On the Suitability of Viking Differenced Range to the Determination of Relative Z-Distance

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Radiometric differenced range residuals (10 Viking orbiting spacecraft observations) have been used to evaluate the current Deep Space Net adopted relative Z distance between Deep Space Station 43 and Deep Space Station 14. The $\Delta\rho$ noise is approximately twice the predicted noise: $\sigma_{\Delta\rho}$ is 3.4 m relative to a mean of -1 m. This scatter in the $\Delta\rho$ is most likely due to media calibration uncertainties and Deep Space Net hardware noise.

These 10 $\Delta\rho$ residuals yield an estimated Z distance between DSS 43 and DSS 14 of 7351803.6 m. The standard deviation of that estimate is 10.1 m.

I. Introduction

Radiometric differenced range residuals (10 Viking orbiting spacecraft observations) have been used to evaluate the current Deep Space Net adopted relative Z distance between Deep Space Station 43 and Deep Space Station 14. The Deep Space Net 'adopted Z coordinates' for Deep Space Stations are a product of the Goddard Space Flight Center. The adopted DSS 43–DSS 14 relative Z distance is 7.351807×10^6 m. JPL has assigned a 15-m standard deviation to these Z coordinates. These 10 Viking differenced range observations, $\Delta\rho$, do not reveal a relative Z error of statistical significance: the ΔZ correction equals 1/3 the computed $\sigma_{\Delta Z}$. The relative Z correction is -3.4 m and the sigma is 10.1 m. Although this is encouraging, it is not definitive. This range sample is so small that systematic effects due to ΔZ, Deep Space Station longitude, and spacecraft declination errors may not have emerged above the range nosie.

The $\Delta\rho$ noise is approximately twice the predicted noise: $\sigma_{\Delta\rho}$ is 3.4 m relative to a mean of - 1 m. This scatter in the $\Delta\rho$ is most likely due to media calibration uncertainties and Deep Space Net hardware noise. Additionally, the influences of ΔZ, Deep Space Station longitude, and spacecraft declination errors on $\Delta\rho$ are discussed.

II. The Differenced Range Data Set from the DSS 43—DSS 14 Baseline

Near simultaneous range measurements were obtained from the DSS 43–DSS 14 baseline (Ref. 2, Ref. 3): seven at an S-band frequency and three at an X-band frequency. The DSN acquired these observations from Viking-Mars orbiting spacecraft from January to April 1977 in a dual-station ranging demonstration. Usually only minutes separated the measurements of different DSSs (Figs. 1–6).

The positions and velocities of a spacecraft in the solar system are estimated usually by a least squares, differential correction technique (Orbit Determination Program, Ref. 4). The Doppler shift experienced by round-trip radio transmission from the earth to the spacecraft and back is the prime observable used in the least squares technique. A part of the procedure computes the difference between the observed range and the range computed from numerical integrations of the equations of motion of all the significant solar system bodies.

When compressed range residuals are computed from the raw range residuals (Fig. 1–6) and differenced, the compressed differenced range residuals do not show any systematic signatures in time (Fig. 7) or as a function of declination (Fig. 8). In these figures, a label of 'S' or 'X' indicates the frequency of the observation and the bar specified the one-sigma dispersion of each $\Delta\rho$ value. As previously stated, the dispersion of the combined 'S' and 'X' frequency $\Delta\rho$ is 3.4 m with a mean of -1 m. Alone 'S' band $\Delta\rho$ has a $\sigma_{\Delta\rho}$ = 2.6 m and a mean of -1 m.

III. Additional Australia—Goldstone Baseline Data

Besides these 10 $\Delta\rho$ observations, 4 additional $\Delta\rho$ observations were obtained from 3 different Canberra–Goldstone baselines: one from the DSS 42–DSS 14 baseline on 31 January; two from the DSS 43–DSS 11 baseline on 31 January and 16 February; and one from the DSS 42–DSS 11 baseline on 16 February.

