Estimated Displacements for the VLBI Reference Point of the DSS 14 Antenna

H. McGinness
DSN Engineering Section

The maximum displacement of the defined VLBI reference point is shown to be primarily a function of the ambient air temperature.

I. Introduction

When using the 64 meter azimuth-elevation antenna at DSS 14 for very long base interferometry (VLBI), it is necessary either to establish that a defined reference point will have displacements less than a tolerable amount, or to be able to evaluate these displacements as functions of measurable quantities. These quantities would include the environmental quantities of temperature and wind together with the runouts of the antenna axes. The reference point is defined as the nominal intersection of the azimuth and elevation axes. This article describes three different sets of measurements which relate to the thermal displacement of the reference point and shows how the total displacement magnitude can be estimated from the ambient temperature during the VLBI experiment.

II. Description of the Experiments

A. Alidade Leg Temperatures

The surface temperatures of the front right alidade leg, together with the ambient air temperature, were measured during the period of April 1976 to March 1977. On most days, three measurements were made near the hours 0600, 1500, and 2300. Six surface thermometers were distributed along an inner surface of the leg (so as to avoid direct sunlight). Generally, the variation among the thermometer readings was small. A more detailed description of the gage locations can be obtained from Reference 1.

The vertical displacement of the reference point caused by ambient temperature changes will be proportional to the average temperature of the alidade legs. Therefore, it is necessary to compare the ambient air temperature range to the alidade leg surface temperature range and then subsequently estimate the relationship between the leg surface temperature and the leg average temperature. From the alidade leg temperature records, those showing large daily ambient ranges are displayed in Table 1. Table 1 shows the ambient difference between the hours 0600 and 1500, the leg surface temperature difference between these same hours, and the maximum observed range of the leg temperature. The last two columns of the table show the ratios of leg temperature ranges to ambient temperature range. It is believed that the minimum and maximum ambient temperatures usually occur near the hours 0600 and 1500 respectively. The conclusion is that on days having large ambient temperature excursions, the leg surface temperature usually undergoes somewhat less excursion. Since the average of the last column of Table 1 is 75%, this value will be applied subsequently when estimates for total displacements are made.

B. Displacements of Reference Point

During the operational shutdown of DSS 14 in May and June 1977, an experiment was conducted which directly measured most of the reference point displacement occurring over the time of the experiment. Figure 1 shows the configuration of the test apparatus. Two spring loaded styluses at right angles to each other were mounted in a frame positioned only 23 centimeters above the reference point. This frame was fixed to an inverted tripod, which in turn, was temporarily clamped to the frame of the intermediate reference structure Optical Assembly, which is firmly attached to the antenna dish backup structure. Two pieces of carbon paper were mounted on another bracket attached to the Master Equatorial mirror frame, which in turn is firmly mounted on the top of the shielded instrument tower. Each piece of carbon paper was covered with plain white paper and was perpendicular to its stylus.

This experiment was begun at hour 1630 on May 27, 1977. The antenna elevation and azimuth angles were held at 88.95° and 246° respectively. The styluses were carefully retracted, the papers removed, and examined at hour 0940 on June 2, 1977. During this seven-day period, the ambient temperature ranged from 12° to 35° celsius. At DSS 12, approximately 18 kilometers away, there were winds of 13 meters per second with gusts up to 19 meters per second within the time period. Examination of the stylus traces showed that the total horizontal excursion along the Y axis was 2.0 mm, the total horizontal excursion along the X axis was 1.0 mm, and the total vertical excursion along the Z axis was 2.50 mm.

The styluses were reset against new paper at hour 0945 of June 2, 1977. The traces were examined at hour 1215 of June 4, 1977. During this 2-day period, the ambient air temperature ranged from 17° to 36° celsius. The total horizontal excursion along the Y axis was 1.75 mm, the total horizontal excursion along the X axis was 1.0 mm, and the total excursion along the vertical Z axis was 2.50 mm. Wind records were not obtained during this period; however, persons at the antenna said the winds were weak.

Daily ambient temperature ranges of 16.7, 16.7, 17.7, and 17.7°C occurred during the displacement experiment.

Between the hours 1515 and 1635 of June 4, 1977, the antenna was rotated a full turn about the azimuth axis. The wind during this period did not exceed 4.5 meters per second. The antenna was started and stopped several times and a noticeable motion of the vertical stylus was observed at each acceleration and estimated to be ±0.25 mm. The vertical stylus was located approximately 7 mm from the center of rotation and produced a closed trace on the paper. Perfect antenna bearings would have produced a 7 mm radius perfect circle. The actual trace varied approximately 0.50 mm from the best-fitted circle. The conclusion is that the combined effect of imperfect radial and hydrostatic thrust bearings produces a horizontal motion of ±0.50 mm.

