A Model of SNR Degradation During Solar Conjunction

S. T. Rockwell
DSN Network Operations Section

The downlink signal from spacecraft in solar conjunction phases suffers a drastic reduction in signal-to-noise ratio (SNR). Responsible in large part for this effect is the increase in system noise temperature (SNT) in the ground antenna-receiver system. This article presents an empirical model of SNR degradation due to increasing SNT during solar conjunction phases.

I. Introduction

One of the most important parameters in a telecommunication signal processing and detection system is the signal-to-noise ratio or SNR. When a spacecraft undergoes either a superior or inferior solar conjunction this parameter becomes greatly reduced, the reduction spanning several orders of magnitude for small Sun-Earth-Probe (SEP) angles. This change in the SNR, called the SNR degradation, results from several different causes.

During superior conjunctions the signal transmitted by the spacecraft will pass close to the Sun, traversing dense regions of the solar corona. Under these conditions, complicated plasma effects corrupt the signal and degrade the SNR. Additionally, just pointing the antenna in the vicinity of the Sun causes an increase in the system noise temperature (SNT) in the ground antenna-receiver system. Since the noise power is proportional to the SNT, the SNT increase will cause a decrease in the SNR.

In an inferior conjunction, the coronal plasma effects are minimized, leaving most of the SNR degradation due to the increasing SNT. This report is concerned with the modelling of the SNR degradation during inferior solar conjunction, the assumption being made that all of the degradation can be attributed to increases in the SNT. Although the model will be developed using data from an inferior conjunction, it should be clear that the results obtained can be used in estimating that portion of the SNR degradation due to the increasing SNT during superior conjunction.

In order to develop a model of SNR degradation, we first obtain an expression for this quantity. Begin with the definition of the symbol *SNR*:

$$SNR \equiv \frac{ST_S}{kT} \tag{1}$$

where

S = data power

T_S = symbol time

k = Boltzmann constant

T = system noise temperature

Furthermore, when the Sun is neglected, T is the sum of the zenith system noise temperature and an antenna elevation angle correction:

$$T = T_Z + T_{EL} \tag{2}$$

T is known completely since T_Z is a measured quantity and T_{EL} is readily computed from the empirical formula:

$$T_{EL} = A'e^{-B'(EL)} \tag{3}$$

where A' and B' are antenna-dependent constants (see Appendix A) and EL is the antenna elevation angle.

If the antenna is tracking a spacecraft near inferior conjunction, the Sun can be considered an additional noise source. In this case the *SNR* would be written as:

$$SNR' = \frac{ST_S}{kT'} \tag{4}$$

where T' includes the increase in *SNT* due to the Sun. The total *SNT* is now written:

$$T' = T + T_{SUN} \tag{5}$$

where T_{SUN} is to be interpreted as the increase in system noise temperature due to the Sun.

From Eqs. (1) and (4) we can obtain an expression for the SNR degradation. If ΔSNR is the *SNR* degradation in dB due to the increase in SNT during inferior conjunction, then:

$$\Delta SNR = 10 \log_{10}\left(\frac{SNR}{SNR'}\right) = 10 \log_{10}\left(\frac{T'}{T}\right)$$

$$\Delta SNR\ (\text{dB}) = 10 \log_{10}\left(\frac{T + T_{SUN}}{T}\right) \tag{6}$$

It is important to note that the Δ_{SNR} is a function of two variables, T and T_{SUN}.

II. T_{SUN} – The Ambiguous Parameter

When tracking a spacecraft near solar conjunction, the ΔSNR increases as the *SEP* angle decreases. It seems reasonable, then, to try to model the ΔSNR as a function of *SEP*. However, in attempting to model the ΔSNR, we will have to find a related parameter that depends solely on the *SEP* angle. The reason for this becomes apparent if we remember that the ΔSNR varies with both T_{SUN} and T. We assume T_{SUN} depends on the *SEP* angle and, as stated, T depends on the system and antenna elevation angle. It is important to note at this point that if T_{SUN} could be determined as a function of *SEP*, constructing a model of ΔSNR would be simplified.