Geodetic surveys within the Goldstone DSN complex have determined the relative $Z(\Delta Z)$ for the short baselines to <1 m (σ). The same is true for the DSS of the Canberra complex. Thus, all long baselines between the Canberra and Goldstone complexes should have a nearly common ΔZ error. Although a plot of all 14 $\Delta\rho$ points for the Canberra–Goldstone baselines is consistent with this view, the noise level is so high ($\sigma_{\Delta\rho}$ = 4.1 m) that little weight is given to this observation. To dramatize the noise level of this data, Fig. 9 exhibits what the signature in $\Delta\rho$ from an error in ΔZ (equal to 18 m, 3 times what the Voyager project specified as acceptable, Ref. 5) looks like.

IV. Error Analysis for $\Delta\rho$

This scatter in the $\Delta\rho$ values is most likely due to the 'media' calibration uncertainties (plasma, ionospheric ion, and atmospheric refraction calibration uncertainties) and DSN ranging system hardware perturbations. It can be shown that $\Delta\rho$ scatter seen in Fig. 9 does not stem from geometrical parameters, such as: relative Z distance (ΔZ); Mars declination (δ); DSS distance off the earth's spin axis (r_s); or, DSS longitude errors. Consider the following discussion.

The topocentric range from a DSS to a distant spacecraft may be approximated (Ref. 6) by

$$\rho(t) = R(t) - Z \sin \delta(t) - r_s \cos \delta(t) \cos [\omega t - \alpha(t) + \lambda]$$

where

$\rho(t)$ = the topocentric range at time 't'

$R(t)$ = geocentric range

Z = distance of DSS from earth's equator

δ = spacecraft declination

r_s = distance of DSS from earth's spin axis

ω = earth rotation rate

λ = DSS longitude

α = spacecraft right ascension

For differenced range from two DSS, which are time synchronized

$$\rho_2 - \rho_1 = (Z_1 - Z_2) \sin \delta$$

$$- (r_{s_2} - r_{s_1}) \cos \delta \, [\cos(\omega t - \alpha) \{\cos \lambda_2 - \cos \lambda_1\}$$

$$+ \sin(\omega t - \alpha) \{\sin \lambda_1 - \sin \lambda_2\}]$$

The errors in the differenced range per unit error in ΔZ, $\Delta\delta$, Δr_s, and λ are then

$$\frac{\epsilon(\Delta\rho)}{\epsilon(\Delta Z)} = \sin \delta$$

$$\frac{\epsilon(\Delta\rho)}{\epsilon(\Delta\delta)} = \Delta Z \cos \delta + \Delta r_s \sin \delta \, [\cos(\omega t - \alpha) \{\cos \lambda_2 - \cos \lambda_1\}$$

$$+ \sin(\omega t - \alpha) \{\sin \lambda_1 - \sin \lambda_2\}]$$

$$\frac{\epsilon(\Delta\rho)}{\epsilon(\Delta r_s)} = \cos\delta \; [\cos(\omega t - \alpha) \{\cos\lambda_2 - \cos\lambda_1\}$$

$$+ \sin(\omega t - \alpha) \{\sin\lambda_1 - \sin\lambda_2\}]$$

$$\frac{\epsilon(\Delta\rho)}{\epsilon(\lambda)} = - \Delta r_s \cos\delta \; [\cos(\omega t - \alpha) \{\sin\lambda_1 - \sin\lambda_2\}$$

$$+ \sin(\omega t - \alpha) \{\cos\lambda_1 - \cos\lambda_2\}]$$

For the DSS 43–DSS 14 baseline,

$$\Delta Z = 6 \times 10^6 \text{ m}$$

$$\Delta r_s = 1 \text{ m}$$

$$\lambda_1 = 149°, \lambda_2 = 243°, \Delta\lambda = 94°$$

$$\epsilon(\Delta Z) = 15 \text{ m } (\sigma)$$

$$\epsilon(\Delta r_s) = 0.6\sqrt{2} = 0.8 \text{ m } (\sigma)$$

$$\epsilon(\lambda) = 2\sqrt{2} = 2.8 \text{ m } (\sigma)$$

and where $(\omega t - \alpha) = 196°$. The spacecraft is over the sub-earth point equally distant from both DSS 43 and DSS 14 when $\omega t - \alpha = 196°$. For a spacecraft declination of 20° and an $\epsilon(\delta)$ of 4×10^{-7} radians,[1] all the parameters of the $\Delta\rho$ equations are defined.