C. Instrument Tower Temperatures

Instrument tower temperatures were recorded in December 1966 as part of a tower motion study (Ref. 2). Over a 36-hour period, the ambient air varied from 4.0°C to 13.0°C and the lower portion of the steel tower varied from 17.1 to 18.9°C. During this time, the greatest temperature difference across 180° of the tower was 0.54°C and this figure was based on the average of three differential thermocouples at various elevations on the tower. Also, during this time the angular displacement of the tower top had a total excursion of 6.7 arc seconds. The experimentally determined relationship between the angular displacement, θ in arc seconds, and the horizontal displacement, X in millimeters, of the tower top is, from Ref. 3, $X = 0.0613\theta$. This relationship produces a horizontal displacement of 0.41 mm when the angular displacement is 6.7 arc seconds.

If it is assumed that there is a temperature difference ΔT of 0.54°C across the diameter of the tower and that the temperature is linear with respect to the radius, R, of the tower, the horizontal displacement, X, may be estimated by the following equation:

$$X = \frac{\beta \ell^2 \Delta T}{2R} \tag{1}$$

where β is the linear expansion coefficient, and ℓ is the length of the steel portion of the tower.

For $\beta = 10.8 \times 10^{-6}$ per degree Celsius, $\ell = 23$ m, $R = 3.5$ m, equation (1) yields the value of 0.44 mm which checks well with the above value of 0.41 mm.

The ratio, R, between the observed change in instrument tower temperature and the change in ambient air temperature is, using the above values:

$$R = \frac{18.9 - 17.1}{13 - 4} = 0.20 \tag{2}$$

For subsequent calculations, it will be assumed that the change in tower temperature will be 20% of the change in ambient air temperature. Furthermore, it will be assumed that the horizontal displacement of the instrument tower top will be 0.41 mm multiplied by the ratio of ambient air temperature change in degrees Celsius to the value 9 which was the ambient air temperature change during these observations. Such calculations are necessary to modify the measured displacements of the VLBI reference point so as to have the displacements referenced to earth rather than to the top of a moving instrument tower. It is practical to determine experimentally the displacements of the instrument tower top with respect to the bottom of the tower foundation, but this has not yet been done.

III. Heat Transfer Analysis

In order to obtain a relationship between the average temperature of the alidade leg and the ambient air temperature, it is necessary to obtain solutions for two transient heat transfer problems. The first problem is to estimate the time required for the interior of the leg to reach a certain temperature, given that the surface temperature has suddenly changed. The second problem is to estimate the surface temperature of the leg, given that the ambient air temperature has suddenly changed.

The alidade leg cross section and some of its properties are shown in Figure 2. Since the ratio of surface area to volume is large, it seems reasonable to treat it as a wall of finite thickness, initially at a uniform temperature θ_0, and subsequently subjected to a sudden change in the temperature of both surfaces to θ_s. Reference 4 gives the following solution:

$$\theta = \theta_s - (\theta_s - \theta_0) f_3 \left(\frac{\alpha t}{d^2}, \frac{X}{d}\right) \tag{3}$$

where

- θ is the temperature at distance X from the center of the wall
- d is the wall thickness
- α is the thermal diffusivity
- t is the time
- $f_3\left(\frac{\alpha t}{d^2}, \frac{X}{d}\right)$ is a dimensionless function of the indicated dimensionless parameters and is given in graph form in Reference 4.

It may be seen that the temperature at the center of the leg flange (center of wall) is almost the same as the surface temperature after only a few minutes by evaluating equation (3) for $t = 120$ sec. Using the graph of Reference 4, the following value of f_3 is obtained:

$$f_3\left(\frac{\alpha t}{d^2}, \frac{X}{d}\right) = f_3 \left(\frac{0.000012(120)}{0.072^2}, \frac{0}{0.072}\right)$$

$$= f_3 (0.277, 0) = 0.10$$

The substitution of this value into equation (3) yields:

$$\theta = \theta_s - (\theta_s - \theta_0)0.10 = 0.90\,\theta_s + 0.10\,\theta_0 \tag{4}$$

which shows that the innermost temperature is more than 90% of the surface temperature after only two minutes from the sudden change of surface temperature. For the case at hand, the surface temperature will change sufficiently slowly to justify the assumption that the effective leg temperature is the same as its surface temperature.