Before investigating the T_{SUN} parameter further, it is worth mentioning another parameter related to the ΔSNR and varying with *SEP*. This is the total operating system noise temperature T_{OP}. This quantity is measured on a strip chart at the site and is written as

$$T_{OP} = T_Z + T_{ELOP} + T_{SUNOP} \tag{7}$$

T_{SUNOP} is the increase in operating system noise temperature due to the Sun and T_{ELOP} is the actual elevation effect. Since T_{OP} and T_Z are measured and T_{ELOP} is approximated by T_{EL} it would, at first, appear that T_{SUNOP} is an attractive modelling parameter. Unfortunately in addition to measuring the changes in the *SNT*, the strip chart recording also reflects variations in maser gain and atmospheric conditions. This then renders the T_{SUNOP} parameter unusable for our purposes.

Now, continuing with the investigation of T_{SUN}, solve Eq. (6) for T_{SUN}. Some rearranging yields:

$$T_{SUN} = T\left(10^{\frac{\Delta SNR}{10}} - 1\right) \tag{8}$$

One method of developing our model would be to try to construct a set of ΔSNRs from actual measurements and then calculate a corresponding set of T_{SUN} values. The actual data will also yield a corresponding set of *SEP* angles. An empirical model could then be constructed providing the desired T_{SUN}, *SEP* relationship. We will adopt this approach in building the model. However, before calculating the ΔSNR set it will first be necessary to take a closer look at the system.

Before the spacecraft's signal reaches the ground antenna it has associated with it an incoming *SNR* (denoted SNR_{IN}). In a conjunction situation, with the Sun introduced as an extra noise source, the incoming *SNR* is degraded by an amount ΔSNR, producing a degraded incoming *SNR* (denoted $DSNR_{IN}$). See Fig. 1.

Mathematically, the above quantities are related as:

$$DSNR_{IN} = SNR_{IN} - \Delta SNR \tag{9}$$

Once in the system the $DSNR_{IN}$ undergoes the usual system losses (L_S) with a still more degraded *SNR* emerging (denoted $DSNR_{OUT}$). The L_S is estimated from the Telemetry Analysis Program (TAP) using an iterative method. The losses in the degraded SNR due to the system are expressed mathematically by the equation:

$$DSNR_{IN} = DSNR_{OUT} + L_S \tag{10}$$

The $DSNR_{OUT}$ is a quantity measured by the telemetry system and, therefore, $DSNR_{IN}$ can be determined.

With the above interpretation it is possible to calculate ΔSNR and hence T_{SUN} by starting with the measured quantity $DSNR_{OUT}$ and working backwards through the system. The only quantity unaccounted for is the SNR_{IN}. This can be estimated using the formula (see Appendix B):

$$SNR_{IN} = P_C + 20 \log \tan \phi - 10 \log k + 10 \log T_S - 10 \log T \tag{11}$$

where

P_C = downlink carrier power (dBm)

ϕ = modulation index

k = Boltzmann constant $\left(\frac{\text{mW} - \text{s}}{\text{K}}\right)$

T_S = symbol time

Putting the above ideas together, we can compute T_{SUN} as follows. Starting with the measured value of $DSNR_{OUT}$, Eq. (10) can be iterated to yield an approximate value of $DSNR_{IN}$. Combining Eqs. (11) and (9) gives for ΔSNR:

$$\Delta SNR = P_C + 20 \log \tan \phi + 10 \log T_S - 10 \log k - 10 \log T - DSNR_{IN} \tag{12}$$

Finally, substitution of this into Eq. (8) yields T_{SUN}:

$$T_{SUN} = T\left(10^{\frac{\Delta SNR}{10}} - 1\right) \tag{8}$$

This parameter, T_{SUN}, will be used as the modelling parameter. It is ambiguous inasmuch as it is subject to the interpretation of how the Sun actually affects the signal and the antenna-receiver system. One interpretation (the one used here) is to consider the Sun as an external noise source. Another would be to consider the system performance degradation by the Sun. In this latter case a more complicated set of equations would have been needed to find T_{SUN}. The interpretation adopted here is that illustrated in Fig. 1.

III. The Data

The data used in this study were obtained during the inferior conjunctions of Helios-1 and Helios-2 in early 1976. Specifically, the data span the following time periods:

Helios-1 DOY 066 – DOY 083 1976

Helios-2 DOY 074 – DOY 098 1976

Helios-1 achieved a SEP = 0 on DOY 074

Helios-2 achieved a SEP = 0 on DOY 084

Data were taken at both 26 and 64-meter-diameter antenna sites, each treated as an independent data set. Identical modelling methods were used on both sets of data to produce two separate models.

