epsilon'(\tau)$ and the ratio $\epsilon'(\tau)/\epsilon$.

It is easy to show (see the Appendix) that:

$$\left[R \int_0^R \{n(R)\}^2\, dR\right]^{1/2} \approx \epsilon \int_0^R N_e(R)\, dR$$

$$\approx \epsilon I$$

where

$$I = \int_0^R N_e(R)\, dR$$

so that if:

$$\phi_{rms} \equiv 2.8 \times 10^{-15}\, [\lambda] \left\{R \int_0^R \{n\}^2\, dR\right\}^{1/2} \left[\frac{\epsilon'(\tau)}{\epsilon}\right]$$

then:

$$\phi_{rms}\,(\text{rad}) = 2.8 \times 10^{-15}\, [\lambda]\, \{\epsilon I\} \left[\frac{\epsilon'(\tau)}{\epsilon}\right]$$

$$= 2.8 \times 10^{-15}\, [0.131]\, \{\epsilon'(\tau) I\}$$

$$= 3.66 \times 10^{-16}\, \epsilon'(\tau) I$$

$$\phi_{rms}\,(\text{cycles}) = 3.66 \times 10^{-16}\, \epsilon'(\tau) I \left[\frac{\text{cycle}}{2\pi\ \text{rad}}\right]$$

$$= 5.8 \times 10^{-17}\, \epsilon'(\tau) I$$

$$\phi_{rms}\,(\text{Hz}) = 5.8 \times 10^{-17}\, \epsilon'(\tau) I \left[\frac{1}{60\ \text{s}}\right]$$

$$= 0.97 \times 10^{-18}\, \epsilon'(\tau) I$$

$$\phi_{rms}\,(\text{mHz}) = 0.97 \times 10^{-15}\, \epsilon'(\tau) I$$

now:

$$\text{S-band } \Delta R,\ \text{meters} = \frac{I}{1.31 \times 10^{17}}$$

so that:

$$\frac{\Delta R,\ \text{meters}}{\phi_{rms},\ \text{mHz}} = \frac{I}{(1.31 \times 10^{17})(0.97 \times 10^{-15})\, \epsilon'(\tau) I}$$

$$= \frac{0.0079}{\epsilon'(\tau)}\ \frac{\text{meter}}{\text{mHz}}$$

using the expression found in Section II, one has for 60-s sample interval doppler noise:

$$\frac{0.0079}{\epsilon'(60)}\ \frac{\text{meter}}{\text{mHz}} = 10\ \frac{\text{meter}}{\text{mHz}}$$

or:

$$\epsilon'(60) = 0.00079$$

This value for $\epsilon'(\tau)$ is about 1/25th the value commonly assigned to ϵ(no time-scale specified) in the literature:

$$\epsilon \approx 0.02$$

An interpretation of these two parameters is made as follows:

$\epsilon'(\tau)$: the ratio of observed electron density fluctuation to total columnar electron density (rms meters/meter).

$\epsilon'(\tau)/\epsilon$: the ratio of observed electron density fluctuation to local electron density fluctuation.

If the above interpretation proves correct, then the usefulness of these two parameters in concert would be:

> If one has in situ measurements of ϵ (really $\epsilon(\tau)$) and Earth observed measurements of $\epsilon'(\tau)$, then theory should be able to correctly predict the correspondence between the two.

This author has recently determined (Ref. 5) the dependence between doppler sample interval and doppler noise to be:

$$\text{noise}(\tau) = \{\text{noise}(60\text{ s})\}\left[\frac{60}{\tau}\right]^{0.3}$$

or

$$\text{rms phase}(\tau) = \{\text{rms phase}(60)\}\left[\frac{\tau}{60}\right]\left[\frac{60}{\tau}\right]^{0.3}$$

where:

τ = doppler sample interval, s

One therefore obtains for the relationship between $\epsilon'(\tau)$ and τ:

$$\epsilon'(\tau) = \epsilon'(60)\left[\frac{\tau}{60}\right]\left[\frac{60}{\tau}\right]^{0.3}$$

$$= 0.00079\left[\frac{\tau}{60}\right]^{0.7}$$

This relationship is seen in Fig. 3, as well as in the tabulated values below:

τ, s	$\epsilon'(\tau)$
1.0	4.5×10^{-5}
10.0	2.3×10^{-4}
60.0	7.9×10^{-4}
600.0	4.0×10^{-3}
1800.0	8.5×10^{-3}
3600.0	1.4×10^{-2}

IV. Doppler Noise Observations Compared to Results Derived From the Thin Phase-Changing Screen Approximation

Derivations using the thin phase-changing screen approximation (for instance Salpeter, Ref. 6, and Ducher, Ref. 7) produce an expression for rms phase as follows:

$$\phi_{rms} = \left\{L\int_0^R n^2\,dR\right\}^{1/2}(2\pi)^{1/4}\,r_e\,\lambda$$

$$\propto \left\{L\int_0^R N_e^2\,dR\right\}^{1/2}$$

$$\propto \frac{1}{a^{1.8}}$$

where:

L = "correlation length", "irregularity scale size", etc.

$r_e = 2.8 \times 10^{-15}$ meters

a = closest approach distance

Obviously, considering L as a constant produces an rms phase fluctuation ($\propto 1/a^{1.8}$) which is inconsistent with the radial dependence of the observed data ($\propto 1/a^{1.3}$). Considered values of L range from 100 km (Ducher, Ref. 7) to 10^6 km or larger (Jokipii and Hollweg, Ref. 8, Callahan, Ref. 9).

However, some investigators have proposed that L is a function of closest approach distance

$$L \propto a$$

Examples are Cohen and Gundermann, Ref. 10, where:

$$L \propto a^{1.05}$$

and Houminer and Hewish, Ref. 11, where:

$$L \propto a^{1.0}$$

L proportional to a would be exactly the condition required to bring doppler noise observations into correspondence with rms phase derivations from the thin phase-changing screen approximation.

The thin phase-changing screen expression can be compared to the expression for phase fluctuation presented in Section II, but with the integration limited to $\pm a$ (as performed in the Appendix):

$$\phi_{rms} = \left\{ 2a \int_{-a}^{a} n^2 \, dR \right\}^{1/2} r_e \lambda \left[\frac{\epsilon'(\tau)}{\epsilon} \right]$$

ignoring the difference between $\sqrt{2\pi}$ and 2, one obtains for the scale size L:

$$\sqrt{L} = \sqrt{a} \left[\frac{\epsilon'(\tau)}{\epsilon} \right]$$

or

$$L = a \left[\frac{\epsilon'(\tau)}{\epsilon} \right]^2$$

Hence one concludes that the thin phase-changing screen scale size must be proportional to closest approach distance and is strongly effected by the time-scale over which observations are taken. Using the expression for $\epsilon'(\tau)$, one obtains for L:

$$L = a \left[\frac{7.9 \times 10^{-4}}{\epsilon} \right]^2 \left[\frac{\tau}{60} \right]^{1.4}$$

Figures 4, 5, 6, and 7 present plots of L versus a for various conditions.

It has been previously suggested that, like $\epsilon'(\tau)$, the local fluctuation to density ratio ϵ should be considered as $\epsilon(\tau)$. If one speculates that $\epsilon(\tau)$ has approximately the same dependence upon time-scale (hence τ) as does $\epsilon'(\tau)$, one would write:

$$\epsilon(\tau) = \epsilon(60) \left[\frac{\tau}{60} \right]^{0.7}$$

and thus the scale size L would simply become a function of closest approach distance:

$$L = a \left[\frac{\epsilon'(\tau)}{\epsilon(\tau)} \right]^2$$

$$= a \left[\frac{7.9 \times 10^{-4}}{\epsilon(60)} \right]^2$$

This (postulated) relationship for various values of $\epsilon(60)$ is seen in Fig. 8 as well as in the tabulated values below (L in km, a in solar radii):

$\epsilon(60)$	L
10^0	$4.3 \times 10^{-1}\, a$
10^{-1}	$4.3 \times 10^{1}\, a$
10^{-2}	$4.3 \times 10^{3}\, a$
10^{-3}	$4.3 \times 10^{5}\, a$
10^{-4}	$4.3 \times 10^{7}\, a$

