The Application of Differential VLBI to Planetary Approach Orbit Determination

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The development of multistation tracking techniques has resulted in improved orbit determination accuracies. Differential Very Long Baseline Interferometry (VLBI), which involves performing measurements of a spacecraft and nearby extra galactic radio source and differencing, potentially offers at least an order of magnitude improvement over data types currently implemented. In this article, the application of differential VLBI to planetary approach orbit determination is described.

I. Introduction

The accuracy of planetary approach orbit determination using radio metric data types is generally limited by station location and planet ephemeris errors. With conventional doppler and range data taken over a single station pass, station location errors translate into right ascension and declination errors in a geocentric frame. The accuracy of this data type is approximately 0.5 μrad and require relatively long data arcs to resolve spacecraft state errors.

The development of multistation tracking techniques has resulted in a considerable reduction in angle measurement error. Using the relative positions of Deep Space Stations (DSS) on Earth as a baseline, precise angle measurements are obtained by simply measuring the difference in time of arrival of a spacecraft radio signal at two or more stations. This technique is referred to as Very Long Baseline Interferometry (VLBI). A recent refinement involves eliminating common effects such as station location errors and transmission media delay by differencing the spacecraft radio signal with a nearby fixed radio source such as a quasar. This technique, referred to as differential VLBI (ΔVLBI), yields angle measurement precision of potentially 0.025 μrad.

There are two methods of implementing ΔVLBI. The first method involves the development of radio source catalogs and the determination of planet ephemerides in an inertial radio source fixed coordinate frame. A spacecraft orbit may thus be determined relative to a planet with the accuracy inherent in the ΔVLBI measurement. This method is often referred to as global ΔVLBI. The second method involves measuring angle rates and determining the planet position error from the observed acceleration of the spacecraft. This method, referred to as local ΔVLBI, results in less accurate spacecraft orbit determination ultimately attainable by global ΔVLBI, but is insensitive to planet ephemeris errors.

The development of radio source catalogs and determination of planet ephemerides to the required accuracy for global ΔVLBI will not be complete in the near future. However, local ΔVLBI, which uses only the inherent accuracy of the ΔVLBI data type is nearly available.

In this article, the application of local ΔVLBI to planetary approach orbit determination is described. A simplified model based on conic motion is developed for predicting the orbit determination error given the ΔVLBI measurement error. This simplified model is applied to the large outer planets for an overall assessment of the effectiveness of ΔVLBI orbit determination. For Jupiter and Saturn the results are encouraging. As early as 61 days before Jupiter closest approach, the ΔVLBI measurement error is sufficiently accurate to detect the planet ephemeris error. As early as 10 days before encounter the orbit determination error may be reduced to within 200 km, permitting an approach trajectory correction maneuver if required. The results for Uranus and Neptune are less encouraging but may permit a late update of camera pointing for reconnaissance or antenna pointing for relay link. The critical parameter limiting the usefulness of ΔVLBI is the magnitude of approach velocity. Tour type missions with large values of V_∞ (greater than 10 km/sec) approach the planet with a mean motion too great to be detected until near encounter. Orbiter missions favor ΔVLBI orbit determination since approach velocity must be low for fuel economy.

As an example of the application of ΔVLBI, the proposed Jupiter Orbiter Probe (JOP) mission is selected. A comparison of ΔVLBI with current baseline radio metric data (e.g., doppler and range) revealed a dramatic improvement in approach orbit determination. Ten days before encounter, the orbit determination error is reduced from 600 to 310 km with a conservative data acquisition strategy. Doppler and range data does not achieve this level of accuracy until within 3 days of encounter with continuous spacecraft tracking coverage.

