Electron Density and Doppler RMS Phase Fluctuation in the Inner Corona

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Previous work has developed a self-consistent set of solar wind descriptors for the extended corona ($5\,r_{\odot} \lesssim r \lesssim 1\,AU$). In this article, observations of the radial dependence of electron density and RMS phase fluctuation are used to construct a similar and symmetrical set of descriptors for the vastly different regime of the inner corona ($r_{\odot} \lesssim r \lesssim 5\,r_{\odot}$). The article concludes that the applicability of symmetrical coronal descriptors for both the inner and extended corona argues forcefully for the basic validity of the description.

I. Introduction

In a previous article (Ref. 1), extensive work by J. V. Hollweg (Refs. 2 through 4) was shown to provide a framework in which observations of the radial dependence of phase fluctuation and electron density in the extended corona ($5r_{\odot} \lesssim r \lesssim 1$ AU) could be combined with various assumptions and conservation of particle flow to produce an internally consistent set of solar wind descriptors. These are reproduced from Ref. 1 for the extended corona as follows:

$$\phi(a) \propto a^{-1.3}$$

$$N_e(r) \propto r^{-2.3}$$

$$L_t(r) \propto r$$

$$n(r) \propto r^{-2.3}$$

$$n/N_e = \epsilon;\ \epsilon \neq \epsilon(r)$$

$$v(r) \propto r^{0.3}$$

where

ϕ = RMS phase fluctuation

N_e = electron density

L_t = transverse fluctuation scale

n = electron density fluctuation

ϵ = ratio of electron density fluctuation to (mean) density

v = radial component of the solar wind velocity

a = signal closest approach distance

r = radial distance

$r_{\odot}$ = solar radius

It should be noted that the value for $\phi(a)$ assumes a signal path integration of $(-\infty,\infty)$ about the signal closest approach point; due to the finite limits of the Earth and the spacecraft, the observed radial dependence of actual data will always be somewhat steeper than $a^{-1.3}$.

The inner corona (here to be defined as $r_\odot \lesssim r \lesssim 5r_\odot$) forms a sharply distinct regime ("solar wind acceleration region") from the extended corona in regard to the radial dependence of electron density, solar wind radial velocity, etc., and hence provides an excellent opportunity for validating the extended corona description. It is therefore the purpose of this article to explore the relationship of the various coronal descriptors of the inner corona. Experimental observations of phase fluctuation and electron density in the inner corona will be shown to provide a self-consistent set of coronal descriptors which is symmetrical with those of the extended corona. The fact that the same description prevails in both (vastly distinct) coronal regimes is interpreted as compelling evidence for the validity of the description.

II. Phase Fluctuation Observations in the Inner Corona

The equatorial electron density model derived by Berman and Wackley (Refs. 5 through 7) from Viking Doppler noise and concurrent measurements of dual frequency (S minus X) range is:

$$N_e(r) = \frac{2.39 \times 10^8}{r^6} + \frac{1.67 \times 10^6}{r^{2.30}}, \text{ electrons/cm}^3$$

r = radial distance, solar radii

The doppler noise model ("ISEDC"; Ref. 5) which is the (signal path integrated) equivalent of the above electron density model is:

$$\text{ISEDC, Hz} = A_0 \left[\frac{\beta}{(\sin \alpha)^{1.30}} \right] F(\alpha,\beta) + A_1 \left[\frac{1}{(\sin \alpha)^5} \right]$$

$$F(\alpha, \beta) = 1 - 0.05 \left\{ \frac{(\beta - \pi/2 + \alpha)^3 - (\alpha - \pi/2)^3}{\beta} \right\}$$

$$- 0.00275 \left\{ \frac{(\beta - \pi/2 + \alpha)^5 - (\alpha - \pi/2)^5}{\beta} \right\}$$

where

α = Sun-Earth-probe (SEP) angle, radians

β = Earth-Sun-probe (ESP) angle, radians

$$A_0 = 1.182 \times 10^{-3}$$

$$A_1 = 4.75 \times 10^{-10}$$

For the above models, the electron density terms become equal valued at approximately $r = 4r_\odot$, while the corresponding Doppler noise terms become equal valued at approximately $r = 3r_\odot$. Doppler noise observations in the region $r < 4r_\odot$ are naturally quite sparse; however 20 pass average Doppler values have been accumulated for this inner coronal region from various Solar Conjunctions of Helios 1, Helios 2, and Viking. The method of computing "pass average" values is described in detail in Ref. 8. In deriving the ISEDC model, a three parameter (essentially simultaneous) least squares minimization was performed. The three parameters solved for were the inner corona term coefficient (A_1), and the extended corona term coefficient (A_0) and radial index (-1.30). Figure 1 presents the inner corona data as compared to the inner corona term, the extended corona term, and the combined ISEDC model. The data in the region $r < 4r_\odot$ are clearly seen to respond to the (integrated) inner coronal electron density term ($\propto a^{-5}$). Figure 2 presents the correlation between the data and the ISEDC model. Again the correlation is seen to be quite strong. The data in Figures 1 and 2 provide forceful evidence that $\phi(a) \propto a^{-5}$ in the inner corona.