V. Summation

It has been periodically suggested by this author that observed doppler noise is proportional to integrated signal path electron density. The comparison of differenced Viking S-X range to Viking S-band doppler noise has lent added weight to this conviction and has resulted in the approximate determination of the constant of proportionality between the two. Incorporating this result with the observed functional dependance of doppler noise upon data sample interval allows the ratio of net observed rms phase fluctuations to integrated electron density to be explicitly exhibited as a function of time-scale (here 15 times the data sample interval). These results are then applied to define the condition necessary to bring thin phase-changing screen theory into agreement with observed doppler rms phase fluctuations:

$$L \propto a$$

and to establish the relationship between scale size, time-scale, and the ratio of local electron density fluctuation to electron density.

Table 1. Plasma range error and plasma induced doppler noise conversion factors

Plasma/noise combinations	Conversion Value
$\frac{\text{Downlink S-band plasma range error}}{\text{Downlink S-band doppler noise}}$	$10.0 \ \frac{\text{meters}}{\text{mHz}}$
$\frac{\text{Downlink S-X plasma range error}}{\text{Downlink S-band doppler noise}}$	$9.2 \ \frac{\text{meters}}{\text{mHz}}$
$\frac{\text{Downlink S-band plasma range error}}{\text{2-way S-band doppler noise}}$	$7.0 \ \frac{\text{meters}}{\text{mHz}}$
$\frac{\text{Downlink S-X plasma range error}}{\text{2-way S-band doppler noise}}$	$6.5 \ \frac{\text{meters}}{\text{mHz}}$

