Solar Wind Density Fluctuation and the Experiment to Detect Gravitational Waves in Ultraprecise Doppler Data

A. L. Berman
TDA Engineering Office

The experiment to detect gravitational waves in ultraprecise Doppler data requires a total system (DSN Tracking System, spacecraft transponder, and media) fractional frequency fluctuation of approximately 1×10^{-15} for averaging times greater than 1000 seconds. At such levels, solar wind density fluctuation looms as a very difficult error source.

This article presents a detailed examination of solar wind electron (columnar) density fluctuation as it applies to the experiment to detect gravitational waves in ultraprecise doppler data. Expected two-way S-band fractional frequency fluctuation (due to solar wind density fluctuation) at a 1000-second averaging time is considered to be approximately 3×10^{-13}, while the optimum two-way X-band (X-band uplink and downlink) performance is predicted to be (also at a 1000-second averaging time) approximately 1×10^{-14}. The article concludes that both two-way S-band and X-band will be required (simultaneously) so that the predicted two-way X-band performance ($\sim 10^{-14}$) can be improved to a (defined) media goal of 3×10^{-16} via two-way dual frequency calibration of solar wind induced density fluctuation.

I. Introduction

In order to utilize two-way ultraprecise doppler data in the attempt to detect and measure various parameters of (hypothesized) gravitational waves (Refs. 1 and 2), one requires a total measurement system fractional frequency fluctuation on the order of 10^{-15}, over the averaging times of interest (approximately 50 to 5000 seconds). Major elements included in the total measurement system are:

(1) DSN Tracking System.

(2) Media (electron density fluctuation).

(3) Spacecraft transponder.

Since there are multiple elements in the DSN Tracking System which independently influence total system performance, a fractional frequency fluctuation goal of 3×10^{-16}

for each (independent) element will be adopted. At the present time, the two-way[1] S-band media (solar wind density) fractional frequency fluctuation performance appears to be approximately 3×10^{-13} (averaging times > 1000 seconds), or 3 orders of magnitude short of the desired 3×10^{-16} level. The steps which need to be taken (higher frequency band capability, dual frequency band capability) within the DSN Tracking System and on the spacecraft to significantly reduce media effects may begin to rival those necessary to put the DSN Tracking System on an overall fractional frequency fluctuation basis of less than 10^{-15}. Given the importance of solar wind density fluctuation in the usage of ultraprecise doppler data for the detection and measurement of gravitational waves, one requires accurate solar wind density fluctuation data for the design and implementation of such an ultraprecise doppler based gravitational wave detection experiment. It is the purpose of this article to present and analyze measured solar wind electron density fluctuation performance, appropriately formatted as fractional frequency fluctuation. Extensive two-way S-band doppler noise statistics acquired from a variety of spacecraft during 1975 and 1976 (Refs. 3 through 8) are used to construct average solar wind density fractional frequency fluctuation performance.

II. Solar Wind Density Fractional Frequency Fluctuation Performance

Two-way S-band doppler noise results obtained with the Viking spacecraft can be summarized (Refs. 5 and 6) as follows:

$$\sigma_f(\tau) = 1.182 \times 10^{-3} \left[\frac{\beta}{(\sin \alpha)^{1.3}}\right] F(\alpha, \beta) \left[\frac{60}{\tau}\right]^{0.29}$$

where:

$\sigma_f(\tau)$ = doppler noise, Hz

α = Sun-Earth-Probe angle, radians

β = Earth-Sun-Probe angle, radians

τ = doppler sample interval, seconds

$$F(\alpha, \beta) = 1 - 0.05 \left\{\frac{(\beta - \pi/2 + \alpha)^3 - (\alpha - \pi/2)^3}{\beta}\right\}$$

$$- 0.00275 \left\{\frac{(\beta - \pi/2 + \alpha)^5 - (\alpha - \pi/2)^5}{\beta}\right\}$$

The algorithm[2] used to calculate doppler noise (essentially) filters out fluctuation frequencies (ν) lower than ν_0 where (Ref. 8):

$$\nu_0 \approx (30\tau)^{-1}$$

Assuming a similar filtering out of (relatively) low frequency fluctuations in possible gravitational wave data reduction algorithms[3], the (given) doppler noise expression can be rather directly applied to the gravitational wave detection problem, with doppler sample interval approximately equal to averaging time.