II. Analysis

The ΔVLBI measurement technique provides an accurate spacecraft state determination in a geocentric coordinate system relative to fixed radio sources. When coupled with doppler or loosely weighted range data, the 0.025-μrad angular measurement error translates into a 2-km position determination error at Mars opposition or approximately 20 km near Jupiter. Using ΔVLBI measurements, the position of a spacecraft relative to Earth may be determined with this precision. The spacecraft state error relative to a planet is therefore dominated by the planet ephemeris error during approach. In the vicinity of a planet, the motion of a spacecraft may be described by a hyperbola. As a spacecraft approaches a planet, gravity begins to accelerate the spacecraft. The planet position may be determined from the magnitude and direction of the acceleration vector. In the limit of direct impact ($e \to 1$) or far out on the hyperbola asymptote, the time-of-flight or $\hat{S}$ direction is related to the magnitude of acceleration and the B-plane parameters ($\mathbf{B} \cdot \hat{R}$ and $\mathbf{B} \cdot \hat{T}$) are related to the direction of acceleration. Because of the inverse square law of gravity, the time-of-flight or $\hat{S}$ direction is twice as sensitive to acceleration errors as the B-plane parameters. Therefore, the $\hat{S}$ direction position error may be determined with twice the precision as B-plane parameters during initial approach to the planet. The **B** magnitude and normal to **B** position error may be determined with equal precision.

A simplified model may be used to assess the potential accuracy of ΔVLBI orbit determination during planetary approach. Consider an out-of-plane (normal to **B**) position error of the planet during approach (δz_∞). Referring to Fig. 1, at some later time the normal to **B** acceleration will result in a displacement δz_T of the trajectory plane. From the geometry,

$$\delta z_T = \frac{x_T}{|\mathbf{B}|}\,\delta z_\infty, \tag{1}$$

and

$$x_T = |\mathbf{B}| - x_b. \tag{2}$$

From Ref. 1, the trajectory bending (x_b) is given by,

$$x_b = |\mathbf{B}|\left[1 - \frac{\exp(F)}{e}\right], \tag{3}$$

where F is the hyperbolic eccentric anomaly. Substituting Eqs. (2) and (3) into (1) yields

$$\delta z_T = \frac{\exp(F)}{e}\,\delta z_\infty \tag{4}$$

The differential bending of the trajectory (δz_T) is directly observable by ΔVLBI coupled with loosely weighted range data. Initial tracking establishes the direction and position of the approach asymptote with respect to the a priori position of the planet at encounter. The displacement of the planet from the a priori (δz_∞) is the out-of-plane orbit determination error. Referring again to Fig. 1, the differential bending per unit orbit determination error ($\delta z_T/\delta z_\infty$) is plotted as a function of mean anomaly for a parabola ($e = 1$). The mean anomaly is given by

$$M = \frac{V_\infty^3}{\mu}\,(t - t_{ca}) = e \sinh F - F, \tag{5}$$

where t is referenced to time of closest approach (t_{ca}).

Consider as an example the proposed JOP mission. For an approach velocity (V_∞) of 5.73 km/sec, the mean anomaly 10 days before encounter is −1.29 as determined from Eq. (5).

From the curve shown in Fig. 1, the differential bending is approximately 0.16 per unit orbit determination error. Thus, to achieve a 100-km orbit determination error 10 days before Jupiter encounter, a ΔVLBI position determination accuracy of 16 km (.16 × 100) would be required. This result is somewhat optimistic since it ignores the effect of nongravitational accelerations on determining the a priori coordinates of the approach asymptote in the B-plane.

For assessment of approach orbit determination error, a more convenient form of Eq. (4) is the reciprocal. Fig. 2 shows the ratio of orbit determination to ΔVLBI error as a function of mean anomaly with eccentricity as a parameter. The limiting slope of these curves is -2 and is achieved relatively close to encounter for eccentricity less than 10. A locus of minimum values as a function of mean anomaly is plotted with the eccentricity of the minimum identified. The orbit determination accuracy ratio is obtained by interpolation between the locus of minimums and the actual curves shown.