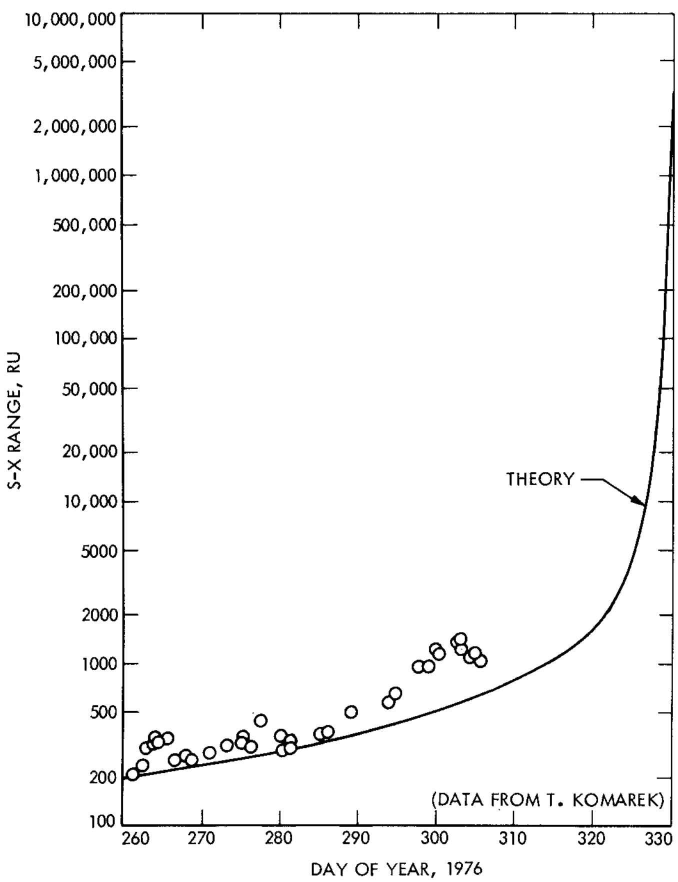


Fig. 1. Viking S-X range vs day of year

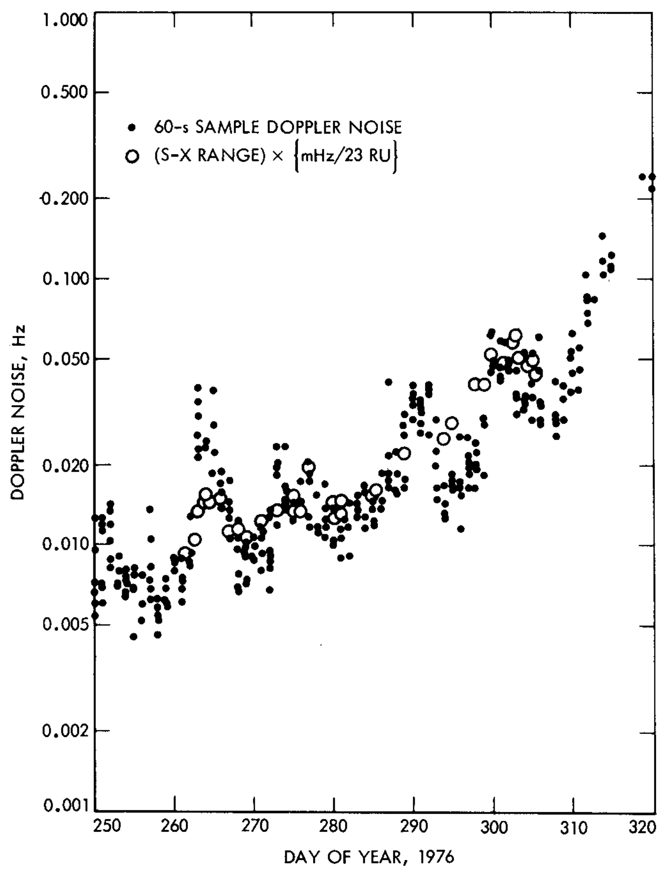


Fig. 2. Viking two-way S-band doppler noise and S-X range vs day of year

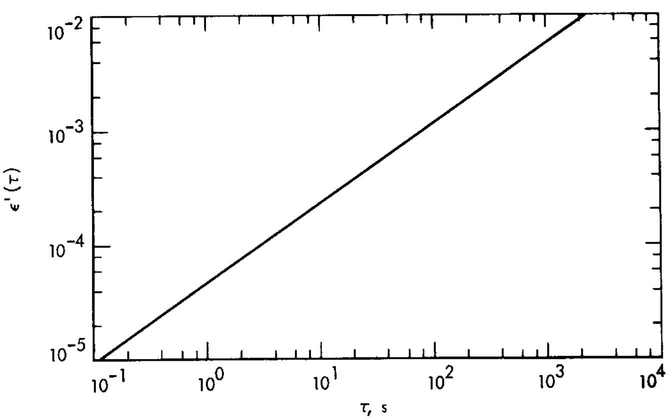


Fig. 3. $\epsilon'(\tau)$ vs τ

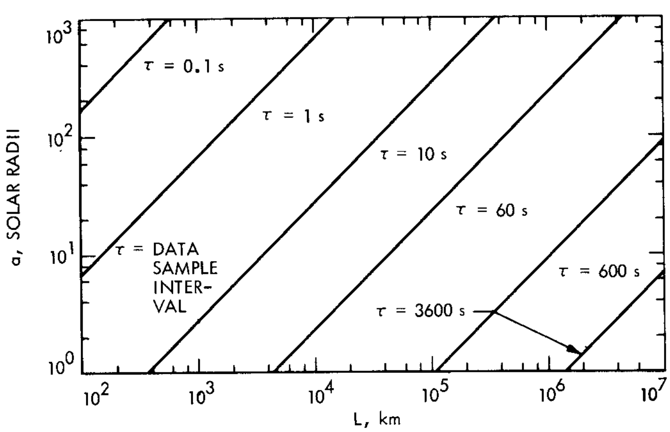


Fig. 4. Scale size for ϵ = 0.01 vs closest approach distance

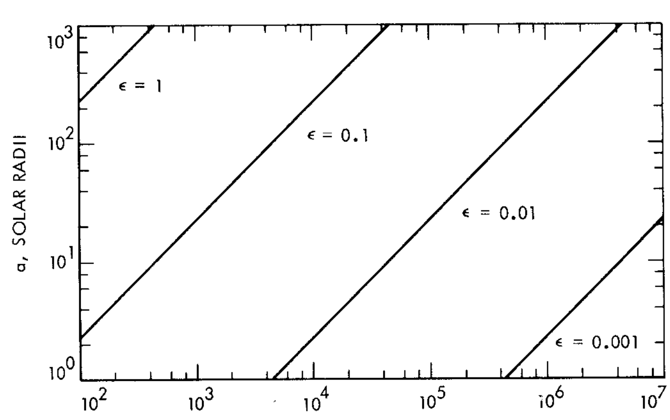


Fig. 5. Scale size for τ = 60 s vs closest approach distance