$$\epsilon(\Delta\rho) = f[\epsilon(\Delta Z)] = 5.1 \text{ m } (\sigma)$$

$$\epsilon(\Delta\rho) = f[\epsilon(\delta)] = 2.3 \text{ m } (\sigma)$$

$$\epsilon(\Delta\rho) = f[\epsilon(\Delta r_s)] = 0.5 \text{ m } (\sigma)$$

$$\epsilon(\Delta\rho) = f[\epsilon(\lambda)] = 1.4 \text{ m } (\sigma)$$

The root-sum-of-square of these error terms yields a combined $\sigma_{\Delta\rho} = 5.8$ m. A $\sigma_{\Delta\rho} = 5.6$ m results when only the $\sin\delta \; \epsilon(\Delta Z) + \Delta Z \cos\delta \; \epsilon(\delta)$ terms are considered. The diurnal terms are insignificant.

Since eight of the 10 DSS 43–DSS 14 $\Delta\rho$ (Fig. 8) involve a spacecraft at near constant δ (the declination is $-23° \pm 1°$ from 11 January through 31 January), it follows

$$\epsilon(\Delta\rho) = f[\Delta Z, \delta, \Delta r_s, \lambda] \approx \text{constant}$$

[1] 0.1 arc second is thought to be a reasonable Mars declination error for the JPL Developmental Ephemeris 96 (Ref. 7).

for these eight observations. Any declination error which might exist of the form '$a + bt$' (that is, a bias plus a drift rate) would be very small ($t < 20$ days) and not evident in the $\Delta\rho$.

The scatter in the $\Delta\rho$ set must stem from media calibration errors and hardware performance. Charged particle environments retard range transmission. The neutral particle atmosphere refracts and retards group propagations. DSS electrical hardware response times are included in range measurements. Different DSS frequency standards also distort $\Delta\rho$. These effects, via calibration, are removed from range data; however, each calibration process has its limitations. The uncertainties in a calibrated $\Delta\rho$ point are

$\sigma_{\Delta\rho} = 0.1\sqrt{2}$ m (σ)	(plasma ion calibration, Ref. 8)
$\sigma_{\Delta\rho} = 0.5\sqrt{2}$ m (σ)	(ionospheric ion calibration, Ref. 9)
$\sigma_{\Delta\rho} = 0.3\sqrt{2}$ m (σ)	(trophospheric refraction calibration, Ref. 10)
$\sigma_{\Delta\rho} = 0.5$ m (σ)	(frequency offset calibration, Ref. 11)
$\sigma_{\Delta\rho} = 1\sqrt{2}$ m (σ)	(DSS hardware relay calibration, Refs. 12, 13)

These nongeometrical uncertainties RSS to 1.5 m. The $\sigma_{\Delta\rho}$ observed is 2.6 m for the S-band $\Delta\rho$ and 3.4 m for the S- and X-band $\Delta\rho$ combined – a noise level almost twice that predicted. This extra noise is detrimental to the determination of ΔZ. This is particularly true for low declination observations.

V. Accuracy of ΔZ Estimates

Each of the 10 $\Delta\rho$ observations (Fig. 8) independently provides an estimate of ΔZ (Fig. 10). The translation is $\Delta Z = \Delta\rho$ cosecant (δ). The mean ΔZ is -3.4 m with a standard deviation of 10 m. $\Delta Z = \Delta\rho/\sin\delta$; $\sigma_{\Delta\rho} = 3.4 \Rightarrow \delta_{\Delta Z} = 10$ for $\delta = 23°$. The lower the declination, the greater the $\sigma_{\Delta Z}$ for a given $\sigma_{\Delta\rho}$.

