Radio-Frequency Carrier Arraying for High-Rate Telemetry Reception

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A method for increasing the sensitivity for radio-frequency reception is to array receiving systems or stations in such a manner as to provide signal-to-noise ratio improvement relative to a single receiving system or station. Radio-frequency carrier arraying for high-rate telemetry that provides signal-to-noise ratio improvement for RF carrier reception and demodulation represents one element of such an array.

I. Introduction

Increasing the sensitivity for radio-frequency reception can be achieved by arraying receiving systems or stations in such a manner as to provide signal-to-noise ratio improvement relative to a single receiving system or station. The following material relates to radio-frequency carrier arraying which provides signal-to-noise ratio improvement for coherent RF carrier reception and RF carrier demodulation. This article is directed toward high-rate telemetry reception with residual RF carrier. A later article will consider RF carrier arraying for low-rate telemetry reception with its attendant smaller RF carrier margin. In this article, the components of equivalent system noise temperature including galactic noise and atmospheric noise are assumed to be statistically independent among the various stations (antennas) of the array. A later article will consider the equivalent system noise temperature in more detail.

II. Receiver Configuration

Figure 1 illustrates the method for achieving RF carrier arraying. Two receiving systems are shown to illustrate the concept. However, additional receiving systems can be added to expand the signal-to-noise improvement capability for RF carrier arraying. In addition, each receiving system can be fed from its own antenna. The input signal to N receiving systems or stations is a radio-frequency signal ($\cos \omega_{RF} t$) phase modulated with telemetry and/or a ranging waveform. Consider, as an example, the case for telemetry where the RF carrier is phase modulated with a square-wave subcarrier ($\text{Cos}\ \omega_{sc} t$) at a peak modulation index m_{pD} that is in turn, biphase-modulated with data $D(t)$. Refer to Fig. 1 during the following development. The signal at the input to the low-noise amplifier in Receiving System 1 is

$$2^{1/2} A_1(t) \cos \left[\omega_{RF} t + \theta_{i1} + D(t + \tau_{i1}) \cdot m_{pD} \cdot \text{Cos}\,(\omega_{sc} t + \theta_{sc_{i1}})\right] + n_{i1}(t) \tag{1a}$$

For a binary modulation waveform, the carrier component becomes

$$2^{1/2}A_1(t) \cdot \cos m_{pD} \cos(\omega_{RF}t + \theta_{i1})$$

The sideband component becomes:

$$+2^{1/2}A_1(t) \sin m_{pD} \cdot D(t + \tau_{i1}) \cdot \text{Cos}(\omega_{sc}t + \theta_{sc_{i1}}) \cdot \sin(\omega_{RF}t + \theta_{i1}) \tag{1b}$$

The noise component is $n_{i1}(t)$.

For Receiving System 2, the corresponding input signal is

$$2^{1/2}A_2(t) \cdot \cos m_{pD} \cos(\omega_{RF}t + \theta_{i2}) + 2^{1/2}A_2(t) \cdot \sin m_{pD} \cdot D(t + \tau_{i2})$$

$$\cdot \text{Cos}(\omega_{sc}t + \theta_{sc_{i2}}) \cdot \sin(\omega_{RF}t + \theta_{i2}) + n_{i2}(t) \tag{2}$$

The other $N - 2$ receiving systems will have corresponding input signals. Since Antenna 1 and Antenna 2 are separate antennas, physically separated, the phase shift on the RF carrier (θ_{i2}) and square wave subcarrier $(\theta_{sc_{i2}})$ and the group delay on the data (τ_{i2}) in Receiving System 2 are different from the corresponding phase shifts and modulation group delay in Receiving System 1. In general, the differences in RF carrier phase shift $(\theta_{i2} - \theta_{i1})$, subcarrier phase shift $(\theta_{sc_{i2}} - \theta_{sc_{i1}})$ and modulation group delay $(\tau_{i2} - \tau_{i1})$ will vary with time during a station pass. The above applies in general to all N receiving systems. The terms $n_{i1}(t)$, $n_{i2}(t)$, etc., represent a combination of galactic noise, atmospheric noise, noise in the antenna sidelobes due to the Earth, noise due to losses in microwave reflectors, and noise due to losses in microwave components all lumped with noise due to input amplifier(s). This combined noise is measured relative to reference temperature load(s) connected to the amplifier input (during the measurement) and designated as the operating equivalent system temperature T_{op_1}, T_{op_2}, etc. The noise term $n_{i1}(t)$ has a double-sided noise spectral density $N_{o1}/2 = (k \cdot T_{op_1} \cdot 1)/2$ watts/Hz where k is Boltzmann's constant, 1.38×10^{-23} joule/°Kelvin. The noise terms $n_{i2}(t)$, etc., have noise spectral densities related to T_{op_2}, etc., as above.