The orbit determination accuracy ratios shown in Fig. 2 assume a data arc starting at minus infinity. It will be necessary to start the data arc at some finite epoch where the gravity gradient is detectable. Consider a data arc beginning at t_1 and ending at t_2. The corresponding mean anomalies are M_1 and M_2. From Fig. 2, the orbit determination accuracy ratios for tracking from minus infinity are $\sigma_{OD_1}/\sigma_{VLBI}$ and $\sigma_{OD_2}/\sigma_{VLBI}$, respectively. The orbit determination accuracy ratio over the tracking interval from t_1 to t_2 is approximately,

$$\frac{\sigma_{OD_{12}}}{\sigma_{VLBI}} = \frac{1}{\dfrac{1}{\sigma_{OD_2}/\sigma_{VLBI}} - \dfrac{1}{\sigma_{OD_1}/\sigma_{VLBI}}} \tag{6}$$

III. Application to Outer Planets

During approach, ΔVLBI is a useful data type when the accuracy of position determination is roughly equal to the planet ephemeris error. At this point, the ΔVLBI orbit determination error decreases linearly with time, as shown in Fig. 2, until close to encounter, where the trajectory dynamics result in a dramatic reduction. Table 1 shows a comparison of ΔVLBI effectiveness for the large outer planets. The first column of Table 1 contains the planet ephemeris error. Current Earth-based optical measurement techniques yield angular accuracies that approach 0.5 μrad. The planet position error scales proportional to range from Earth. The second column contains the ΔVLBI position determination error which also scales proportional to range. The angle measurement accuracy is 0.025 μrad. We are interested in the time from encounter, where the ΔVLBI orbit determination error is equal to the planet position error. Referring to Fig. 2, this occurs at a mean anomaly of approximately -7.5, where the orbit determination accuracy ratio is 20 (Column 1 of Table 1 divided by Column 2) for orbit eccentricities in the range from 1 to 10. It remains to translate mean anomaly into time from encounter. The mean motion is given by

$$\frac{dM}{dt} = \frac{V_\infty^3}{\mu} \tag{7}$$

The hyperbolic approach velocity (V_∞) is dependent on interplanetary trajectory design. Low values of V_∞ favor ΔVLBI orbit determination since the resulting mean motion translates into a longer time for ground data processing and command response. An approximate lower bound for V_∞ may be obtained from the Hohmann transfer orbit. This orbit is the minimum energy transfer orbit from Earth to the target planet. The results of the analysis are shown in Table 1. The time from encounter shown in the last column is obtained by dividing the mean anomaly by the mean motion and may be considered indicative of the time available for ground data processing and command preparation. For Jupiter and Saturn, the time available is sufficient to permit an approach trajectory correction maneuver, if required, to alter the aiming point. The results for Uranus and Neptune are less attractive but may permit a late update of an orbit insertion maneuver or camera pointing sequence.

The above analysis is somewhat optimistic since it assumes a low V_∞ and resulting mean motion. The long flight times associated with the Hohmann transfer orbit make it undesirable from a mission design viewpoint. For tour-type missions, approach velocities of over 10 km/sec may be designed. Because of the cubic relationship of mean motion to V_∞, the effectiveness of ΔVLBI orbit determination would be considerably diminished. It should be noted however, that planet orbiter missions will have approach velocities in the range of Hohmann transfer orbits to minimize orbit insertion fuel expenditure.

IV. Jupiter Orbiter Probe Example

The proposed JOP mission was selected for detailed analysis. The trajectory parameters that pertain to approach orbit determination are:

Encounter date = 15 Nov. 1984

B = 0.1857 E7 km

θ = 20°

V_∞ = 5.73 km/sec

$\alpha_\infty = 196.9^\circ$, $\delta_\infty = -5.81^\circ$ } Approach asymptote in Earth mean equator of 1950 coordinates

The first phase of this analysis was devoted to comparison of ΔVLBI data sets with conventional radio metric data. Because of the newness of the ΔVLBI data type, a conservative data acquisition strategy was simulated. For simplicity, ΔVLBI data is modeled as a geocentric right ascension and declination angle measurement with an accuracy of 0.025 μrad. Since the location of radio sources in the Earth mean equator of 1950 coordinate system may not be known with precision, a bias is added to each angle measurement. It will be necessary to solve for these biases, and the assumed a priori uncertainty of 0.5 μrad is consistent with the error in planet ephemerides. The simulated data rate is one point per day. Deep Space Station 14 (Goldstone, California) and 42 (Australia) were selected for ΔVLBI measurements and a single doppler or range data point is taken from DSS 14 during each pass. The doppler measurement error is 1 mm/sec for a 60-sec count time and is compressed to one hour. Range data is loosely weighted at 20 km. For comparison, continuous doppler at a data rate of one point per hour was taken from the DSS station with the greatest elevation angle.