III. Electron Density Observations in the Inner Corona

The radial dependence of electron density in the inner corona has been determined by eclipse photometry methods (van de Hulst, Ref. 9, Saito, Ref. 10, and Blackwell, Ref. 11) and via spacecraft range delays (Muhleman, Mariner 6, Ref. 12, Edenhofer, Helios 2, Ref. 13); in general these experiments have obtained consistent results which indicate a radially dependent electron density in the inner corona of the form $N_e(r) \propto r^{-6}$. Using the relationship derived by Hollweg (Ref. 2), one has for the assumption of a linear transverse scale[1]:

$$\phi^2(a) \propto \int_a^\infty \frac{1}{(r^6)^2} \frac{r^2 \, dr}{\sqrt{r^2 - a^2}} \propto (a^{-5})^2$$

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[1] The dependence with closest approach distance (a) is obtained by noting the integral:

$$\int_a^\infty \frac{r^\alpha \, dr}{\sqrt{r^2 - a^2}}$$

is transformed via the substitution:

$$r = a (\cos x)^{-1}$$

to:

$$a^\alpha \int_0^{\pi/2} (\cos x)^{-(1+\alpha)} \, dx$$

Assuming conservation of particle flow in the corona (Cuperman and Harten, Ref. 14):

$$N_e(r)v(r)r^2 = K$$

and the proportionality between electron density and electron density fluctuation ($\epsilon = n/N_e$), one obtains an inner corona set of self-consistent descriptors symmetrical to those for the extended corona:

$$\phi(a) \propto a^{-5}$$

$$N_e(r) \propto r^{-6}$$

$$L_t(r) \propto r$$

$$n(r) \propto r^{-6}$$

$$n/N_e = \epsilon; \ \epsilon \neq \epsilon(r)$$

$$v(r) = r^4$$

This set of coronal descriptors is compared to those for the extended corona in Table 1.

IV. Discussion and Conclusions

Previously, observations of the radial dependence of phase fluctuation and electron density in the extended corona were seen to be consistent with the concept of a linear transverse fluctuation scale. The fact that a vastly different regime exists in the inner corona provides a powerful test bed to check the validity of the extended corona description. Observations of doppler noise and electron density in the inner corona, when combined with the assumption of a linear transverse fluctuation scale and conservation of particle flow, are seen to be self-consistent and symmetrical with the extended coronal description. It is hereby concluded that experimental observations in both the inner and extended corona provide compelling evidence for:

(1) The existence of a linear transverse fluctuation scale.

(2) The existence of proportionality between RMS phase fluctuation and integrated electron density.

References

1. Berman, A. L., "Electron Density in the Extended Corona: Two Views," in *The Deep Space Network Progress Report 42-41*, Jet Propulsion Laboratory, Pasadena, California, October 15, 1977.

2. Hollweg, J. V., "A Statistical Ray Analysis of the Scattering of Radio Waves by the Solar Corona," in *The Astronomical Journal*, Volume 73, Number 10, Part 1, December 1968.

3. Hollweg, J. V., and Harrington, J. V., "Properties of Solar Wind Turbulence Deduced by Radio Astronomical Measurements," in *The Journal of Geophysical Research, Space Physics*, Volume 73, Number 23, December 1, 1968.

4. Hollweg, J. V., "Angular Broadening of Radio Sources by Solar Wind Turbulence," in *The Journal of Geophysical Research, Space Physics*, Volume 75, Number 19, July 1, 1970.

5. Berman, A. L., and Wackley, J. A., "Viking S-Band Doppler RMS Phase Fluctuations Used to Calibrate the Mean 1976 Equatorial Corona," in *The Deep Space Network Progress Report 42-38*, Jet Propulsion Laboratory, Pasadena, California, April 15, 1977.

6. Berman, A. L., Wackley, J. A., Rockwell, S. T., and Kwan, M., "Viking Doppler Noise used to Determine the Radial Dependence of Electron Density in the Extended Corona," in *The Deep Space Network Progress Report 42-38*, Jet Propulsion Laboratory, Pasadena, California, April 15, 1977.

7. Berman, A. L., "Proportionality Between Doppler Noise and Integrated Signal Path Electron Density Validated by Differenced S-X Range," in *The Deep Space Network Progress Report 42-38*, Jet Propulsion Laboratory, Pasadena, California, April 15, 1977.