After passing through the low-noise amplifier in Receiving System 1, expression (1) can be written as:

$$k_{LN1}\Big[2^{1/2}A_1(t) \cos m_{pD} \cos(\omega_{RF}t + \theta_{LN10})$$

$$+ 2^{1/2}A_1(t) \sin m_{pD}\, D(t + \tau_{LN10})\, \text{Cos}(\omega_{sc}t + \theta_{sc_{LN10}}) \sin(\omega_{RF}t + \theta_{LN10})\Big] + n_{LN10}(t) \tag{3}$$

where k_{LN1} represents the gain of the low-noise amplifier and $k_{LN1} \cdot A_1(t) \ll 1$. The corresponding expression at the output of the low-noise amplifier in Receiving System 2 is:

$$k_{LN2}\Big[2^{1/2}A_2(t) \cos m_{pD} \cdot \cos(\omega_{RF}t + \theta_{LN20}) + 2^{1/2}A_2(t) \sin m_{pD}$$

$$\cdot D(t + \tau_{LN20})\, \text{Cos}(\omega_{sc}t + \theta_{sc_{LN20}}) \cdot \sin(\omega_R t + \theta_{LN20})\Big] + n_{LN20}(t) \tag{4}$$

where

$$k_{LN2} \cdot A_2(t) \ll 1.$$

The transfer characteristics of low-noise Amplifiers 1 and 2 modify the RF (i.e., $\theta_{LN10} = \theta_{i_1} + \theta_{LN1}$) and subcarrier (i.e., $\theta_{sc_{LN10}} = \theta_{sc_{L1}} + \theta_{sc_{LN1}}$) phase shift as well as data group delay (i.e., $\tau_{LN10} = \tau_{i1} + \tau_{LN1}$) and band limit and amplify by K_{LN} the receiver noise to $n_{LN10}(t)$ and $n_{LN20}(t)$. The other $N - 2$ receiving systems will have corresponding signal expressions at this point in their respective receiving systems.

The first local oscillator signal, which is derived from a voltage-controlled oscillator (VCO_1) through a frequency multiplier (XM) is:

$$2^{1/2} \cos (\omega_{LO} t + \hat{\theta}_{RF}) \tag{5}$$

at the first mixer in Receiving System 1. The term $\hat{\theta}_{RF}$ represents a noisy estimate of the input signal phase to the first mixer. Note that this first local oscillator signal (expression (5)) is also applied to the first mixers of the other receiving systems. In particular, this local oscillator signal will experience an additional phase shift of $Y_2 \cdot 2\pi + \theta_2$ to the first mixer of Receiving System 2, where Y_2 is an integer (a large integer for separate antennas). As a consequence, the first local oscillator signal (Receiving System 2) becomes:

$$2^{1/2} \cos (\omega_{LO} t + \hat{\theta}_{RF} + \theta_2) \tag{6}$$

For Receiving System N, the additional first local oscillator phase shift is $Y_N \cdot 2\pi + \theta_N$ which provides a local oscillator signal $2^{1/2} \cos (\omega_{LO} t + \hat{\theta}_{RF} + \theta_N)$ to its first mixer.

The first IF amplifier in Receiving System 1 has a variable gain including first mixer loss of K_{IFV1} (gain controlled by AGC). The output signal of this first IF amplifier becomes:

$$2^{1/2} A_1 \cos m_{pD} \cos [(\omega_{RF} - \omega_{LO})t + (\theta_{IFV10} - \hat{\theta}_{RF})] + 2^{1/2} A_1 \sin m_{pD} \cdot D(t + \tau_{IFV10}) \cdot \text{Cos} (\omega_{sc} t + \theta_{sc_{IFV10}})$$

$$\cdot \sin [(\omega_{RF} - \omega_{LO})t + (\theta_{IFV10} - \hat{\theta}_{RF})] + n_{IFV10}(t) \tag{7}$$

where

$$A_1 = K_{LN1} \cdot K_{IFV1} \cdot A_1(t)$$

is held essentially constant due to AGC and $A_1 \ll 1$. Using the same convention for notation as above for the low-noise amplifier, the terms θ_{IFV10} and $\theta_{sc_{IFV10}}$ represent the sum of all the RF ($\theta_{i1} + \theta_{LN1} + \theta_{IFV1}$) and subcarrier ($\theta_{sc_{i1}} + \theta_{sc_{LN1}} + \theta_{sc_{IFV1}}$) phase shifts, respectively, and data group delay ($\tau_{IFV10} = \tau_{i1} + \tau_{LN1} + \tau_{IFV1}$) up to this point in the system including coaxial lines. The receiver noise is band limited by the combined bandwidth of the low-noise amplifier and first IF amplifier, and amplified by $k_{LN1} \cdot k_{IFV1}$ to $n_{IFV10}(t)$.

The second local oscillator signal in Receiving System 1, which is derived from a frequency standard common to all N receiving systems, is:

$$2^{1/2} \cos (\omega_{REF1} t) \tag{8}$$

The signal at the output of the second IF distribution amplifier (with gain K_{DIST1} which includes coaxial line and the second mixer loss) becomes:

$$K_{DIST1} \cdot 2^{1/2} A_1 \cos m_{pD} \cos [(\omega_{RF} - \omega_{LO} - \omega_{REF1})t + (\theta_{DIST10} - \hat{\theta}_{RF})] + K_{DIST1} \cdot 2^{1/2} A_1 \sin m_{pD}$$

$$\cdot D(t + \tau_{DIST10}) \text{Cos} (\omega_{sc} t + \theta_{sc_{DIST10}}) \cdot \sin [\omega_{RF} - \omega_{LO} - \omega_{REF1})t + (\theta_{DIST10} - \hat{\theta}_{RF})] + n_{DIST10}(t) \tag{9}$$

where $\theta_{DIST10} = \theta_{IFV10} + \theta_{DIST1}$, $\theta_{sc_{DIST10}} = \theta_{sc_{IFV10}} + \theta_{sc_{DIST1}}$ and $\tau_{DIST10} = \tau_{IFV10} + \tau_{DIST1}$ (the phase shift and group delay up to this point in the system including coaxial lines). In addition, $n_{DIST10}(t)$ is the band limited and amplified (by $k_{LN1} \cdot k_{IF1} \cdot k_{DIST1}$) receiver noise at the output of the second IF distribution amplifier. The term $(\omega_{RF} - \omega_{LO} - \omega_{REF1})$ becomes $(\omega_{REF1} - \omega_{RF} + \omega_{LO})$ for $\omega_{REF1} > \omega_{RF} - \omega_{LO}$. Note that $(\omega_{REF1} - \omega_{RF} + \omega_{LO})$ or $(\omega_{RF} - \omega_{LO} - \omega_{REF1}) = \omega_{REF2}$. Also frequency $(\omega_{REF2} t)$ is the reference frequency for the phase detector in the RF carrier phase tracking loop, the AGC detector, and the telemetry IF channel phase detector (Fig. 1). Reference frequency $(\omega_{REF2} t)$ is also derived from the frequency standard common to all N receiving systems.