Reference 4 also contains the solution for the problem of a wall of finite thickness initially at a uniform temperature θ_0, subsequently exposed on both sides to a liquid or gas at constant temperature θ_f. The surface temperature, θ_s, is given by the following equation:

$$\theta_s = \theta_f + (\theta_0 - \theta_f) f_6 \left(\frac{\alpha t}{d^2}, \frac{hd}{k}\right) \tag{5}$$

where f_6 is a dimensionless function of the dimensionless Fourier and Biot numbers as indicated, where h is the surface conductance, k is the coefficient of conductivity of the alidade leg, α, t, and d are as defined before.

The surface conductance, h, is often times evaluated as follows:

$$h = 5.68\,[1 + 0.74\,V] \tag{6}$$

where V is the wind speed in meters/second and h is in watts/m^2 degree C. Using a wind speed of 4.47 m/sec (10 mph), the value of h is 24.5 watts/m^2 degree C.

Taking a time value of 4320 seconds and using the properties of the alidade leg shown in Figure 2, the Fourier and Biot numbers are respectively 10.0 and 0.039, for which values the function f_6 is 0.48 according to the curves of Reference 4. For these values equation (5) becomes:

$$\theta_s = \theta_f + (\theta_0 - \theta_f)(0.48) = 0.52\,\theta_f + 0.48\,\theta_0 \tag{7}$$

Reference 5 gives a convenient formula for the case of a body of any shape having a surface area A_s and a volume V, namely:

$$\theta_s = \theta_f + (\theta_0 - \theta_f) \exp\left(-\frac{h A_s t}{CV\rho}\right) \tag{8}$$

where C is the specific heat of the body and ρ is its mass density.

When equation (8) is evaluated for $t = 4320$ seconds, $h =$ 24.5 watts/m^2 degree C, and the properties shown in Figure 2, the following is obtained:

$$\theta_s = \theta_f + (\theta_0 - \theta_f)(0.44) = 0.56\,\theta_f + 0.44\,\theta_0 \tag{9}$$

which checks reasonably well with equation (7) derived from the other reference. Using equation (8) and the above values of the parameters, there is obtained:

$$\theta_s = \theta_f + (\theta_0 - \theta_f) \exp(-0.000189\, t) \tag{10}$$

which gives the surface temperature, θ_s, of the leg as a function of time. Here θ_0 is the initial uniform temperature of the leg and θ_f is the new ambient air temperature moving at a speed of 4.47 meters per second which is the average wind speed at DSS 14.

Equation (10) shows that there can be a considerable lag between the leg surface temperature and the ambient air temperature. For instance, two hours after a sudden air temperature change, equation (10) gives:

$$\theta_{s_{t=7200}} - \theta_f = (\theta_0 - \theta_f)(0.256) \tag{11}$$

$$\theta_0 - \theta_{s_{t=7200}} = \theta_0 - [\theta_f + (\theta_0 - \theta_f)(0.256)] \tag{12}$$

$$\theta_0 - \theta_{s_{t=7200}} = 0.744\,(\theta_0 - \theta_f) \tag{13}$$

Thus after two hours, the difference between original and surface temperature is 74.4% of the difference between original and ambient air temperature. This tends to support the last column values of Table 1.

IV. Comparison of Measured and Calculated Vertical Displacements

From the measured leg temperatures of May and June of 1976, for cases where the ambient temperature daily excursions were approximately 15°C, it appears from Table 1 that the average leg surface temperature excursion averages 75% of the ambient excursion. The previous discussion shows that the effective leg temperature approximates very closely the surface temperature. From the December 1966 tower temperature records, it was concluded that the tower temperature excursion was 20% of the ambient temperature excursion.

During the reference point displacement test from June 2 to June 4, 1977, the ambient temperature excursion was 19°C. Using 75% of the ambient, the vertical displacement, ΔZ, is calculated as follows:

$$\Delta Z = 0.75\, \Delta T \beta \ell \tag{14}$$

where ΔT is the ambient excursion, β is the coefficient of linear expansion, ℓ is alidade leg length.

Using the values from Figure 2, equation (14) becomes:

$$\Delta Z = 0.75\,(19^\circ)\, 10.8 \times 10^{-6}\, (25000) = 3.85 \text{ mm} \tag{15}$$

which is the calculated vertical displacement with respect to ground.