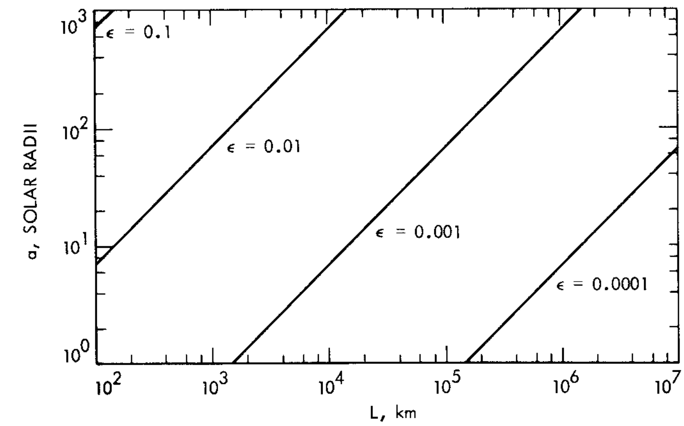


Fig. 6. Scale size for τ = 1 s vs closest approach distance

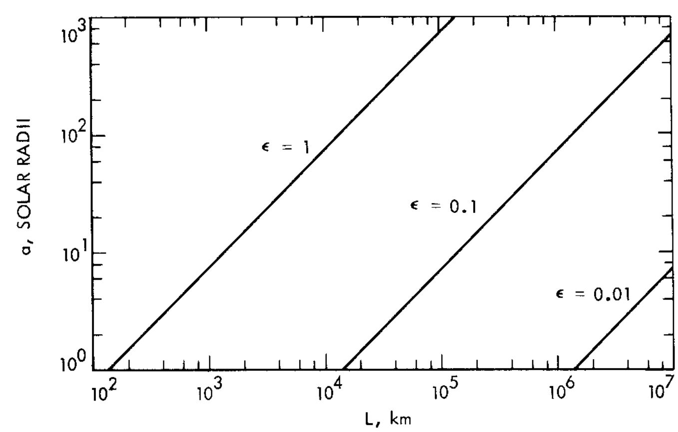


Fig. 7. Scale size for τ = 3600 s vs closest approach distance

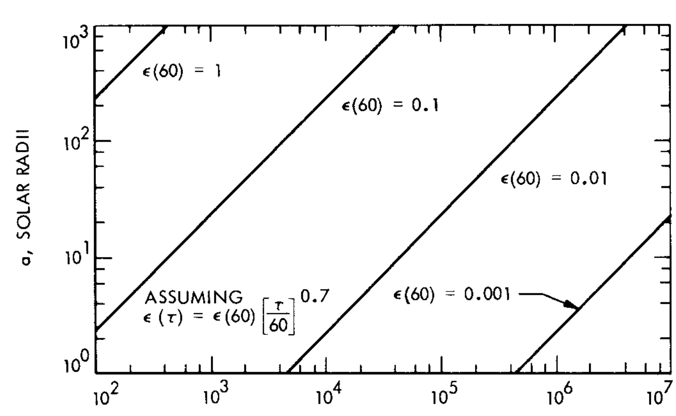


Fig. 8. Scale size vs closest approach distance, assuming

$$\epsilon(\tau) = \epsilon(60)\left[\frac{\tau}{60}\right]^{0.7}$$

Appendix

Proportionality Between Signal Path RMS Phase Fluctuation and Integrated Signal Path Density