The second IF filter F_{A1} is narrow band relative to the telemetry sidebands and consequently filters the sidebands out. After passing through filter F_{A1}, the signal becomes:

$$K_{A1} \cdot K_{DIST1} \cdot 2^{1/2} A_1 \cos m_{pD} \cdot \cos [\omega_{REF2} t + (\theta_{FA10} - \hat{\theta}_{RF})] + n_{FA10}(t) \tag{10}$$

where $\theta_{FA10} = \theta_{DIST10} + \theta_{FA1}$ and $n_{FA10}(t)$ represents the amplified receiver noise related to T_{OP1} in the noise bandwidth of F_{A1}. Note that since the gain from Expression (10) in Fig. 1 to the AGC detector is constant at $K_{2\text{nd } IF_1}$ (minus the summing junction loss), the attenuator (A_1) at the input to the second IF amplifier in Receiving System 1 provides a means for setting the signal level at (9) to provide the required output level for the telemetry subcarrier spectrum in Receiving System 1 when RF carrier arraying with Receiving System 2, etc.

Proceeding in Receiving System 2 as above (for Receiving System 1), the output signal of the first IF amplifier in Receiving System 2 (from expressions (4) and (6)) becomes:

$$2^{1/2} A_2 \cos m_{pD} \cos [(\omega_{RF} - \omega_{LO})t + (\theta_{IFV20} - \hat{\theta}_{RF} - \theta_2)] + 2^{1/2} A_2 \sin m_{pD} \cdot D(t + \tau_{IF20}) \cdot \text{Cos} (\omega_{sc} t + \theta_{sc_{IFV20}})$$

$$\cdot \sin [(\omega_{RF} - \omega_{LO})t + (\theta_{IFV20} - \hat{\theta}_{RF} - \theta_2)] + n_{IFV20}(t) \tag{11}$$

where

$$A_2 = (K_{LN2} K_{IFV2} \cdot A_2(t))$$

and is held essentially constant due to AGC and $A_2 \ll 1$. Note that the receiving systems considered here are similar so that the signal-to-noise ratio at this point (11) in Receiving System 2 is nearly equal to that at the corresponding point in Receiving System 1. (The concept developed herein however, can be applied to arraying receiving systems where the signal-to-noise ratios are not equal.) The second local oscillator signal in Receiving System 2 is derived from a voltage-controlled crystal oscillator (VCO_2) through a frequency multiplier (XQ). From inspection of Fig. 1, the frequency $VCO2\ XQ$ is equal to $(\omega_{REF1} t)$ although its phase is modified by differences in carrier phase as pointed out in the discussion relative to expressions (2), (4), and (6). The second local oscillator signal can be expressed as:

$$2^{1/2} \cos (\omega_{REF1} t - \hat{\theta}_{LO2}) \tag{12}$$

where $\hat{\theta}_{LO2}$ represents the estimate of the RF carrier input phase to the second mixer (Receiving System 2). The phase estimate $\hat{\theta}_{LO2}$ is derived from a phase-locked loop whose closed-loop noise bandwidth is a small fraction of the closed-loop noise bandwidth of the RF carrier phase tracking loop discussed below (expression (16)). Hence the noise on phase estimate $\hat{\theta}_{LO2}$ is much less than that on $\hat{\theta}_{RF}$ (first local oscillator signal). Note that if the carrier signal-to-noise ratio in Receiving System 2 were significantly less than that in Receiving System 1, this effect could be offset by making the closed-loop noise bandwidth in Receiving System 2 a still smaller fraction of that in Receiving System 1.

The signal at the output of the second IF distribution amplifier (with gain K_{DIST2} which includes coaxial line and second mixer loss) is:

$$K_{DIST2} \cdot 2^{1/2} A_2 \cos m_{pD} \cos [(\omega_{RF} - \omega_{LO} - \omega_{REF1})t + (\theta_{DIST20} - (\hat{\theta}_{RF} - \hat{\theta}_{LO2}))]$$

$$+ K_{DIST2} \cdot 2^{1/2} A_2 \sin m_{pD} \cdot D(t + \tau_{DIST20}) \text{Cos} (\omega_{sc} t + \theta_{sc_{DIST20}})$$

$$\cdot \sin [(\omega_{RF} - \omega_{LO} - \omega_{REF1})t + (\theta_{DIST20} - (\hat{\theta}_{RF} - \hat{\theta}_{LO2}))] + n_{DIST20}(t) \tag{13}$$