Since spacecraft declinations are typically less than 20° for distant probes, the expectation that a $\sigma_{\Delta Z} \leqslant 6$ will be realized requires an expectation that $\sigma_{\Delta\rho} \leqslant 2$ m will be achieved.

VI. Range Quality

It is not known why $\sigma_{\Delta\rho}$ is at 3.4 meters for this sample. Maybe the sample is not representative. It certainly is a small sample. If the sample is fair, then the noise may result from 'media' or system perturbations not considered in the calibration process. The 'media' calibrations do not appear a likely explanation. Charged-particle calibrations for radiometric range and Doppler have been compared from such independent sources as Faraday polarimeter measurements (Ref. 9), S- and X-band dispersive Doppler and range measurements (Ref. 10), and S- and X-band DRVID measurements and found consistent to a few tenths of meters. Similarly, tropospheric refraction calibrations consistently yield conventional tracking data modelled to the 2 to 3 decimeter level. This is true even for observations below a 10° elevation angle (Ref. 10).

More $\Delta\rho$ data will provide the base needed to characterize the dependence of $\Delta\rho$ on $\epsilon(\Delta Z)$ and $\epsilon(\delta)$.

Differenced range data is being acquired from the Voyager spacecraft while the spacecraft are in heliocentric cruise. Although only four observations have been processed at this time, the Voyager $\Delta\rho$ appear even noisier than those of Viking. DSN and Voyager personnel are currently investigating the DSS fundamental calibration procedures in an effort to reduce the differenced range noise.

VII. Conclusions

DSN radiometric differenced range has been used to differentially correct the DSS 43–DSS 14 baseline relative Z distance. A correction of −3 m is indicated. However, the standard deviation of that correction is 10 m. The prior relative Z sigma was 15 m. The Voyager project requires a sigma of 6 m.

The 10 m sigma on relative Z stems from the differenced range noise ($\sigma_{\Delta\rho} = 3.4$ m), the low declination angles for the spacecraft used ($\delta < 25^\circ$), and transmission media calibrations.

For $\delta \leqslant 20^\circ$, the $\Delta\rho$ must have a $\sigma_{\Delta\rho} \leqslant 2$ m if relative Z determinations to standard deviations of less than six meters are to be realized.

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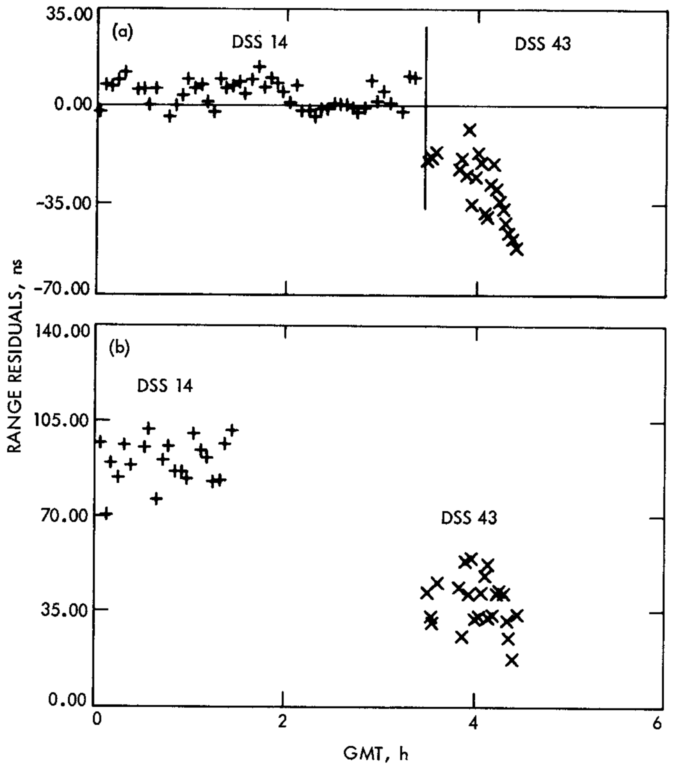