The results of this comparison are shown in Fig. 3. B-plane semi-major axis (SMAA) is plotted as a function of time from encounter. Estimated parameters are spacecraft state, Jupiter ephemeris and mass. Station location errors are considered. The ΔVLBI data arcs begin at encounter minus 50 days (E - 50d) and the estimated B-plane SMAA levels off around 600 km. The combined a priori planet position and ΔVLBI bias errors map into about 600 km SMAA in the B-plane. Around E - 25d the gravity gradient is sufficient to affect the solutions, and SMAA error is reduced as the trajectory bending is detected through encounter. Also shown for comparison is a conventional doppler-only solution. The data arc starts at E - 50d and levels off at 600 km SMAA due to planet ephemeris and station location errors. At about E - 5d the trajectory dynamics resulting from Jupiter's gravity begin to affect the solution. Because of the high data rate (1 pt/hour), the doppler-only solution will overtake the ΔVLBI solution near encounter, providing a more accurate orbit estimate. In this region, a more meaningful comparison would be obtained by increasing the doppler data rate coupled with ΔVLBI measurements from 1 pt/day to 1 pt/hour.

Other parameters that must be modeled and may need to be estimated include Jupiter's gravity harmonics and the masses of the larger inner satellites. The need to solve for these parameters may be ascertained by examining the sensitivity of the ΔVLBI measurement to the a priori parameter uncertainty. An acceleration uncertainty of 10^{-12} km/sec^2 approaches the threshold of ΔVLBI measurement accuracy when acting on the spacecraft for around 50 days. It becomes necessary to estimate or consider a gravitational parameter when the spacecraft trajectory passes sufficiently close to a body that the perturbative effect of the parameter uncertainty is sufficient to cause an acceleration uncertainty of 10^{-12} km/sec^2 or greater. We may define the distance from a body where the uncertainty in acceleration is sufficient to influence the spacecraft state solution as the radius of orbit determination corruption r_c. Table 2 contains a tabulation of r_c for the gravitational parameters of interest relative to Jupiter approach. Included are the mass and dominant zonal harmonics of Jupiter and the masses of Io, Europa, Ganymede, and Callisto.

The gravitational parameter uncertainties in Table 2 were obtained from analyses of doppler data taken from the Pioneer 10 and Pioneer 11 spacecraft (Ref. 2). With the exception of Jupiter's mass, the radii of orbit determination corruption for gravity parameters are relatively small. Jupiter's zonal harmonics do not influence spacecraft state solutions until within 1.25×10^6 km (less than 3 days before encounter). The satellites may present a problem as early as 14 days before encounter, depending on the position in their respective orbits.

The accuracy of ΔVLBI approach orbit determination is dependent on the data acquisition strategy. In general, the longer the data arc and the greater the number of data points, the smaller the orbit determination error. The final phase of this analysis is devoted to determining the effect of data quantity and density on orbit determination error. Figure 4 shows four data acquisition strategies that tend to bound the range of orbit determination errors that may be experienced during flight operations. The solid curves present results for a short data arc starting at E - 50d. At E - 10d, the one point per day data rate results in a B-plane SMAA of 310 km. With only three data points (taken at E - 50d, E - 30d, and E - 10d) the SMAA is 550 km. An additional data point at E - 5d reduces the error in SMAA to 350 km. The relatively poor performance of these short data arcs may be attributed to the large a priori uncertainty in spacecraft approach velocity assumed for this analysis.