where $\theta_{DIST20} (= \theta_{IFV20} - \theta_2 + \theta_{DIST2}), \theta_{sc_{DIST20}}$ and τ_{DIST20} represent the phase shift and group delay up to this point in System 2 including coaxial lines. Note from Fig. 1 that $(\omega_{RF} - \omega_{LO2} - \omega_{REF1}) = \omega_{REF2}$. The second IF distribution amplifier provides its output signal to the IF filter F_{A2} and to the telemetry IF channel in Receiving System 2. The second IF filter F_{A2} in Receiving System 2 has the same noise bandwidth as IF filter F_{A1} in Receiving System 1 (by design). The signal at the output of IF filter F_{A2} (with gain K_{A2}) can be expressed as:

$$K_{A2} \cdot K_{DIST2} \cdot 2^{1/2} A_2 \cos m_{pD} \cdot \cos [\omega_{REF2} t + (\theta_{FA20} - (\hat{\theta}_{RF} - \hat{\theta}_{LO2}))] + n_{FA20}(t) \tag{14}$$

where $\theta_{FA20} = \theta_{DIST20} + \theta_{FA2}$ and $n_{FA20}(t)$ represents the amplified receiver noise in the noise bandwidth of F_{A2} at the operating equivalent system noise temperature T_{op2}. This signal is provided as an input to the summing junction (Fig. 1) as well as to the RF carrier loop and AGC in Receiving System 2. Note that the phase shift from the output of IF filter F_{A2} (expression 14) in Fig. 1 to the phase detector which provides the error signal to the tracking filter (Receiving System 2) is a constant by design. Consequently the phase shifter marked A, which is in series with the reference 2 input $(\omega_{REF2} t)$ in Receiving System 2 provides a means for setting the RF phase of expression (14) equal to the RF phase of expression (10) in Receiving System 1 at the summing junction. The second IF amplifier gain $K_{2nd\ IF_2}$ is designed so that the signal level $(K_{A2} \cdot K_{DIST2} \cdot 2^{1/2} A_2)$ can be set as required relative to the signal level $(K_{A1} \cdot K_{DIST1} \cdot 2^{1/2} A_1)$ in Receiving System 1 at the input to the summing junction.

III. Predetection Signal-to-Noise Ratio and Resultant Phase Noise

A. Receiving System 1

Assume for the moment that the second receiving system shown in Fig. 1 is not connected to the summing junction. The predetection carrier signal-to-noise power ratio in Receiving System 1 represented by expression (10) is:

$$P_{c1}/P_{n1} = \frac{A_1^2 \cos^2 m_{pD}}{NBW_{F_{A1}} \cdot N_{o1}} \quad \text{or} \quad \frac{P_{c1}}{NBW_{F_{A1}} \cdot N_{o1}} \tag{15}$$

where $NBW_{F_{A1}}$ represents the noise bandwidth of F_{A1} and N_{o1} is the one-sided noise spectral density for Receiving System 1 which was defined in the discussion relative to expression (1). The receiving system contains a second-order RF carrier phase tracking loop which utilizes a band pass limiter and a sinusoidal phase detector. Utilizing the information in Refs. 1 and 2 and limiting $\sigma_n \leqslant 1$ radian, the rms phase noise σ_{ϕ_n} at the output of the RF carrier tracking loop (i.e., on the first local oscillator signal) can be expressed as:

$$\sigma_{\phi_{n1}} = \left(\frac{\frac{N_{o1}}{2} \cdot 2B_{L1}}{P_{c1}}\right)^{1/2} \cdot \left[\frac{1 + \frac{P_{c1}}{NBW_{F_{A1}} N_{o1}}}{0.862 + \frac{P_{c1}}{NBW_{F_{A1}} N_{o1}}}\right]^{1/2} \cdot \left[\frac{N_{o1} B_{L_1}}{P_{c_1}} \cdot \frac{e^{\left(\frac{N_{o1} B_{L1}}{P_{c1}}\right)}}{\sinh\left(\frac{N_{o1} B_{L1}}{P_{c1}}\right)}\right]^{1/2} \quad \text{radians, rms}$$

or collecting terms.

$$\sigma_{\phi_{n1}} = \left[\frac{\frac{N_{o1}}{2} \cdot 2B_{L1}}{P_{c1}} \cdot \frac{1 + \frac{P_{c1}}{NBW_{F_{A1}} \cdot N_{o1}}}{0.862 + \frac{P_{c1}}{NBW_{F_{A1}} \cdot N_{o1}}} \cdot \frac{e^{\left(\frac{N_{o1} B_{L1}}{P_{c1}}\right)}}{\sinh\left(\frac{N_{o1} B_{L1}}{P_{c1}}\right)}\right]^{1/2} \quad \text{radians, rms} \tag{16}$$

The two-sided closed-loop noise bandwidth $2B_{L1}$ can be expressed as:

$$2B_{L1} = \frac{2B_{LO1}}{r_o + 1}\left(1 + r_o \frac{\alpha}{\alpha_{o1}}\right)$$

where $r_o = 2$ by design (0.707 damping) and $2B_{LO1}$ is the design point (threshold) two-sided closed-loop noise bandwidth in Receiving System 1. The term α_1 is the limiter suppression factor resulting from the noise-power-to-carrier-power ratio in NBW_{FA1}, α_1 has a value of α_{o1} at design point (threshold). At threshold, the predetection carrier signal-to-noise ratio in a noise bandwidth equal to the threshold closed-loop noise bandwidth ($2B_{LO}$) is unity (i.e., $P_c/2B_{LO} \cdot N_o = 1$).