Selecting a standard case of the spacecraft-Earth line perpendicular to the Sun-Earth line (with the spacecraft at infinity), so that:

$$\alpha = 90^\circ$$

$$\beta = 90^\circ$$

the doppler noise expression reduces to:

$$\sigma_f(\tau) = 1.60 \times 10^{-3} \left[\frac{60}{\tau}\right]^{0.29}, \text{ Hz}$$

which yields a fractional frequency fluctuation ($\sigma_f(\tau)/f$) for two-way S-band (uplink and downlink) of:

$$\sigma_f(\tau)/f = 2.28 \times 10^{-12}\ \tau^{-0.29}$$

Scaling by $(3/11)^2$, one obtains the equivalent expected two-way X-band performance:

$$\sigma_f(\tau)/f = 1.70 \times 10^{-13}\ \tau^{-0.29}$$

[1]For the remainder of this article, "two-way" will specifically connote "two-way (coherent) tracking mode, both uplink and downlink at the stated frequency".

[2]A "running" standard deviation computed from a least squares linear curve fit to 15 samples of actual minus predicted (average) doppler frequency.

[3]The design and implementation of algorithms to reduce and detect gravitational waves in ultraprecise doppler data will be a major part of the experiment.

Figures 1 and 2 present average two-way S- and X-band fractional frequency fluctuation versus doppler sample interval.

Considering averaging periods of approximately 3 hours ("pass average"), Refs. 3, 4, and 6 found the average one standard deviation of (pass-average) doppler noise (σ_p) about a mean geometric value ($\bar{\sigma}_p$) established over several months of data to be (expressed in terms of $10 \log_{10}(\sigma_p/\bar{\sigma}_p)$):

$$\sigma(\sigma_p/\bar{\sigma}_p) = 1.92 \text{ dB } (-36\%, +56\%)$$

This value is applied as limits to the average fractional frequency fluctuation data presented in Figures 1 through 4.

III. Additional Factors in the Reduction of Solar Wind Density Fluctuation

Expected levels of fractional frequency fluctuation can be reduced somewhat by consideration of three additional circumstances:

(1) Density variations with solar cycle.

(2) Columnar density minimization in the antisolar direction ($\beta = 0°$).

(3) Columnar density fluctuation to (mean) density ratio minimization in the antisolar direction.

These conditions are briefly discussed below.

A. Density Variations with Solar Cycle

It is shown in Ref. 9 that the ratio of density and density fluctuation between solar cycle maximum and minimum (for Solar Cycle 20) was approximately 0.65 (value at maximum divided by value at minimum). The data used to derive the results of Section II were obtained at Solar Cycle minimum, hence one might expect to find at the next Solar Cycle (21) maximum (1979–1981) a reduction in average fractional frequency fluctuation by approximately 0.65.

B. Columnar Density Minimization in the Antisolar Direction (β = 0°)

The antisolar direction provides the minimum columnar density and hence (Ref. 7) the minimum columnar density fluctuation also. The columnar density reduction between the standard geometry chosen in Section II:

$$\alpha = 90°$$

$$\beta = 90°$$

and the antisolar direction:

$$\alpha = 180°$$

$$\beta = 0°$$

is approximately (assuming an r^{-2} density falloff for $r > 1$ AU):

$$2/\pi$$

and hence the density fluctuation can be expected to be reduced by a similar amount. Combination of this reduction with that from Subsection A yields a total reduction by a factor of 0.414. Figures 3 and 4 present the two-way S- and X-band average fractional frequency fluctuation for these conditions.