8. Berman, A. L., "Phase Fluctuation Spectra — New Radio Science Information to Become Available in the DSN Tracking System, Mark III-77," in *The Deep Space Network Progress Report 42-40*, Jet Propulsion Laboratory, Pasadena, California, August 15, 1977.

9. van de Hulst, H. C., "The Electron Density of the Solar Corona," *Bulletin of the Astronomical Institutes of The Netherlands*, Volume XI, Number 410, February 2, 1950.

10. Saito, K., "A Non-Spherical Axisymmetric Model of The Solar K Corona of The Minimum Type," *Annals of The Tokyo Astronomical Observatory*, University of Tokyo, Second Series, Volume XII, Number 2, Mitaka, Tokyo, 1970.

11. Anderson, J. D., Esposito, P. B., Martin, W. L., Thornton, C. L., and Muhleman, D. O., "Experimental Test of General Relativity Using Time Delay Data from Mariner 6 and Mariner 7," in *The Astrophysical Journal*, Volume 200, August 15, 1975.

12. Muhleman, D. O., Esposito, P. B., and Anderson, J. D., "The Electron Density Profile of the Outer Corona and the Interplanetary Medium from Mariner 6 and Mariner 7 Time Delay Measurements," *The Astrophysical Journal*, 211, February 1, 1977.

13. Edenhofer, P., Esposito, P. B., Hansen, R. T., Hansen, S. F., Lueneburg, E., Martin, W. L., and Zygielbaum, A. I., "Time Delay Occultation Data of the Helios Space-

crafts and Preliminary Analysis for Probing the Solar Corona," in the *Journal of Geophysics* 42, 1977.

14. Cuperman, S., and Harten, A., "Some Physical Implications of Recent Solar Wind Measurements" in *Solar Wind*, edited by Sonett, C. P., Coleman, P. J. Jr., and Wilcox, J. M., National Aeronautics and Space Administration, Washington, D. C., 1972.

Table 1. Coronal descriptions for the inner and extended corona

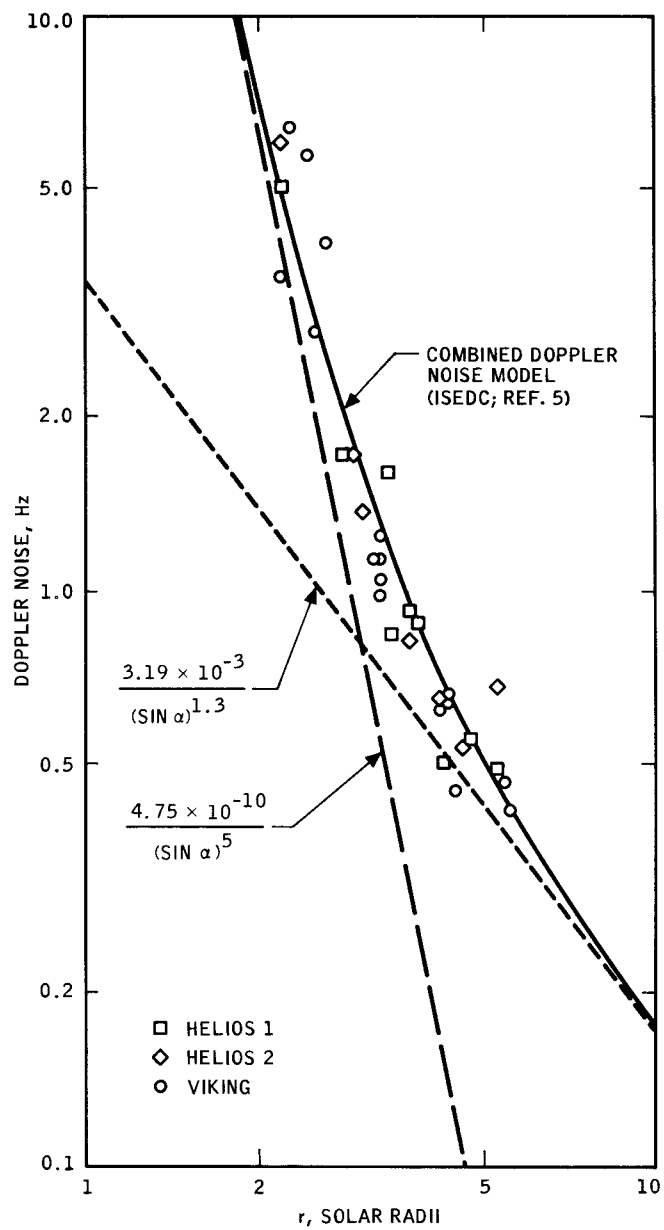| Parameter	Inner corona $(r_\odot \leqslant r \leqslant 5\, r_\odot)$	Extended corona $(5\, r_\odot \leqslant r \leqslant 1\ \mathrm{AU})$
$\phi(a) \propto$	$a^{-5.0}$	$a^{-1.3}$
$N_e(r) \propto$	$r^{-6.0}$	$r^{-2.3}$
$L_t(r) \propto$	r	r
$n(r) \propto$	$r^{-6.0}$	$r^{-2.3}$
$n/N_e =$	$\epsilon;\ \epsilon \neq \epsilon(r)$	$\epsilon;\ \epsilon \neq \epsilon(r)$
$v(r) \propto$	$r^{4.0}$	$r^{0.3}$