B. Receiving System 1 Arrayed with Receiving System(s) 2 Through N

Consider connection of the second Receiving System to the summing junction. Designate the voltage coupling of Receiving System 2 into the summing junction as β_2 (relative to Receiving System 1). In addition, designate the carrier power-to-noise spectral density ratio of Receiving System 2 as γ_2^2 of that in Receiving System 1 where $\gamma_2 \leqslant 1$. Note that the carrier phase noise that is fed from Receiving System 2 into the summing junction is small (because of much narrower closed loop noise bandwidths as discussed above) relative to the phase noise error on the first local oscillator. Consequently, the Receiver 1 and 2 carrier signals fed into the summing junction are coherent with a small differential phase jitter. However, their receiver noise voltages fed into the summing junction are statistically independent. As a result, the summed predetection carrier signal-to-noise power ratio at the output of the summing junction becomes:

$$\frac{P_{c_{1\Sigma 1,2}}}{P_{n_{1\Sigma 1,2}}} = \frac{(A_1 \cos m_{pD} + \beta_2 A_2 \cos m_{pD})^2}{NBW_{F_{A1}} \cdot N_{o1} + \beta_2^2 \, NBW_{F_{A2}} \cdot N_{o2}}$$

which can be expressed as

$$\frac{P_{c_{1\Sigma 1,2}}}{P_{n_{1\Sigma 1,2}}} = \frac{A_1^2 \cos^2 m_{pD}}{NBW_{F_{A1}} \cdot N_{o1}} \cdot \frac{(1 + \beta_2 \gamma_2)^2}{1 + \beta_2^2}$$

or

$$\frac{P_{c_{1\Sigma 1,2}}}{P_{n_{1\Sigma 1,2}}} = \frac{P_{c1}}{NBW_{F_{A1}} \cdot N_{o1}} \cdot \frac{(1 + \beta_2 \gamma_2)^2}{1 + \beta_2^2} \tag{17}$$

The rms phase noise $\sigma_{\phi_{n\Sigma 1,2}}$ at the output of the RF carrier tracking loop (i.e., on the first local oscillator signal) in Receiving System 1 due to predetection carrier signal-to-noise ratio can now be determined for two receiving systems arrayed by incorporating the information in expression (17) above into the expression for σ_{ϕ_n} (see expression (16)). The resulting expression becomes:

$$\sigma_{\phi_{n1\Sigma 1,2}} = \frac{\frac{N_{o1}}{2} \cdot 2B_{L1}}{P_{c1}} \cdot \frac{1}{\eta_2} \left[\frac{1 + \dfrac{P_{c1} \cdot \eta_2}{NBW_{F_{A1}} \cdot N_{o1}}}{0.862 + \dfrac{P_{c1} \cdot \eta_2}{NBW_{F_{A1}} \cdot N_{o1}}} \cdot \frac{\exp\left(\dfrac{N_{o1} B_{L1}}{P_{c1} \cdot \eta_2}\right)}{\sinh\left(\dfrac{N_{o1} \cdot B_{L1}}{P_{c1} \cdot \eta_2}\right)} \right]^{1/2} \text{radians, rms} \tag{18}$$

where

$$\eta_2 = \frac{(1 + \beta_2 \gamma_2)^2}{1 + \beta_2^2}$$

for two receiving systems arrayed. Note that for N similar receiving systems arrayed, the predetection carrier signal-to-noise power ratio in Receiving System 1 becomes

$$\frac{P_{c1\Sigma 1, \cdots N}}{P_{n1\Sigma 1, \cdots N}} = \frac{P_{c1}}{NBW_{F_{A1}} \cdot N_{o1}} \frac{(1 + \beta_2 \gamma_2 + \beta_3 \gamma_3 + \cdots + \beta_N \gamma_N)^2}{1 + \beta_2^2 + \beta_3^2 + \cdots + \beta_N^2} \tag{19}$$

The corresponding $\sigma_{\phi_{1n\Sigma 1 \cdots N}}$ can now be determined and becomes

$$\sigma_{\phi_{n1\Sigma 1, \cdots, N}} = \frac{\frac{N_{o1}}{2} \cdot 2B_{L1}}{P_{c1}} \cdot \frac{1}{\eta_N} \left[\frac{1 + \frac{P_{c1} \cdot \eta_N}{NBW_{F_{A1}} N_{o1}}}{0.862 + \frac{P_{c1} \cdot \eta_N}{NBW_{F_{A1}} N_{o1}}} \cdot \frac{\exp\left(\frac{N_{o1} \cdot B_{L1}}{P_{c1} \cdot \eta_N}\right)}{\sinh\left(\frac{N_{o1} \cdot B_{L1}}{P_{c1} \cdot \eta_N}\right)} \right]^{1/2} \text{ radians, rms} \tag{20}$$

where

$$\eta_N = \frac{(1 + \beta_2\gamma_2 + \cdots + \beta_N\gamma_N)^2}{1 + \beta_2^2 + \cdots + \beta_N^2}$$

for N receiving systems arrayed.