The data actually collected were the parameters needed to calculate T_{SUN}. These parameters include the downlink signal strength, modulation index, symbol rate, zenith noise temperature, elevation angle, *SEP* angle and $DSNR_{OUT}$. The working data base consisted of the two T_{SUN} sets and corresponding

SEP angles and was calculated from the actual data. A listing and a semilogarithmic plot of T_{SUN} vs *SEP* for both 26-m and 64-m data (Figs. 2, 3) are found in Appendix C.

IV. The Fit

In the previous sections we have seen how the T_{SUN} parameter will be used to model the *SNR* degradation during inferior conjunctions. Now what is needed is the relation between T_{SUN} and *SEP*. To obtain this relation it was decided to fit the data empirically, concentrating on the region spanned from 0 to 5 deg *SEP*.

Upon inspection of the graphical data (semilogarithmic plot) it was observed that T_{SUN} and *SEP* might be inversely proportional. It was, therefore, decided to try fitting a function of the form:

$$\ln T_{SUN} \propto \frac{1}{SEP}$$

Guided by the above functional form, we can construct the function:

$$T_{SUN} = A \exp \left\{ \frac{B}{SEP + C} \right\} \tag{13}$$

where A, B, and C are coefficients to be determined. Note that each of the coefficients affects a different characteristic of the curve. A scales the curve along the vertical axis, and B and C scale and translate the curve along the horizontal axis, respectively.

In fitting the curve, the coefficient C was set to zero and A and B were determined by a least squares fit. C was then varied through a range of values until a minimum standard deviation was found. With this new value of C, A and B were redetermined by least squares.

Because of the spread in the data (several orders of magnitude), the residuals and the standard deviation were expressed in logarithmic form:

$$\Delta(\text{dB}) = \text{residual in dB} = 10 \log \left(\frac{ACTUAL\ T_{SUN}}{PREDICTED\ T_{SUN}} \right)$$

$$\sigma(\text{dB}) = \text{standard deviation in dB} = \sqrt{\sum_{i=1}^{N} (\Delta_i)^2 / N}$$

The values of the coefficients yielding the best fit in each case are presented in the following table:

ANT	A	B	C
26	2.75	8.97	0.90
64	5.60	4.57	0.28

The standard deviation and maximum residual for these fits are:

ANT	σ (dB)	Δ_{MAX} (dB)	SEP(°)
26	1.737	-6.118	2.89
64	2.383	+5.571	0.74

By way of comparison, the following statistics are offered from a previous study utilizing a least squares curve fit of the function:

$$\ln T_{SUN} = A + B(SEP) + C(SEP)^2 + D(SEP)^3$$

$$\sigma\ (\text{dB}) = 2.7813$$

$$\Delta_{MAX}\ (\text{dB}) = -10.228$$

This maximum residual occurred at $SEP = 4^{\circ}\!.37$.

In their final form, the equations relating T_{SUN} and *SEP* are:

$$T_{SUN} = 2.75 \exp \left\{ \frac{8.97}{SEP + .90} \right\} \quad \text{26-m case} \tag{14a}$$

$$T_{SUN} = 5.60 \exp \left\{ \frac{4.57}{SEP + .28} \right\} \quad \text{64-m case} \tag{14b}$$

where *SEP* is in degrees.

V. T_{SUN} vs T_{SUNOP}

In an effort to obtain T_{SUN} more easily than by the method of Section II, the T_{SUN} data were plotted against the T_{SUNOP} data (obtained from T_{OP} strip chart measurements). Both the 26-m and 64-m data were plotted (see Figs. 4 and 5).

The data were plotted on log-log paper and exhibited a high degree of correlation:

$r = 0.88 \quad 26\ \text{m}$

$r = 0.97 \quad 64\ \text{m}$

A linear regression on the data yielded the following fit:

$$\log_{10} T_{SUN} = A \log_{10} T_{SUNOP} + B \tag{15}$$

where

ANT	A	B
26-m	1.052	-0.043
64-m	1.148	-0.141

Some rearranging yields the following expressions,

$$T_{SUN} = (0.91)\, T_{SUNOP}^{(1.05)} \qquad 26\ \text{m} \tag{16a}$$

$$T_{SUN} = (0.72)\, T_{SUNOP}^{(1.15)} \qquad 64\ \text{m} \tag{16b}$$

These equations enable a user to estimate a T_{SUN} value from a readily obtainable T_{SUNOP}. It is interesting to note the departure of the data from the slope 1 line. This is probably due to maser gain variation.