This is far too conservative. With conventional radio metric data taken over a long data arc of 100 days or more, the approach velocity should be determined to the same level of accuracy as the planet velocity (on the order of 10^{-5} km/sec). Referring again to Fig. 4, the dashed curves are representative of the results expected for a "long" data arc. The a priori on the approach velocity is set to virtually zero, permitting a precise determination of the B-plane coordinates of the approach asymptote. The one point per day data rate results in an SMAA of 190 km at E - 10d, and the three data point solution is 270 km.

The above results for the "long" data arc may be compared with the orbit determination error predicted by Eq. (6). For the JOP approach velocity of 5.73 km/sec, the mean anomaly is -6.45 at $E - 50d$ and -1.29 at $E - 10d$. From Fig. 2, the orbit determination accuracy ratios for $e = 1.1$ are $\sigma_{OD_1}/\sigma_{VLBI} = 19$ and $\sigma_{OD_2}/\sigma_{VLBI} = 6.5$, respectively. An orbit determination accuracy ratio of $\sigma_{OD_{12}}/\sigma_{VLBI} = 9.88$ is obtained from Eq. (6). For the specific case being analysed here, the ΔVLBI position measurement accuracy is 18 km and the expected orbit determination error is 178 km SMAA at $E - 10d$. This result is consistent with the long arc results shown in Fig. 4.

References

1. Russell, R. K., "Gravity Focussing of Hyperbolic Trajectories," TM 391-424, Jet Propulsion Laboratory, Pasadena, Calif., March 30, 1973 (an internal document).
2. Null, G. W., "Gravity Field of Jupiter and Its Satellites from Pioneer 10 and Pioneer 11 Tracking Data," *Astron. J.*, Vol. 18, No. 12, Dec. 1976.

Table 1. Application of ΔVLBI to outer planet approach orbit determination

Planet	Planet position error[a]	ΔVLBI position error[b]	ΔVLBI = Planet Position Error: Mean anomaly M	Hohmann transfer V_{∞}	Mean motion dM/dt	Time from encounter
	km	km	rad	km/sec	rad/day	day
Jupiter	400	20	-7.5	5.6	0.123	-61
Saturn	740	37	-7.5	5.4	0.368	-20
Uranus	1480	74	-7.5	4.7	1.51	-5
Neptune	2324	117	-7.5	4.1	0.833	-9

[a]Assumes approximately 0.5 μrad error.

[b]Assumes 0.025 μrad measurement error.

Table 2. Radius of orbit determination corruption

Body	Parameter	σ_p	σ_a^{a}	r_c
Jupiter	GM_5	484 km^3/sec^2	10^{-12} km/sec^2	22 × 10^6 km
Jupiter	J_2	4 × 10^{-6}	10^{-12} km/sec^2	1.25 × 10^6 km
Jupiter	J_3	7 × 10^{-6}	10^{-12} km/sec^2	0.35 × 10^6 km
Jupiter	J_6	50 × 10^{-6}	10^{-12} km/sec^2	0.41 × 10^6 km
Io	GM_1	28 km^3/sec^2	10^{-12} km/sec^2	5.29 × 10^6 km
Europa	GM_2	32 km^3/sec^2	10^{-12} km/sec^2	5.66 × 10^6 km
Ganymede	GM_3	37 km^3/sec^2	10^{-12} km/sec^2	6.1 × 10^6 km
Callisto	GM_4	24 km^3/sec^2	10^{-12} km/sec^2	4.9 × 10^6 km

[a]Sensitivity threshold of ΔVLBI measurement to acceleration.