The preceding paragraph has addressed determination of the rms phase noise $\sigma_{\phi n}$ at the output of the RF carrier tracking loop (first local oscillator signal) in Receiving System 1 due to the predetection carrier signal-to-noise ratio for two (up to N) receiving systems arrayed. Inspection of Fig. 1 and expression (14) above indicates that an additional phase noise term due to $\hat{\theta}_{LO2}$ (noise at the output of the carrier tracking loop in Receiving System 2) coupled into the summing junction must also be considered for two receiving systems arrayed. Note that since the closed-loop noise bandwidth of Receiving System 1 carrier tracking loop is much wider than that in Receiving System 2, the phase noise on $\hat{\theta}_{LO2}$ is tracked by this wider phase tracking loop and consequently appears as an (extremely small) error signal at the output of the summing junction. As a result of the above (and from inspection of Fig. 1), the term $\hat{\theta}_{RF}$ for the first oscillator signal (see expression (5) and (6)) now becomes

$$\hat{\theta}_{RF} + \frac{\beta_2\hat{\theta}_{LO2}}{1 + \beta_2}$$

Consequently expressions (7), (9), and (10) in Receiving System 1 and expressions (11), (13) and (14) in Receiving System 2 now have the term $\hat{\theta}_{RF}$ replaced by

$$\hat{\theta}_{RF} + \frac{\beta_2\hat{\theta}_{LO2}}{1 + \beta_2}$$

As a result, the total rms phase noise at the output of the principal (Receiving System 1) carrier tracking loop (i.e., on the first local oscillator signal) becomes:

$$\left(\sigma^2_{\phi_{n1\Sigma 1,2}} + \left(\frac{\beta_2\sigma_{\phi_{n2}}}{1 + \beta_2}\right)^2\right)^{1/2} \tag{21}$$

for two receiving systems arrayed. The first term, $\sigma_{\phi_{n1\Sigma1,2}}$ is the rms phase noise due to predetection carrier signal-to-noise ratio at the output of the summing junction (i.e., expression (18)). The rms phase noise $\sigma_{\phi_{n2}}$ contained in the second term above will be developed in the following material. Note that for N receiving systems arrayed the total rms phase noise on the first local oscillator signal becomes:

$$\left(\sigma^2_{\phi_{n1\Sigma1,\cdots,N}} + \left(\frac{\beta_2\sigma_{\phi_{n2}}}{1+\beta_2}\right)^2 + \cdots + \left(\frac{\beta_N\sigma_{\phi_{nN}}}{1+\beta_N}\right)^2\right)^{1/2} \tag{22}$$

where $\sigma_{\phi_{n1\Sigma1,\cdots,N}}$ is expression (20) above. The expressions for $\sigma_{\phi_{n2}}$ (and $\sigma_{\phi_{nN}}$) will be developed in the following material.

The total rms phase noise shown in expressions (21) and (22) can be considered as due to an equivalent predetection carrier signal-to-noise ratio for two up to N receiving systems arrayed. Comparison of this equivalent predetection carrier signal-to-noise ratio with the initial predetection carrier signal-to-noise ratio in a single Receiving System (i.e., system 1) provides the improvement due to carrier arraying for high-rate telemetry.

C. Receiving System 2 Through N

Consider next the carrier phase tracking loop in Receiving System 2. Since the closed-loop noise bandwidth of the carrier tracking loop in Receiving System 2 is much narrower than that in Receiving System 1, the phase noise in Receiving System 1 carrier tracking loop (which has an rms value $\sigma_{\phi_{n\Sigma1,2}}$ for two systems arrayed) produces a predetection carrier signal voltage reduction which has an expected value of

$$1 - \frac{\sigma^2_{\phi_{n\Sigma1,2}}}{2}$$

The predetection carrier signal-to-noise power ratio in Receiving System 2 represented by expression (14) is:

$$\frac{P_{c_{2\Sigma1,2}}}{P_{n2}} = \frac{A_2\cos^2 m_{pD}\left(1 - \dfrac{\sigma^2_{\phi_{n1\Sigma1,2}}}{2}\right)^2}{NBW_{F_{A2}}\cdot N_{o2}} \quad \text{or} \quad \frac{P_{c_2}\left(1 - \dfrac{\sigma^2_{\phi_{n1\Sigma1,2}}}{2}\right)^2}{NBW_{F_{A2}}\cdot N_{o2}} \tag{23}$$

The carrier tracking loop in Receiving System 2 is a second-order loop ($r_o = 2$) which also utilizes a band pass limiter and a sinusoidal phase detector. The rms phase noise $\sigma_{\phi_{n2}}$ in Receiving System 2 carrier tracking loop becomes

$$\sigma_{\phi_{n2\Sigma1,2}} = \frac{\dfrac{N_{o2}}{2}\cdot 2B_{L2}}{P_{c_{2\Sigma1,2}}}\cdot\left[\frac{1 + \dfrac{P_{c_{2\Sigma1,2}}}{NBW_{F_{A2}}N_{o2}}}{0.862 + \dfrac{P_{c_{2\Sigma1,2}}}{NBW_{F_{A2}}\cdot N_{o2}}}\cdot\frac{\exp\left(\dfrac{N_{o2}\cdot B_{L2}}{P_{c_{2\Sigma1,2}}}\right)}{\sinh\left(\dfrac{N_{o2}\cdot B_{L2}}{P_{c_{2\Sigma1,2}}}\right)}\right]^{1/2} \text{ radians, rms} \tag{24}$$

where

$$P_{c_{2\Sigma 1,2}} = P_{c2}\left(1 - \frac{\sigma^2_{\phi_{n1\Sigma 1,2}}}{2}\right)^2$$

for two receiving systems arrayed.