VI. Summary

As we have seen, it is possible to model *SNR* degradation as a function of *SEP* (due to increasing *SNT*) during solar conjunctions to a reasonable degree of accuracy. To do so, however, requires the introduction of an intermediate parameter that depends only on the *SEP* angle, since the *SNR* degradation depends on both the SEP and antenna elevation angles. From Eq. (6) it was seen that T_{SUN} is a reasonable choice for this modelling parameter.

In addition, from a theoretical standpoint, it was seen that the explicit expression for T_{SUN} was dependent on the physical interpretation of the Sun-signal-system interaction. In this study, the point of view taken is that the solar effects are treated as a noise source.

From measured degraded *SNR* data, a T_{SUN} vs *SEP* data base was constructed using the procedure of Section II. This data base was divided into two groups: 26-m data and 64-m data. These were treated as independent data. A curve fit of the data provided a functional relationship between T_{SUN} and *SEP*.

What we now have is a model for *SNR* degradation as a function of *SEP*. The only inputs required are T_Z (a pretrack measurement), antenna elevation angle, symbol bit rate, and antenna size. The complete model is:

$$\Delta SNR\ (\text{dB}) = 10 \log_{10} \left(\frac{T_Z + T_{EL} + T_{SUN}}{T_Z + T_{EL}} \right)$$

$$T_{EL} = A' \exp - B' (EL)$$

$$T_{SUN} = A \exp \left\{ \frac{B}{SEP + C} \right\}$$

The assumptions made in developing this model have been, admittedly, oversimplified.

In reducing the data, effects due to the weather and the quadrapod structure have been neglected. However, the most important improvements probably lie in the area of Sun-signal-system interpretation. An understanding of how the Sun actually affects the signal and system during inferior conjunction will, undoubtedly, be one of the more interesting approaches to building a better model.