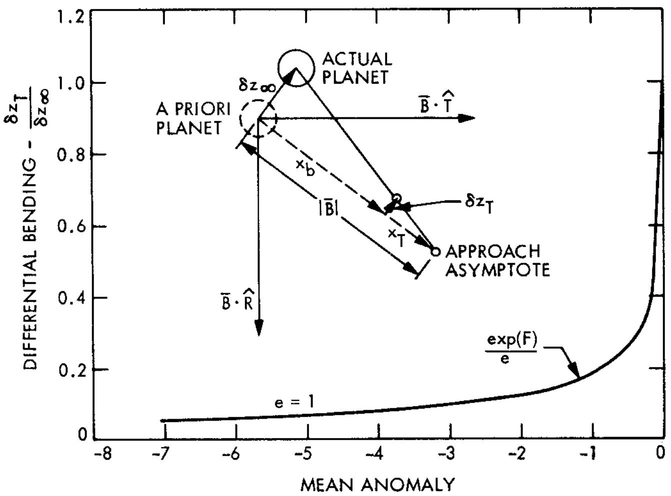

Fig. 1. Differential bending as a function of mean anomaly

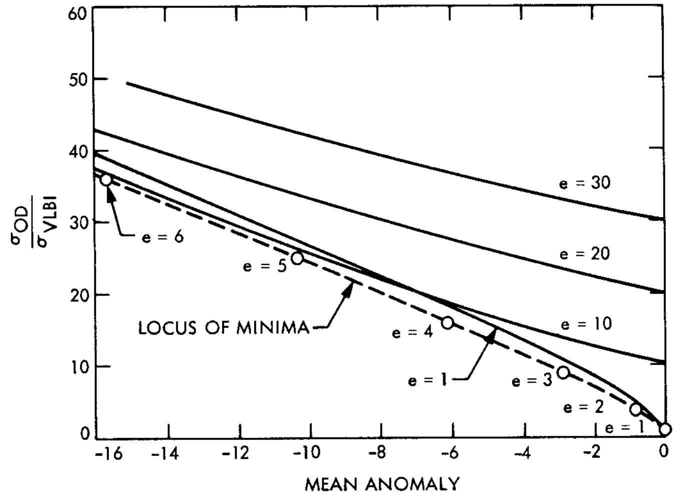

Fig. 2. Approach orbit determination ΔVLBI accuracy ratio as a function of mean anomaly

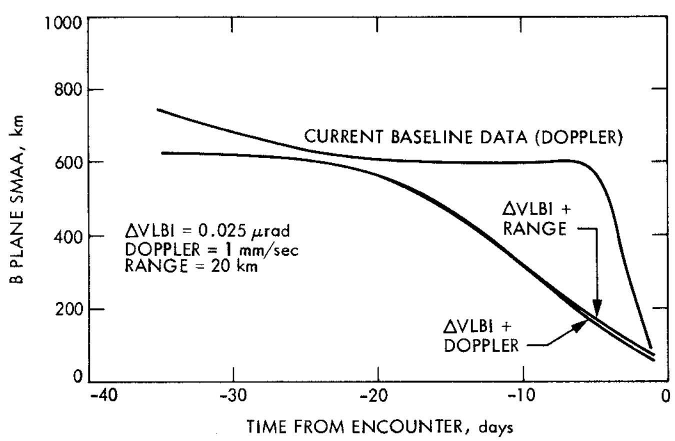

Fig. 3. Comparison of conventional and ΔVLBI data sets (JOP example)

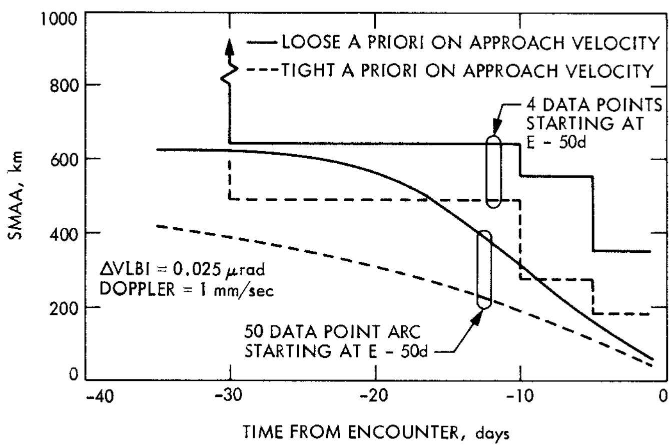

Fig. 4. ΔVLBI + doppler accuracy as a function of days from encounter and data rate (JOP example)