For N receiving systems arrayed, the predetection carrier signal-to-noise power ratio in Receiving System 2 becomes:

$$\frac{P_{c_{2\Sigma 1,\cdots N}}}{P_{n2}} = \frac{P_{c2}\left(1 - \frac{\sigma^2_{\phi_{n1\Sigma 1,\cdots N}}}{2}\right)^2}{NBW_{F_{A2}} \cdot N_{o2}} \tag{25}$$

and the corresponding $\sigma_{\phi_{n2\Sigma 1,\cdots N}}$ can be determined by substitution of $P_{c_{2\Sigma 1,\cdots N}}$ in place of $P_{c_{2\Sigma 1,2}}$ in expression (24). Note that the term $\sigma_{\phi_{n1\Sigma 1,\cdots N}}$ in expression (25) above should be replaced by an rms value similar to that shown in expression (22) with the term

$$\frac{\beta_2 \sigma_{\phi_{n2}}}{1 + \beta_2}$$

deleted. This iteration provides a very small effect on carrier signal-to-noise ratio improvement for the design parameters and number of receiving systems considered in this report. The predetection carrier signal-to-noise power ratio in Receiving System N becomes:

$$\frac{P_{c_{N\Sigma 1,\cdots N}}}{P_{n_N}} = \frac{P_{c_N}\left(1 - \frac{\sigma^2_{\phi_{n1\Sigma 1,\cdots,N}}}{2}\right)^2}{NBW_{F_{AN}} \cdot N_{o_N}} \tag{26}$$

The resultant rms phase noise $\sigma_{\phi_{nN}}$ in Receiving System N carrier tracking loop is:

$$\sigma_{\phi_{nN\Sigma 1,\cdots,N}} = \frac{\frac{N_{o_N}}{2} \cdot 2B_{LN}}{P_{c_{N\Sigma 1,\cdots,N}}} \cdot \left[\frac{1 + \frac{P_{c_{N\Sigma 1,\cdots,N}}}{NBW_{F_{AN}} \cdot N_{o_N}}}{0.862 + \frac{P_{c_{N\Sigma 1,\cdots,N}}}{NBW_{F_{AN}} \cdot N_{o_N}}} \cdot \frac{\exp\left(\frac{N_{o_N} \cdot B_{L_N}}{P_{c_{N\Sigma 1,\cdots,N}}}\right)}{\sinh\left(\frac{N_{o_N} \cdot B_{L_N}}{P_{c_{N\Sigma 1,\cdots,N}}}\right)}\right]^{1/2} \text{radians, rms} \tag{27}$$

IV. Acquisition

The following discussion relates to RF carrier acquisition for the arrayed receiving systems shown in Fig. 1. Prior to RF carrier acquisition, receiving systems 2 through N are disconnected (switched out) from the summing junction. The first local oscillator ($MxVCO_1$) in Receiving System 1 is swept or set in frequency to the expected (or predicted) reception frequency. Radio frequency carrier acquisition is obtained for carrier levels from threshold (defined in discussion following expression (16)) to strong signal levels. RF carrier acquisition in Receiving System 1 provides the first local oscillator signal for receiving systems 2 through N.

The carrier tracking loop in Receiving System 2 is acquired by setting the oscillator frequency ($VCO_2 xQ$) equal (zero beat frequency) to reference frequency 1 (Fig. 1). Receiving System 2 is now connected into the summing junction to accomplish RF carrier arraying of two systems. This procedure is repeated in rapid succession until all remaining receiving systems are acquired and RF carrier arrayed for reception.

V. Performance

The improvement in carrier signal-to-noise ratio that can be achieved by arraying receiving systems for high-rate telemetry can now be determined for a representative set of design parameters using the preceding development. The following parameters are used for Receiving System 1:

Threshold two-sided noise bandwidth $(2B_{LO1}) = 12$ Hz

Predetection IF filter noise bandwidth $(NBW_{FA1}) = 2200$ Hz

The parameters used for Receiving System 2 through N are:

Threshold two-sided noise bandwidth $(2B_{LO2,\cdots,N}) = 0.1$ Hz

Predetection IF filter noise bandwidth $(NBW_{FA2,\cdots,N}) = 2200$ Hz

Figure 2 shows the carrier signal-to-noise ratio improvement resulting from arraying up to twelve similar receiving systems for high-rate telemetry reception. Performance is shown for γ_N values of 1.0, 0.95, 0.9 and 0.84 where β_N is equal to γ_N. As developed earlier in this report, γ_N^2 is the carrier signal-to-noise spectral density ratio of Receiving System N relative to Receiving System 1. Also β_N is the voltage coupling of Receiving System N into the summing junction relative to Receiving System 1.

Figure 3 shows the effect on carrier signal-to-noise ratio improvement for γ_N values of 1.0, 0.95, 0.9 and 0.84 when β_N is varied. Performance is shown for two, three, four and ten receiving systems arrayed for high-rate telemetry. The effect of the carrier phase noise from Receiving Systems 2 through N (see expression (22)) coupled into the principal (Receiving System 1) RF carrier tracking loop can be seen in Fig. 3. This effect becomes more apparent as the number of systems arrayed is increased. Note that, for ten receiving systems arrayed for high-rate telemetry reception, reducing the coupling β from unity results in an increase in carrier signal-to-noise ratio improvement.

Some initial measurements of radio-frequency carrier signal-to-noise ratio improvement have been made in the laboratory by arraying two, three, and four receivers. For convenience, these initial measurements were made with the following parameters; Receiver 1:

Threshold two-sided noise bandwidth $(2B_{LO1}) = 152$ Hz

Predetection IF filter noise bandwidth $(NBW_{FA1}) = 2200$ Hz

The parameters for Receivers 2, 3 and 4 were:

Threshold two-sided noise bandwidth $(2B_{LO2,3,4}) = 1$ Hz

Predetection IF filter noise bandwidth $(NBW_{FA2,3,4}) = 2200$ Hz

These parameters provide essentially the same performance as shown in Figs. 2 and 3 for two, three, and four receiving systems arrayed.