C. Columnar Density Fluctuation to (Mean) Density Ratio Minimization in the Antisolar Direction

Although columnar density (and hence columnar density fluctuation) is minimized in the antisolar direction, there may be a further reduction in the columnar density fluctuation because of a reduction in the columnar density fluctuation to (mean) density ratio (defined as ϵ', in Ref. 7). Quite simply, in the $\alpha = 90°$ case, density enhancements propagating radially cross the signal path perpendicularly near 1 AU (region of maximum density) with attendant (solar wind) velocities of approximately 440 km/second, as in Figure 5. As the antisolar direction is approached, the density enhancement propagation paths line up with signal path, and hence the density enhancements can be expected to move across the signal path with greatly reduced velocity (i.e., shifting the columnar fluctuation spectrum to lower frequencies).

The ground tracking system (with Rubidium frequency standard) doppler noise level for 60-second sample interval data is approximately (Ref. 5):

$$3 \times 10^{-3} \text{ Hz}$$

while the solar wind density S-band frequency fluctuation in the antisolar direction is expected to be:

$$1 \times 10^{-3} \text{ Hz}$$

It would therefore not have been previously possible to test this hypothesis (of a further reduction of density fluctuation in the antisolar direction). However, with the current hydrogen maser frequency standard implementation at the 64-m subnet and the attendant large reduction in the ground tracking system noise level, it may now be possible to easily test this hypothesis.

IV. Conclusions

Two-way X-band (uplink and downlink), under even the most optimum conditions, results in a fractional frequency fluctuation of no better than 10^{-14} for averaging periods of at least 1000 seconds. To achieve the goal of 3×10^{-16}, it is here considered that solar wind density fluctuation will have to be further removed via dual frequency calibration. It may be possible to achieve the 1-1/2 orders of magnitude improvement from the two-way X-band performance level ($\sim 10^{-14}$) to the desired level (3×10^{-16}) via use of an additional S-band downlink (only) in combination with two-way X-band; in this regard, Wu, et al. (Ref. 10), have developed sophisticated techniques for calibration of uplink range when dual frequency is only available on the downlink. However, to more adequately guarantee the required improvement (to 3×10^{-16}) and to substantially reduce data reduction costs (i.e., by not requiring complex calibration algorithms such as the Wu uplink calibration technique) associated with the experiment, it seems reasonable to additionally require two-way S-band. It is thus here concluded that the implementation of a viable gravitational wave detection experiment based on ultraprecise doppler data will require simultaneous two-way X-band and two-way S-band capability.

References

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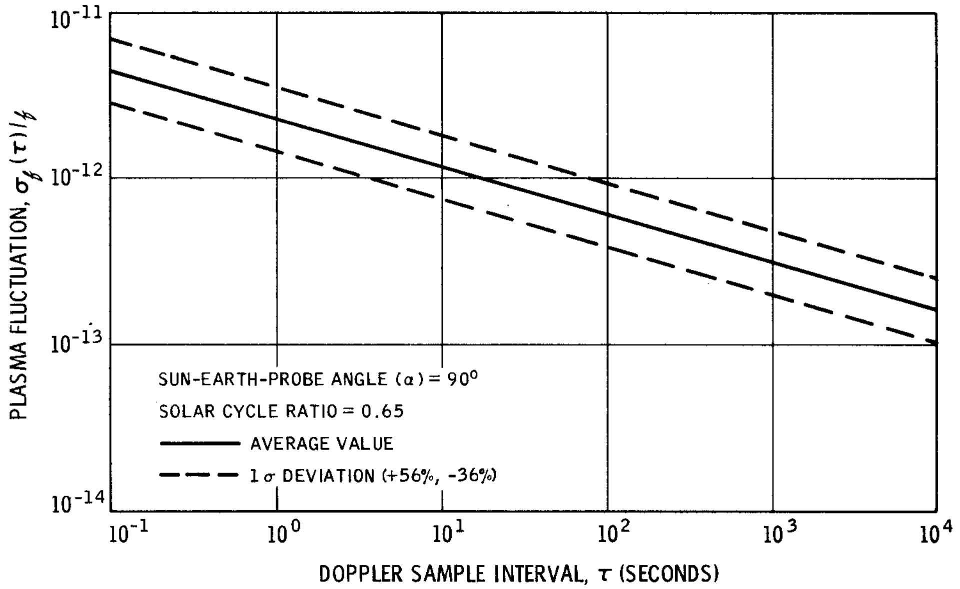