Fig. 1. Residuals as of January 11, 1977: (a) S-band, (b) X-band (1 m ≃ 7 ns)

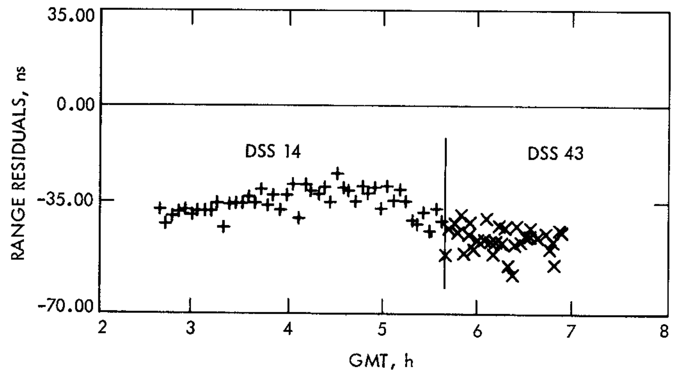


Fig. 2. S-band range residuals, January 19, 1977 (1 m ≃ 7 ns)

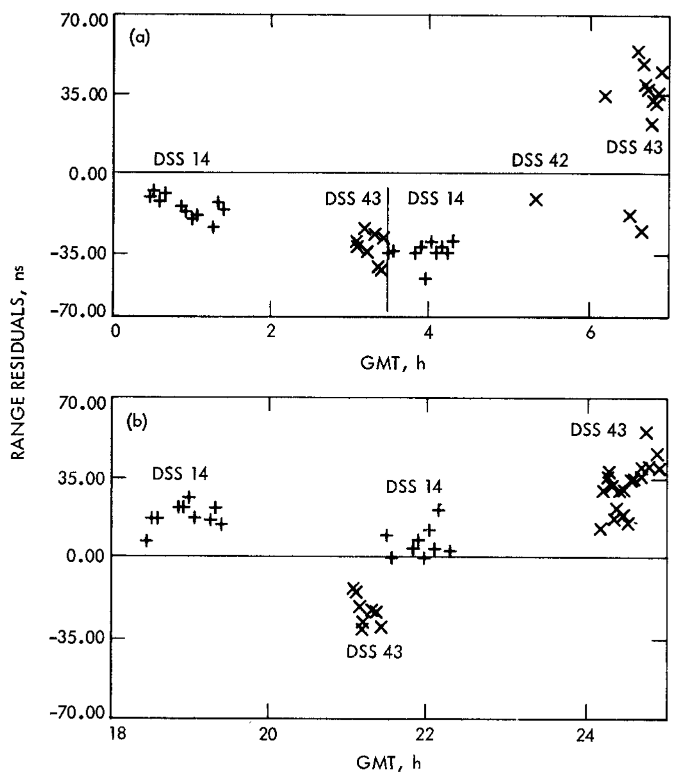


Fig. 3. Residuals as of January 20, 1977: (a) S-band, (b) X-band (1 m ≃ 7 ns)