The calculated vertical displacement of the instrument tower top with respect to ground is:

$$\Delta Z_{TOWER} = 0.20\, \Delta T \beta L \tag{16}$$

where L is the length of the steel tower and is also 25000 mm.

$$\Delta Z_{TOWER} = 0.20\,(19^\circ)\, 10.8 \times 10^{-6}\, (25000)$$

$$= 1.03 \text{ mm} \tag{17}$$

The calculated relative vertical displacement between reference point and tower top is ΔZ_{REL}:

$$\Delta Z_{REL} = \Delta Z - \Delta Z_{TOWER} = 3.85 - 1.03 = 2.82 \text{ mm} \tag{18}$$

which is to be compared to the measured relative vertical displacement of 2.50 mm.

V. Estimation of Total Displacement of Reference Point

The analyses and tests described above allow an estimation of the magnitude of the total reference point displacement with respect to ground to be made, provided the extreme daily mean ambient temperatures are known.

The following assumptions are made:

(1) The maximum daily mean ambient temperature is 35°C and the temperature ranges from 27° to 43°C.

(2) The minimum daily mean ambient temperature is 2°C and the temperature ranges from -6° to 10°C.

(3) The reference point is defined as the intersection point of the antenna axes at an ambient temperature of 18.5°C and for a condition of zero wind.

(4) The alidade leg temperature is assumed to be equal to the daily ambient mean plus (or minus) 75% of the instantaneous ambient variation from the daily ambient mean.

(5) The maximum leg temperature is 35° + 0.75 (8) or 41°C which is 22.5°C above the reference temperature.

(6) The minimum leg temperature is 2° - 0.75 (8) or - 4°C which is 22.5°C below the reference temperature.

The maximum vertical displacement, ΔZ from the zero reference condition is:

$$\Delta Z = \Delta T \beta \ell = 22.5\,(10.8 \times 10^{-6})\,25000 = 6.1 \text{ mm} \tag{19}$$

The maximum horizontal displacement caused by temperature change, ΔH, is estimated by forming the ratio between the expected ΔT and the modified ΔT observed during the reference point displacement experiment. From the experiment, the resultant horizontal displacement was $\sqrt{1.75^2 + 1.0^2}$ or 2.02 mm over an ambient temperature range of 19°C. This, by assumption number 4 above, corresponds to a leg temperature range of (0.75) (19) or 14.25°C. Therefore, the expected maximum horizontal displacement of the reference point caused by thermal effects, ΔH_T, is:

$$\Delta H_T = 2.02 \left(\frac{22.5}{14.25}\right) = 3.19 \text{ mm} \tag{20}$$

The horizontal displacement caused by azimuth bearing runout, ΔH_B, is, from the experiment:

$$\Delta H_B = \pm 0.50 \text{ mm} \tag{21}$$

It is believed that the runout of the elevation bearing will not exceed this value and will be ignored.

The calculated horizontal displacement of the reference point caused by a 13.4 m/sec (30 mph) wind, when the dish is faced into the wind at the horizon look, is:

$$\Delta H_W = 1.6 \text{ mm} \tag{22}$$

Assuming that these three horizontal components of displacement are arithmetically additive, the resultant horizontal component, ΔH_R, is:

$$\begin{aligned} \Delta H_R &= \Delta H_T + \Delta H_B + \Delta H_W \\ &= 3.19 + 0.50 + 1.6 = 5.30 \text{ mm} \end{aligned} \tag{23}$$

The total displacement, D, is obtained by combining the vertical and horizontal components geometrically, obtaining:

$$D = \sqrt{(\Delta Z)^2 + (\Delta H_R)^2} = \sqrt{6.1^2 + 5.3^2} = 8.1 \text{ mm} \tag{24}$$

If the effects of the 13.4 meter per second wind and the bearing runout are ignored, the displacement, D_0, is:

$$D_0 = \sqrt{(\Delta Z)^2 + (\Delta H_T)^2} = \sqrt{6.1^2 + 3.19^2} = 6.9 \text{ mm} \tag{25}$$

Since D_0 is almost as large at D, and since D_0 is proportional to the difference between the reference temperature of 18.5°C and the mean daily ambient of the day of a VLBI experiment, an approximate expression for the displacement of the reference point, D_A, is:

$$D_A = 6.9\,(t - 18.5), \text{ in millimeters} \tag{26}$$

where t is the mean daily ambient temperature on the day of the VLBI experiment.

References

1. Hung, Phillips, and Zanteson, *JPL Deep Space Network Progress Report* 42-36, Page 41.
2. H. McGinness, *JPL Space Programs Summary* 37-50, Vol. II, Page 151.
3. H. McGinness, *JPL Technical Report* 32-1526, Vol. VI, Page 142.
4. Brown, and Marco, *Introduction to Heat Transfer*, McGraw Hill, 1958.
5. F. Kreith, *Principles of Heat Transfer*, Intext Educational Publishers, 1973, Page 141.