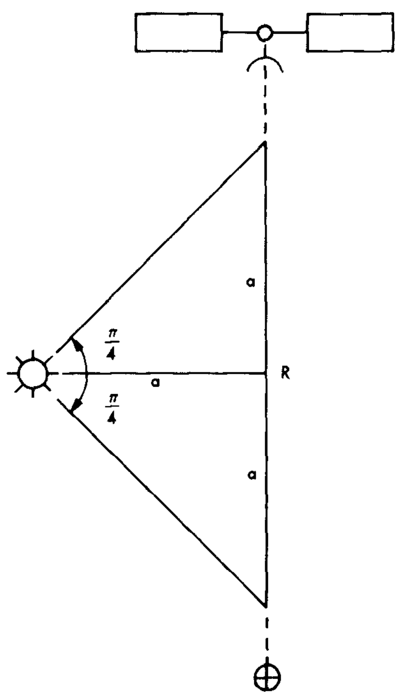
Proof that if:

$$\epsilon \equiv \frac{n}{N_e}$$

then:

$$\left\{ R \int_0^R n^2 \, dR \right\}^{1/2} \approx \epsilon I$$

Let the interval of integration be $-a \leqslant R \leqslant a$ as in diagram below:



where:

$a = r_e \sin \alpha$

r_e = Earth-Sun Distance

α = Sun-Earth-Probe Angle

The signal path rms phase fluctuation is then:

$$R \int_{-a}^{a} n^2 \, dR = 2a \int_{-a}^{a} (\epsilon N_e)^2 \, dR$$

$$= 2a \, \epsilon^2 \int_{-a}^{a} N_e^2 \, dR$$

now (assuming):

$$N_e \cong K/r^{2.3}$$

so that:

$$R \int_{-a}^{a} n^2 \, dR = 2a \, \epsilon^2 K^2 \int_{-a}^{a} \frac{1}{r^{4.6}} \, dR$$

after the fashion of Ref. 12:

$$R \int_{-a}^{a} n^2 \, dR = 2a \, \epsilon^2 K^2 \int_{-a}^{a} \frac{dx}{(x^2 + a^2)^{2.3}}$$

$$= \frac{2a \, \epsilon^2 K^2}{a^{4.6}} \int_{-\pi/4}^{\pi/4} \frac{a \sec^2 w \, dw}{(\tan^2 w + 1)^{2.3}}$$

$$= \frac{2a^2 \epsilon^2 K^2}{a^{4.6}} \int_{-\pi/4}^{\pi/4} (\cos w)^{2.6} \, dw$$

$$\cong \frac{2\epsilon^2 K^2}{a^{2.6}} \int_{-\pi/4}^{\pi/4} \cos^2 w (1 - 0.3 w^2) \, dw$$

$$= \frac{2\epsilon^2 K^2}{a^{2.6}} \left[\int_{-\pi/4}^{\pi/4} \cos^2 w \, dw - 0.3 \int_{-\pi/4}^{\pi/4} w^2 \cos^2 w \, dw \right]$$

$$= \frac{2\epsilon^2K^2}{a^{2.6}} \left[\left\{ 1/2\, w + 1/4 \sin 2w \right\}_{-\pi/4}^{\pi/4} \right.$$

$$-0.3 \left\{ \frac{w^3}{6} + \left(\frac{w^2}{4} - \frac{1}{8} \right) \sin 2w \right.$$

$$\left. \left. + \frac{w \cos 2w}{4} \right\}_{-\pi/4}^{\pi/4} \right]$$

$$= \frac{2\epsilon^2K^2}{a^{2.6}} \left[\frac{\pi}{4} + \frac{1}{2} \right.$$

$$\left. -0.3 \left\{ \frac{1}{3} \left(\frac{\pi}{4} \right)^3 + \frac{1}{4} \left(2 \left[\frac{\pi}{4} \right]^2 - 1 \right) \right\} \right]$$

$$= \frac{2\epsilon^2K^2}{a^{2.6}} \, [1.22]$$

$$= 2.44 \left[\frac{\epsilon^2K^2}{a^{2.6}} \right]$$

or

$$\left\{ R \int_{-a}^{a} n^2 \, dR \right\}^{1/2} = 1.56 \left[\frac{\epsilon K}{a^{1.3}} \right]$$