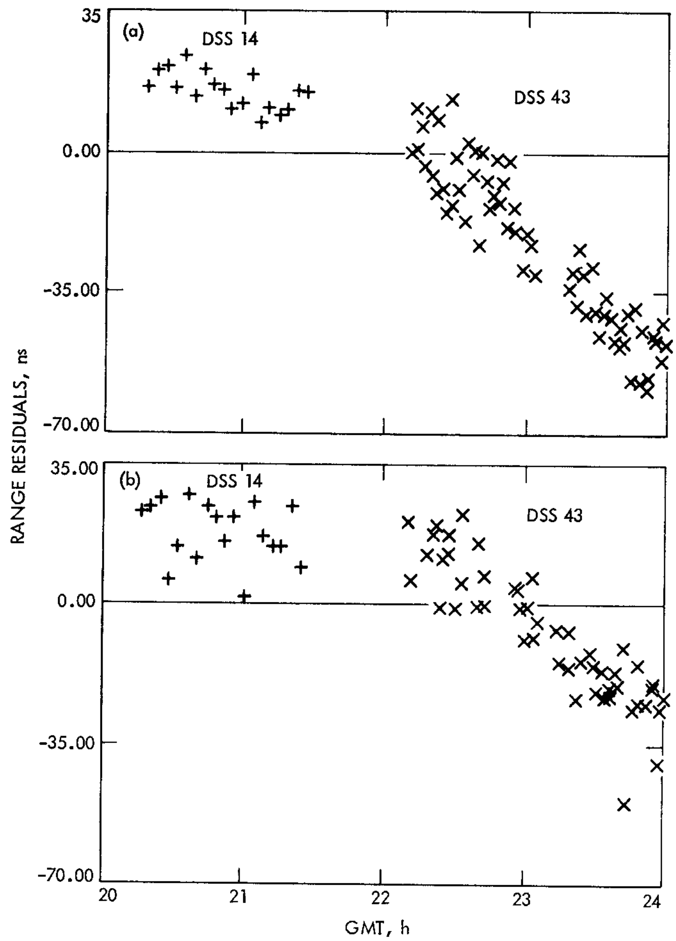


Fig. 4. Residuals as of January 22, 1977: (a) S-band, (b) X-band (1 m ≃ 7 ns)

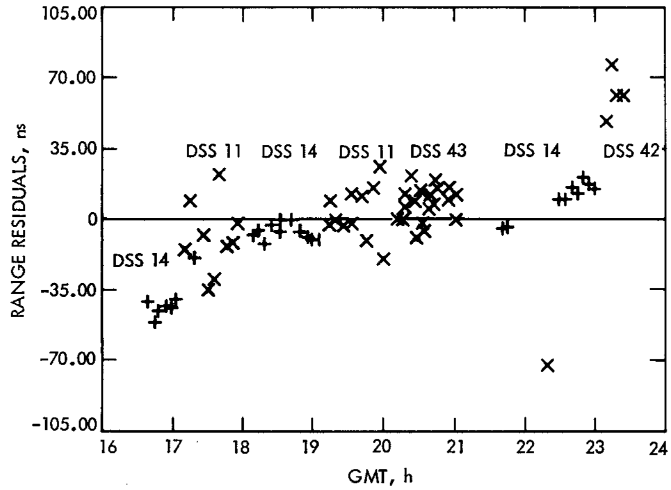
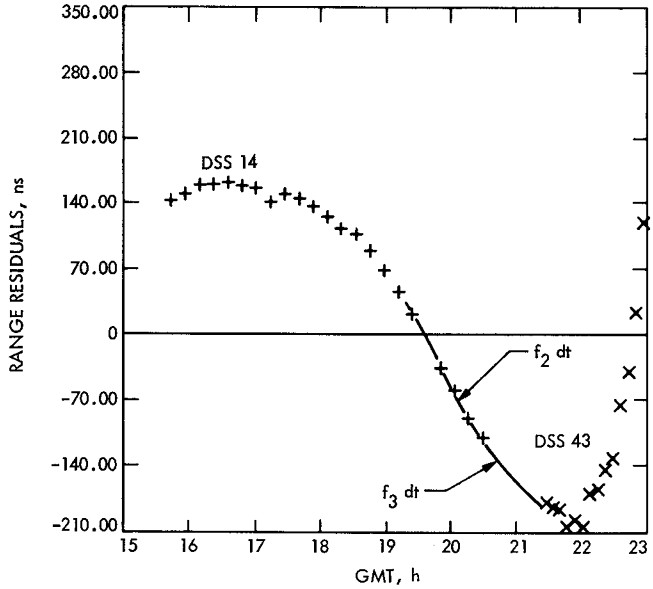