Fig. 1. Two-way S-band plasma fluctuation at solar cycle minimum

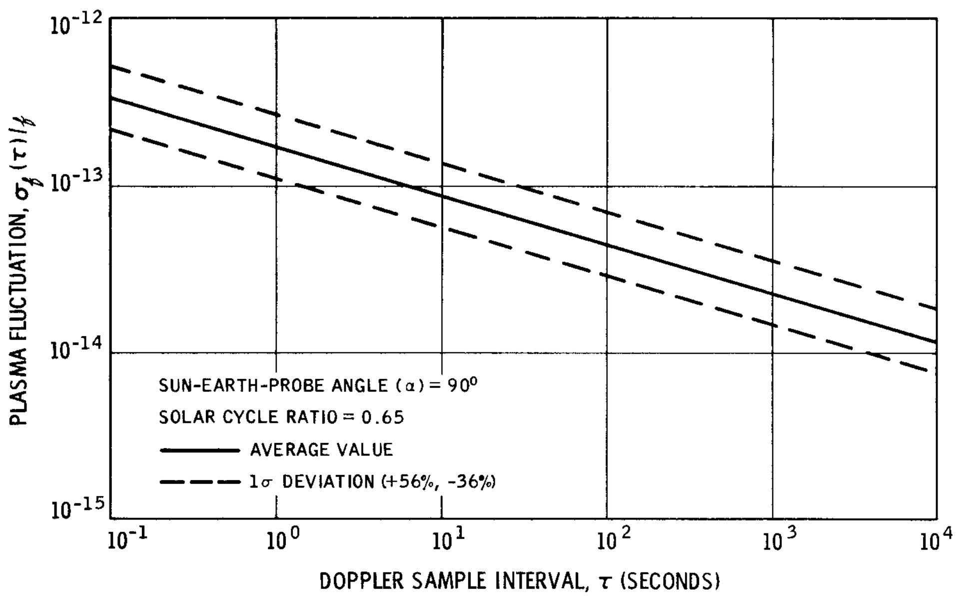

Fig. 2. Two-way X-band plasma fluctuation at solar cycle minimum

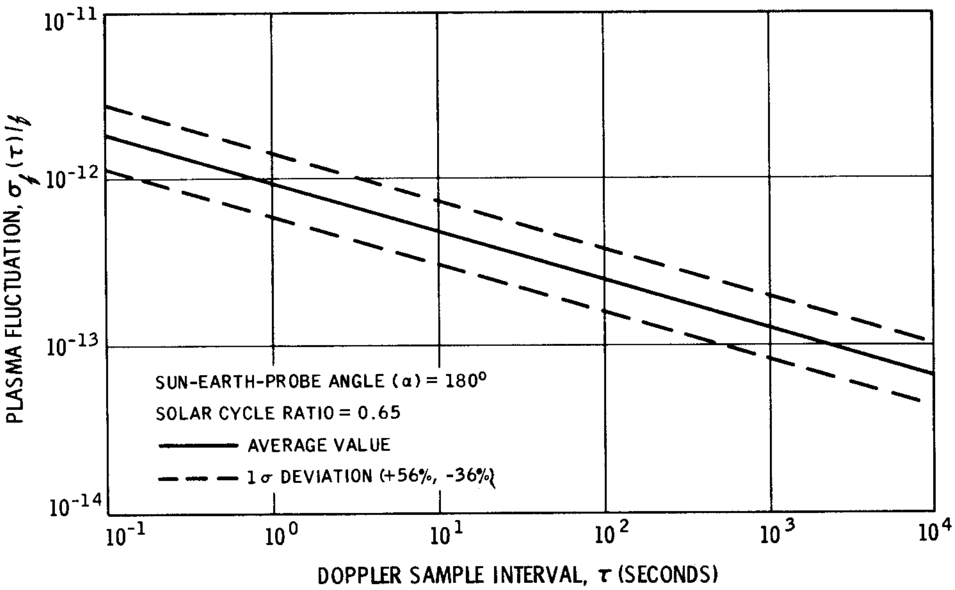

Fig. 3. Two-way S-band plasma fluctuation at solar cycle maximum