Now

$$\int_{-a}^{a} N_e \, dR = \int_{-a}^{a} \frac{K}{r^{2.3}} \, dR$$

$$= K \int_{-a}^{a} \frac{dx}{(a^2 + x^2)^{1.15}}$$

$$= \frac{K}{a^{2.3}} \int_{-\pi/4}^{\pi/4} \frac{a \sec^2 w \, dw}{(\tan^2 w + 1)^{1.15}}$$

$$= \frac{K}{a^{1.3}} \int_{-\pi/4}^{\pi/4} (\cos w)^{0.3} \, dw$$

$$\cong \frac{K}{a^{1.3}} \left[w \left(1 - 0.05\, w^2 \right) \right]_{-\pi/4}^{\pi/4}$$

$$= \frac{K}{a^{1.3}} \left[\frac{\pi}{2} \left(1 - 0.05 \left\{ \frac{\pi}{4} \right\}^2 \right) \right]$$

$$= 1.52 \left[\frac{K}{a^{1.3}} \right]$$

Finally,

$$\frac{\left\{ R \int_{-a}^{a} n^2 \, dR \right\}^{1/2}}{\int_{-a}^{a} N_e \, dR} = \frac{1.56 \left[\frac{\epsilon K}{a^{1.3}} \right]}{1.52 \left[\frac{K}{a^{1.3}} \right]}$$

$$= 1.03\epsilon$$

hence

$$\frac{n}{N_e} \equiv \epsilon \approx \frac{\left\{ R \int_{0}^{R} n^2 \, dR \right\}^{1/2}}{\int_{0}^{R} N_e \, dR}$$

References

1. Brunn, L., "Minutes of the Eleventh Ranging Accuracy Team Meeting", IOM 33RA-76-012, Jet Propulsion Laboratory, Pasadena, Calif., 9 December 1976. (JPL internal document).

2. Efron, L. and Lisowski, R. J., "Charged Particle Effects to Radio Ranging and Doppler Tracking Signals in a Radially Outflowing Solar Wind", in *The Space Programs Summary 37 – 56, Volume II: The Deep Space Network,* Jet Propulsion Laboratory, Pasadena, California, 31 March 1969.

3. Private communication with A. Zygielbaum.

4. Cronyn, Willard M., "The Analysis of Radio Scattering and Space-Probe Observations of Small-Scale Structure in the Interplanetary Medium", *Astrophysical Journal,* 161, pp. 755-763, 1 August 1970.

5. Berman, A. L. "A Comprehensive Two-way Doppler Noise Model for Near-Real-Time Validation of Doppler Data" in *The Deep Space Network Progress Report 42-37,* Jet Propulsion Laboratory, Pasadena, California, 15 February 1977.

6. Salpeter, E. E., "Interplanetary Scintillations. I. Theory", *Astrophysical Journal,* 147, 1967.

7. Dutcher, G. L., *A Communication Channel Model of the Solar Corona and the Interplanetary Medium,* CSRT-69-1, Center for Space Research, Massachusetts Institute of Technology, 1969.

8. Jokipii, J. R., and Hollweg, J. V., "Interplanetary Scintillations and the Structure of Solar Wind Fluctuations", *Astrophysical Journal,* 160, May 1970.

9. Callahan, P. S., "Columnar Content Measurements of the Solar Wind Turbulence Near the Sun", *Astrophysical Journal,* 199, July 1, 1975.

10. Cohen, M. H., and Gundermann, E. J., "Interplanetary Scintillations IV. Observations Near the Sun", *Astrophysical Journal* 155, February 1969.

11. Houminer, Z., and Hewish, A., "Long-Lived Sectors of Enhanced Density Irregularities in the Solar Wind", *Planetary Space Science,* Volume 20, 1972.

12. Berman, A. L., and Wackley, J. A., "Doppler Noise Considered as a Function of the Signal Path Integration of Electron Density", in *The Deep Space Network Progress Report 42-33,* Jet Propulsion Laboratory, Pasadena, California, 15 June 1976.