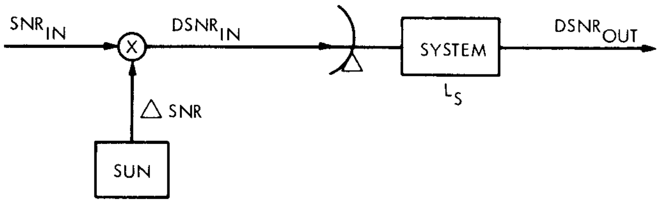

Fig. 1. Sun-signal-system interpretation

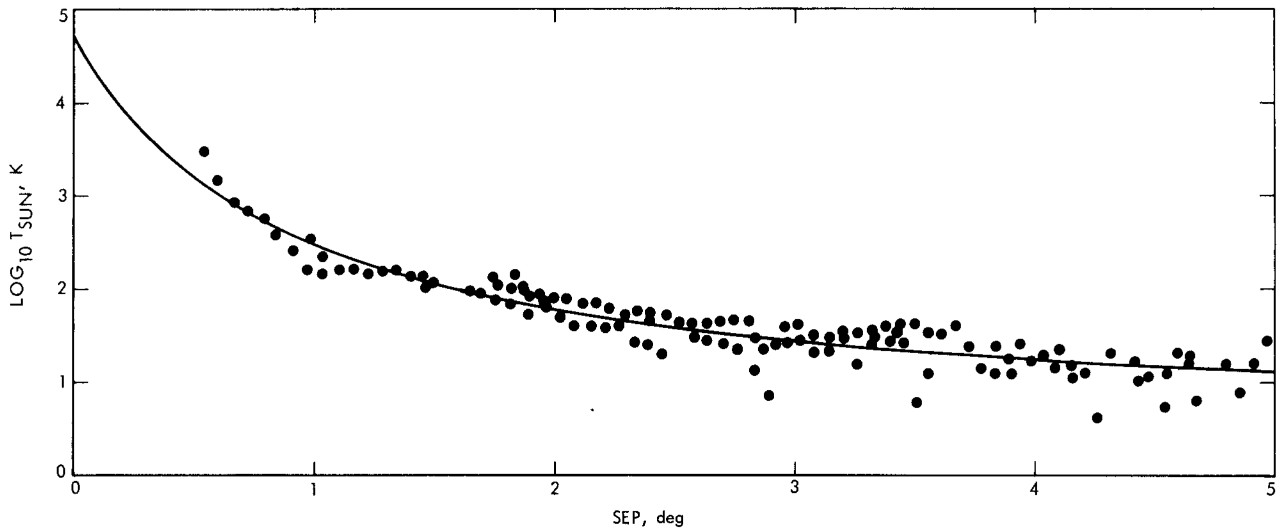

Fig. 2. Helios 1 and 2, T_{SUN} vs *SEP*, 26-m-antenna data

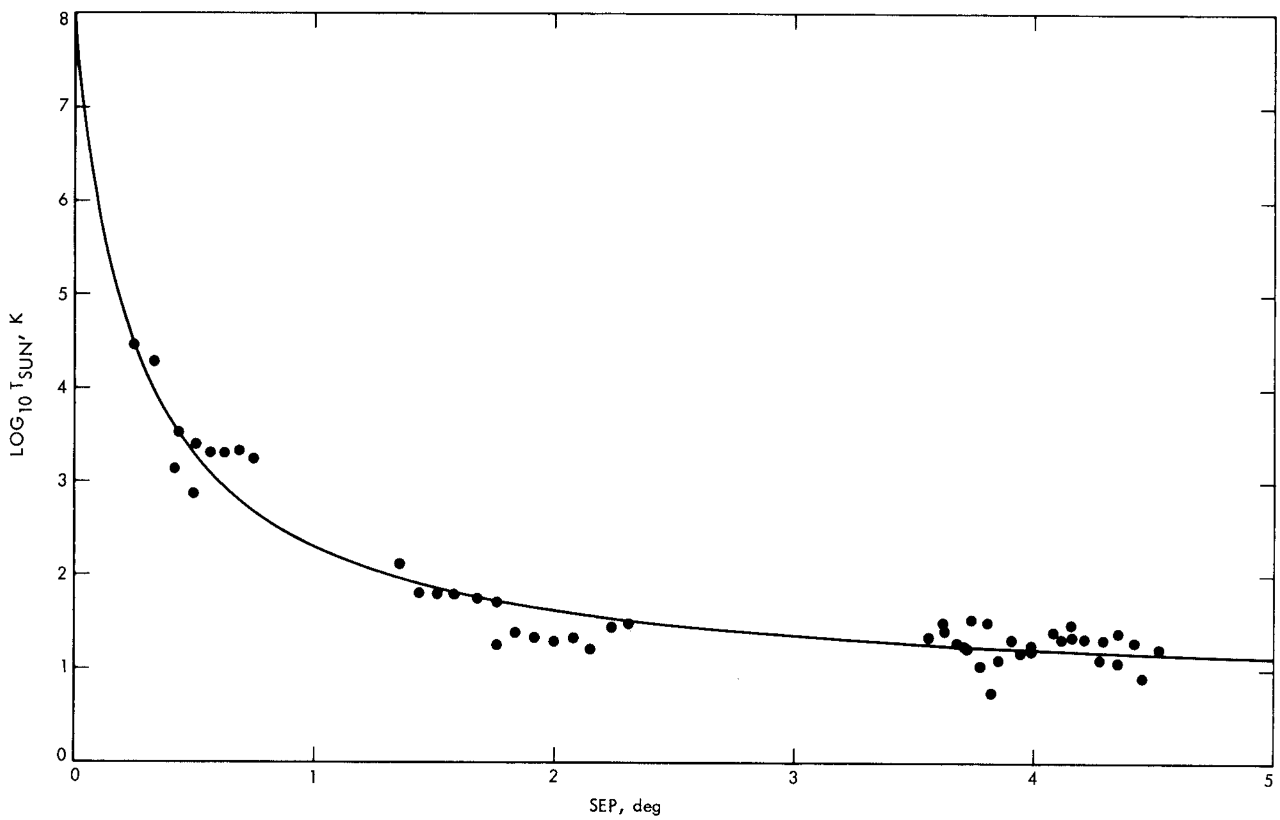

Fig. 3. Helios 1 and 2, T_{SUN} vs *SEP*, 64-m-antenna data

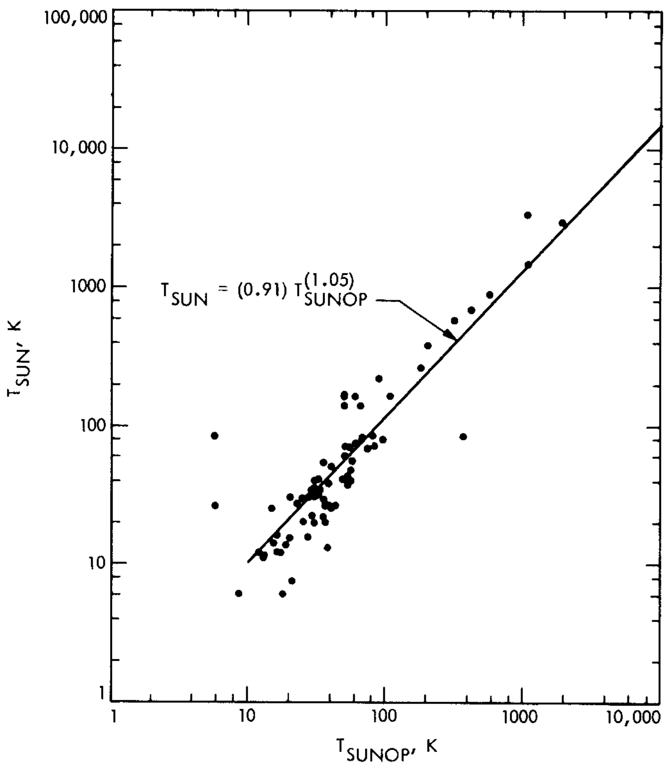

Fig. 4. Helios 1 and 2, T_{SUN} vs T_{SUNOP} 26-m-antenna data

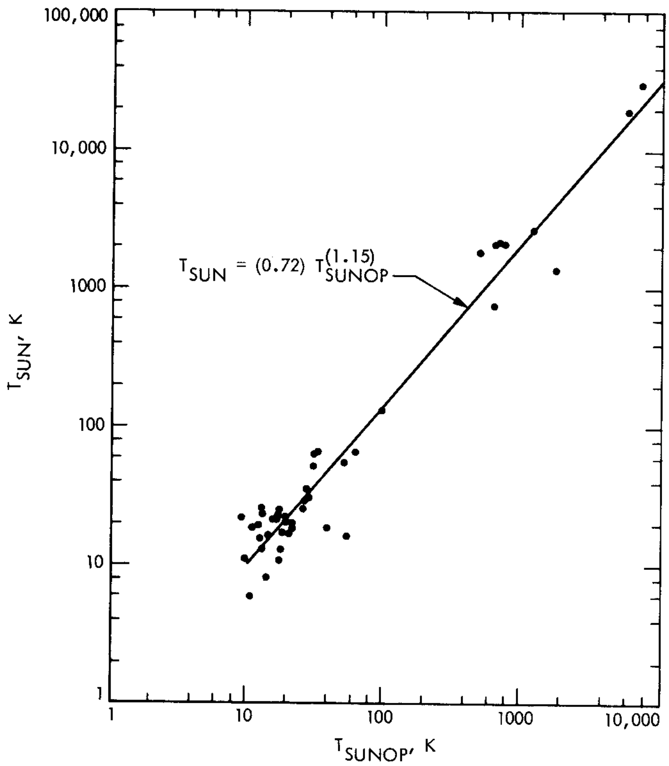

Fig. 5. Helios 1 and 2, T_{SUN} vs T_{SUNOP} 64-m-antenna data

Appendix A
Elevation Correction Coefficients

26-m antenna	64-m antenna
$A' = 29.91$	$A' = 25.90$
$B' = 0.051$	$B' = 0.066$

Appendix B
SNR_{IN} Equation Derivation

To obtain Eq. (11), start with Eq. (1):

$$SNR = \frac{ST_S}{kT}$$

Expressing SNR in dB write

$$10 \log SNR = 10 \log \frac{ST_S}{kT}$$

$$SNR\ (\text{dB}) = 10 \log S + 10 \log T_S - 10 \log k - 10 \log T$$

Now, put $10 \log S$, data power in dBm, in a more useful form. We know:

$$\frac{S_C}{S_T} = \cos^2 \phi \qquad \text{and} \qquad \frac{S_D}{S_T} = \sin^2 \phi$$

where

S_D = data power

S_C = carrier power

S_T = total power

ϕ = modulation index

$$\frac{S_D}{S_C} = \tan^2 \phi \qquad \text{or} \qquad S_D = S_C \tan^2 \phi$$

Expressing S_D in dBm:

$$10 \log S_D = 10 \log S_C + 20 \log \tan \phi$$

$$S_D\ (\text{dBm}) = S_C\ (\text{dBm}) + 20 \log \tan \phi$$

$$SNR\ (\text{dB}) = S_C\ (\text{dBm}) + 20 \log \tan \phi + 10 \log T_S - 10 \log k - 10 \log T$$