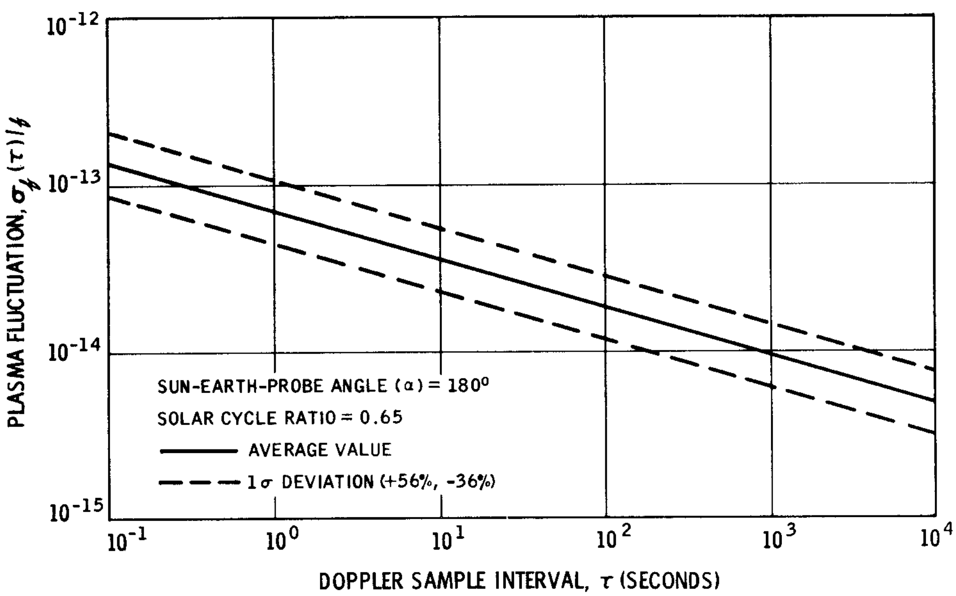

Fig. 4. Two-way X-band plasma fluctuation at solar cycle maximum

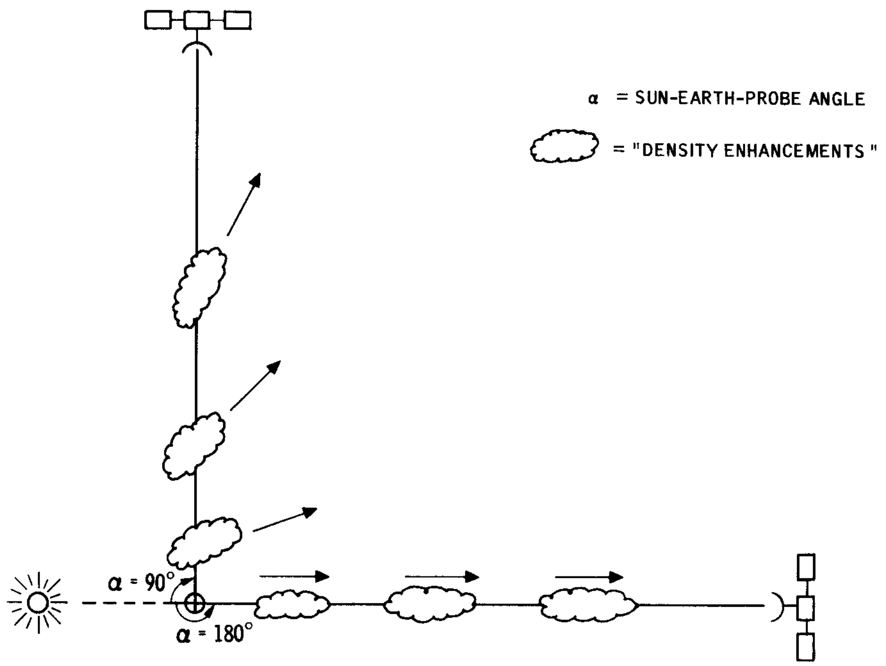

Fig. 5. Possible reduction in the (columnar) density fluctuation/density ratio in the antisolar direction