Table 1. Air Temperatures

Day of Year 1976	Temperature Ranges, Celsius degrees: ① Ambient Air Temp. Range for Hours 0600 and 1500 approx.	② Alidade Leg Surface Temp. Range for Hours 0600 and 1500[a]	③ Alidade Leg Surface Temp. Range for Hours 0600, 1500, and 2300[a]	②/①	③/①
138	15	12.7	12.7	.85	1.00
143	13	6.7	11.3	.52	.87
145	11	5.8	5.8	.53	.53
146	18	12.7	16.7	.71	.93
149	12	8.8		.73	
151	17	11.0	14.0	.65	.82
157	15	13.3	14.0	.89	.93
158	15	13.9	13.9	.93	.93
168	11	8.0	8.0	.73	.73
169	15	10.8	10.8	.72	.72
172	13	8.4	8.4	.65	.65
177	13	6.3	13.3	.48	1.00
181	10	6.0	6.0	.60	.60
183	15	7.1	15.1	.47	1.00
185	18	10.3	10.3	.57	.57
186	13	6.9	14.5	.53	1.00
187	17	9.4	11.2	.55	.66
190	14	9.6	9.6	.69	.69
195	12	7.4	7.6	.62	.63
197	13	8.5	8.5	.65	.65
200	12	6.9	7.0	.58	.58
213	14	5.8	6.8	.41	.49
216	13	10.0	13.0	.77	1.00
217	12	5.8	8.3	.48	.69
222	15	8.5	11.3	.57	.75
233	18	10.5	13.7	.58	.76
241	13	6.0	7.4	.46	.57
243	12	4.7	8.2	.39	.68
245	15	8.6	10.5	.57	.70
			MEAN	.617	.754
			STD. DEVIATION	.136	.163

[a]Based on average of all gages.

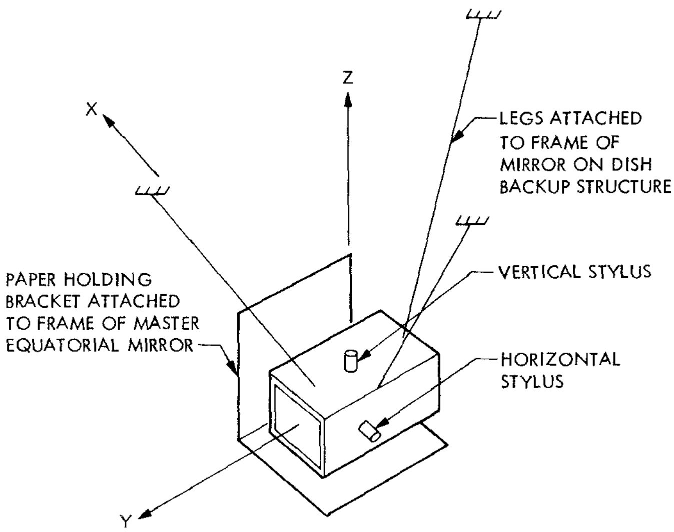

Fig. 1. Schematic diagram of test apparatus

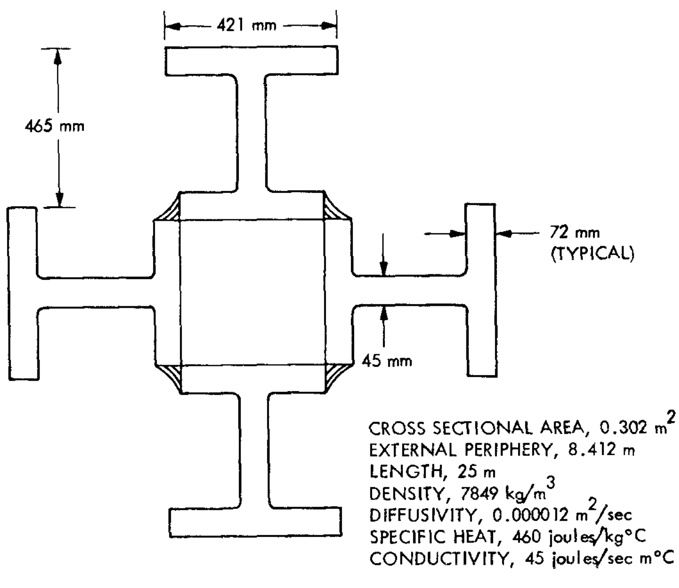

Fig. 2. Alidade leg cross section