Appendix C

T_{SUN} vs SEP

26-m-antenna data					
SEP	T_{SUN}	SEP	T_{SUN}	SEP	T_{SUN}
0.54	2948.67	2.45	20.27	4.04	19.02
0.60	1468.44	2.46	53.16	4.08	13.96
0.67	863.26	2.51	43.19	4.10	21.71
0.73	692.40	2.57	41.93	4.15	10.99
0.79	565.20	2.58	30.87	4.15	14.75
0.85	373.77	2.63	42.97	4.21	12.25
0.91	260.41	2.64	26.92	4.26	4.10
0.97	163.63	2.69	45.02	4.31	20.02
0.98	341.03	2.70	26.03	4.41	16.21
1.04	141.46	2.75	46.56	4.43	10.35
1.04	221.87	2.77	22.44	4.47	11.56
1.10	157.51	2.81	45.95	4.54	12.10
1.16	158.10	2.83	13.65	4.54	5.14
1.22	141.18	2.84	28.52	4.59	20.48
1.28	162.62	2.87	22.75	4.60	13.52
1.34	155.88	2.89	7.17	4.64	15.58
1.40	138.73	2.92	25.65	4.64	18.21
1.46	135.01	2.95	38.19	4.67	6.06
1.46	108.61	2.97	26.51	4.73	13.67
1.49	109.52	3.01	41.03	4.79	14.98
1.64	94.44	3.02	28.78	4.85	7.53
1.70	92.37	3.07	31.61	4.91	15.38
1.74	78.86	3.08	20.82	4.97	28.52
1.76	110.58	3.14	22.65		
1.77	98.94	3.14	29.72		
1.82	68.40	3.20	34.13		
1.82	102.81	3.20	30.77		
1.83	140.07	3.20	34.13		
1.87	102.97	3.26	34.23		
1.88	95.15	3.26	15.52		
1.88	98.43	3.32	26.21		
1.90	55.68	3.32	35.95		
1.90	84.11	3.33	30.60		
1.94	85.77	3.38	39.53		
1.96	73.19	3.40	26.51		
1.97	63.77	3.43	34.07		
1.99	82.22	3.44	40.42		
2.02	48.20	3.45	25.91		
2.05	76.63	3.45	25.91		
2.08	40.39	3.50	40.61		
2.11	68.52	3.50	5.96		
2.15	41.97	3.55	11.99		
2.17	70.36	3.55	33.81		
2.21	37.55	3.61	32.69		
2.23	60.29	3.67	37.42		
2.27	40.99	3.73	23.33		
2.29	53.32	3.77	13.63		
2.33	26.87	3.83	24.45		
2.34	56.00	3.84	11.82		
2.39	24.93	3.88	17.79		
2.40	53.90	3.90	11.90		
2.40	45.21	3.93	25.56		
		3.98	16.61		

64-m-antenna data			
SEP	T_{SUN}	*SEP*	T_{SUN}
0.24	29126.79	3.62	25.50
0.32	18968.85	3.67	18.99
0.41	1352.23	3.70	18.17
0.43	3328.34	3.73	16.83
0.49	740.89	3.73	34.41
0.50	2577.77	3.77	10.59
0.56	2035.71	3.79	31.57
0.62	2035.72	3.82	5.79
0.68	2125.27	3.85	12.85
0.74	1788.51	3.90	20.69
1.35	128.59	3.93	14.96
1.43	64.33	3.98	18.16
1.51	64.70	4.00	16.43
1.59	63.26	4.07	25.08
1.67	54.60	4.11	21.76
1.75	51.76	4.15	30.10
1.75	17.95	4.15	22.56
1.83	25.39	4.21	21.52
1.91	22.02	4.27	12.69
1.99	19.81	4.28	21.03
2.07	21.59	4.35	11.88
2.15	15.97	4.35	24.58
2.23	28.40	4.41	19.49
2.30	30.35	4.44	7.90
3.55	21.58	4.52	16.61
3.61	31.10		