Fig. 5. S-band range residuals, January 31, 1977 (1 m ≃ 7 ns)



NOTE:
THE RANGE RESIDUAL SIGNATURE IS DUE TO UNMODELLED SPACECRAFT MOTION. INTEGRATED TWO-WAY AND THREE-WAY DOPPLER TRACKING DATA WERE USED TO MODEL THE SPACECRAFT TRAJECTORY ERROR DURING THE 56-MIN SEPARATION BETWEEN DSS 14 AND DSS 43 RANGE

Fig. 6. S-band range residuals, February 20, 1977 (1 m ≃ 7 ns)

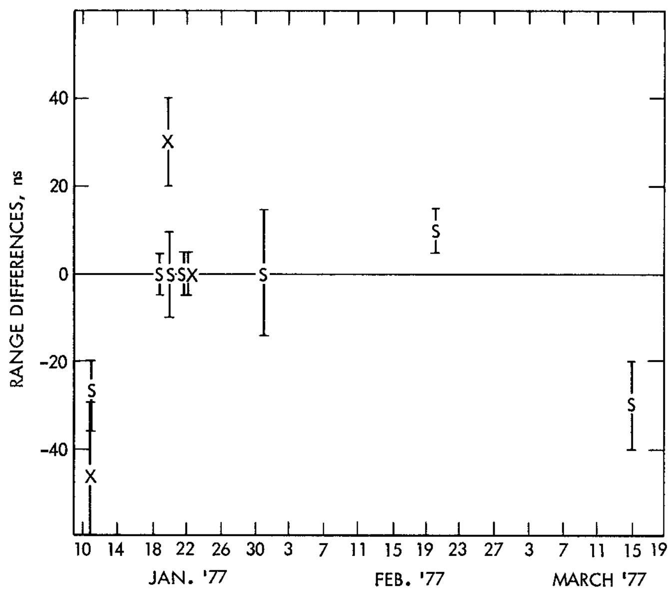


Fig. 7. DSS 43—DSS14 baseline range differences

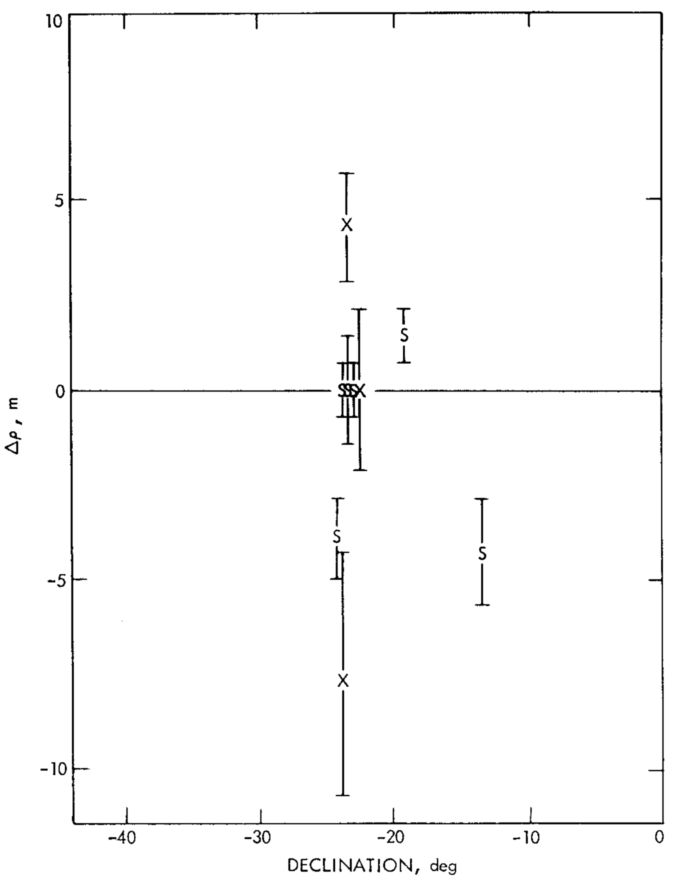


Fig. 8. $\Delta\rho$ as a function of declination

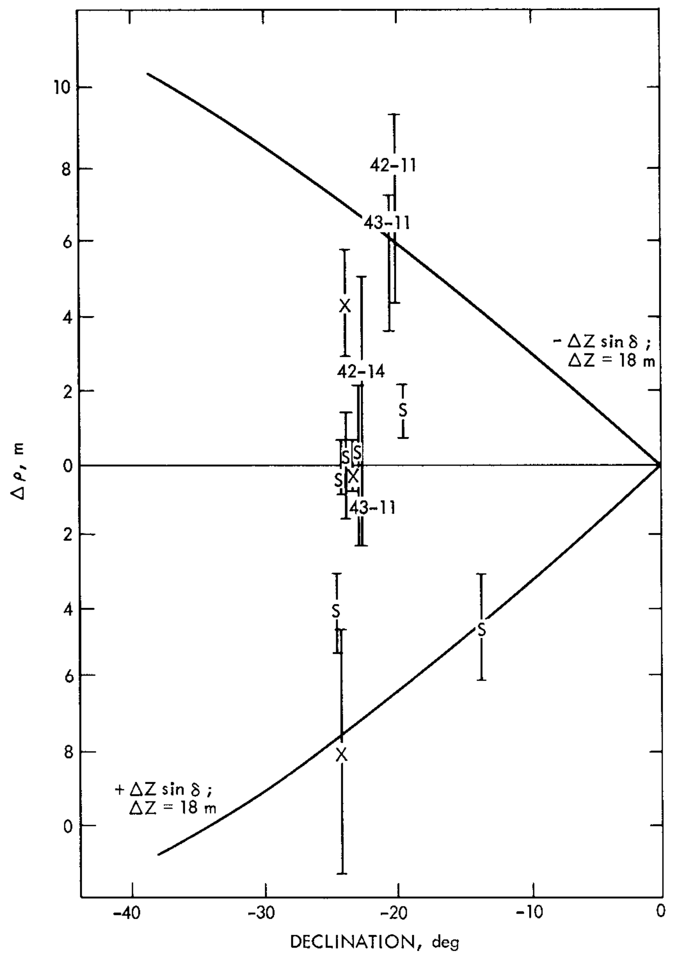


Fig. 9. All $\Delta\rho$ from Canberra—Goldstone Baselines

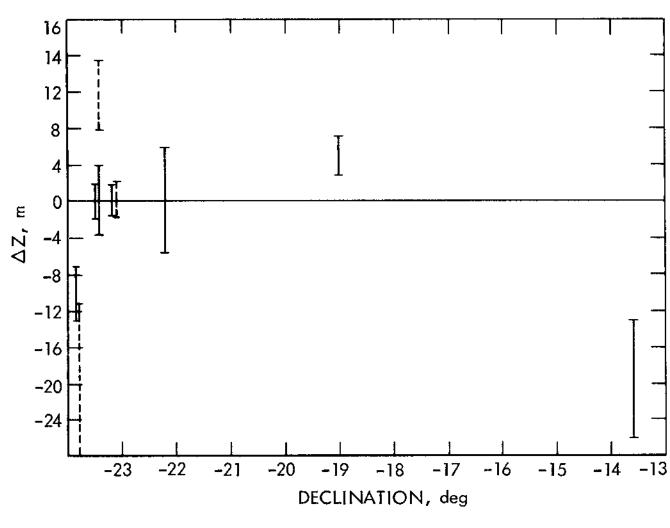


Fig. 10. ΔZ estimates for DSS 43—DSS 14 baseline