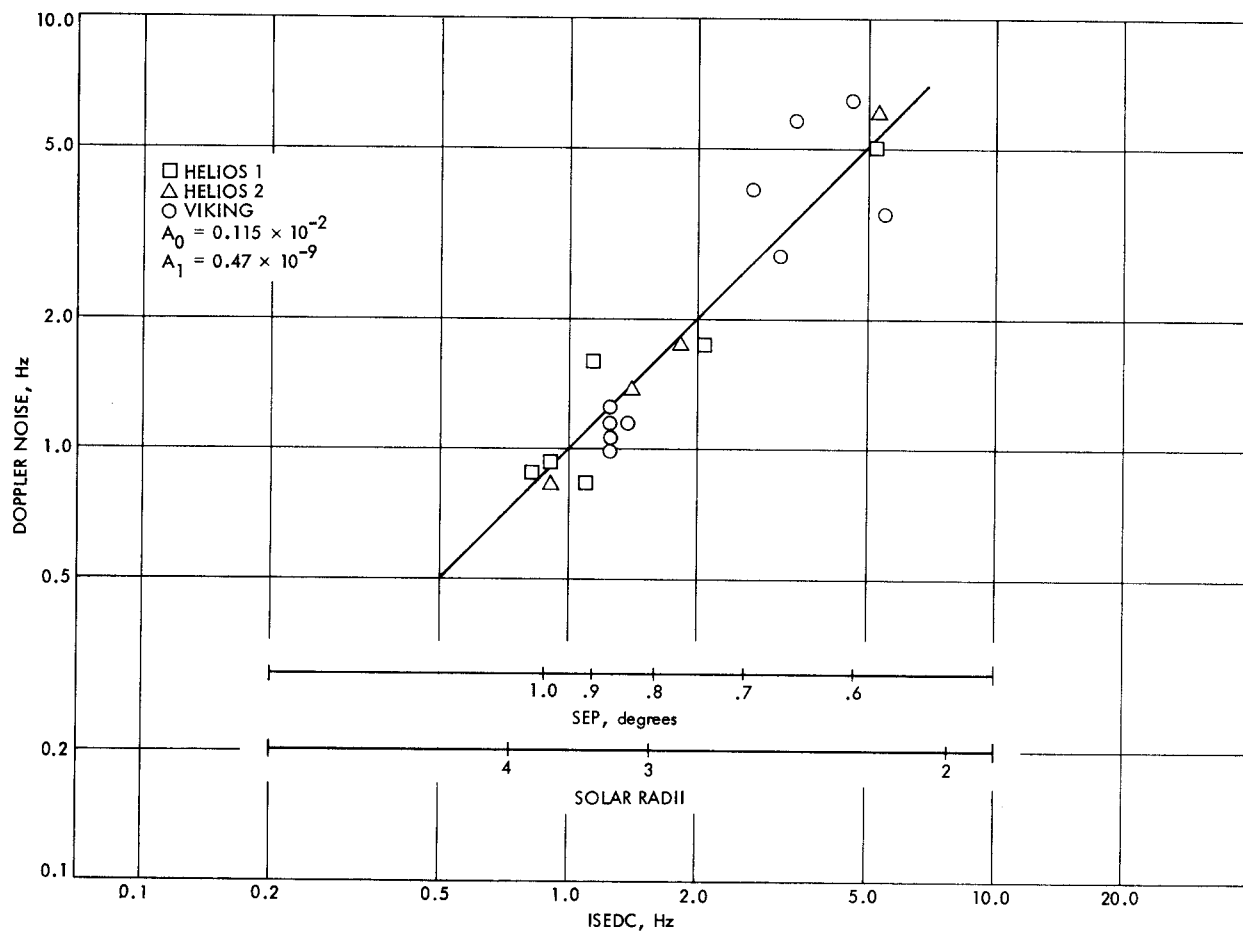**Fig. 1. Doppler noise vs radial distance in the inner corona**

Fig. 2. Correlation of Viking and Helios doppler noise with the ISEDC model between
2 and 4 solar radii