Measurement of two receivers (1 and 2) arrayed, with γ_2 and β_2 equal to one, provided a RF carrier signal-to-noise ratio improvement of 2.7 dB. Calculated improvement is 2.8 dB. Note that a measurement with $\beta_2 = 1$ and $\gamma_2 = 0.88$ provided a carrier signal-to-noise ratio improvement of 2.4 dB, which agrees with predicted performance. Measurement of three receivers (1, 2, and 3) arrayed provided a 4.1-dB improvement in RF carrier signal-to-noise ratio. For this measurement γ_2 and $\gamma_3 = 1$, $\beta_2 = 0.4$ and $\beta_3 = 0.49$. These parameters provide a predicted improvement of 4.1 dB. With four receivers (1, 2, 3, and 4) arrayed, measured improvement in RF carrier signal-to-noise ratio was 4.6 dB. For this measurement γ_2 and $\gamma_3 = 1$, $\gamma_4 = 0.86$, $\beta_2 = 0.3$, $\beta_3 = 0.38$ and $\beta_4 = 0.36$. These parameters provide a predicted improvement of 4.6 dB. For the above measurements, the effective system noise temperatures of receivers 1, 2, 3 and 4 were 3540, 3360, 3360 and 4640 degrees kelvin, respectively. It is planned to make further measurements with parameters optimized (i.e., γ_N and $\beta_N = 1$).

References

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2. Tausworthe, R. C., "Limiters in Phase-Locked Loops: A Correction to Previous Theory," *Space Programs Summary No. 37-54*, Vol. III, pp. 201-203, Jet Propulsion Laboratory, Pasadena, Calif., 1968.

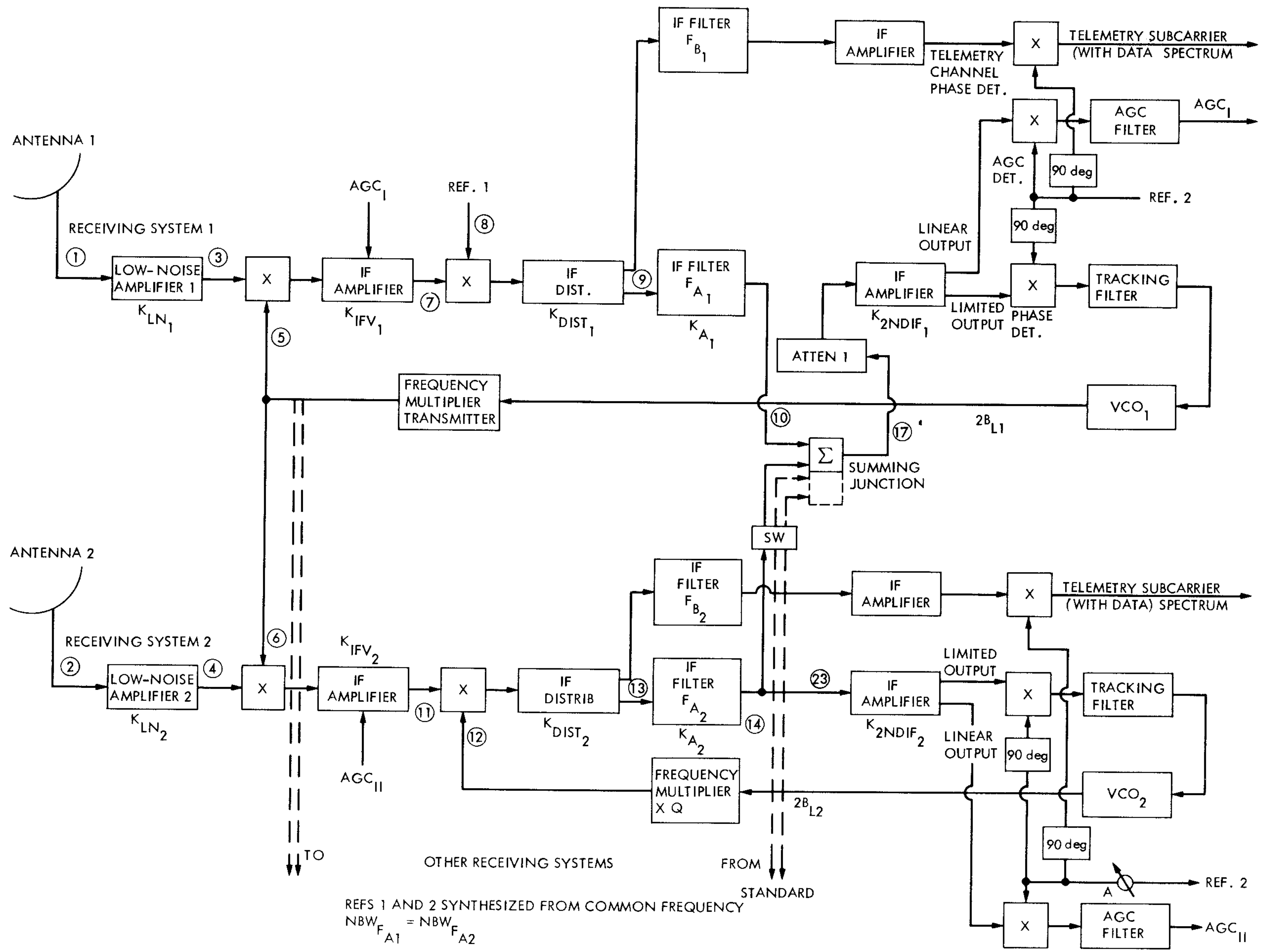

Fig. 1. Radio frequency carrier arraying for high-rate telemetry reception

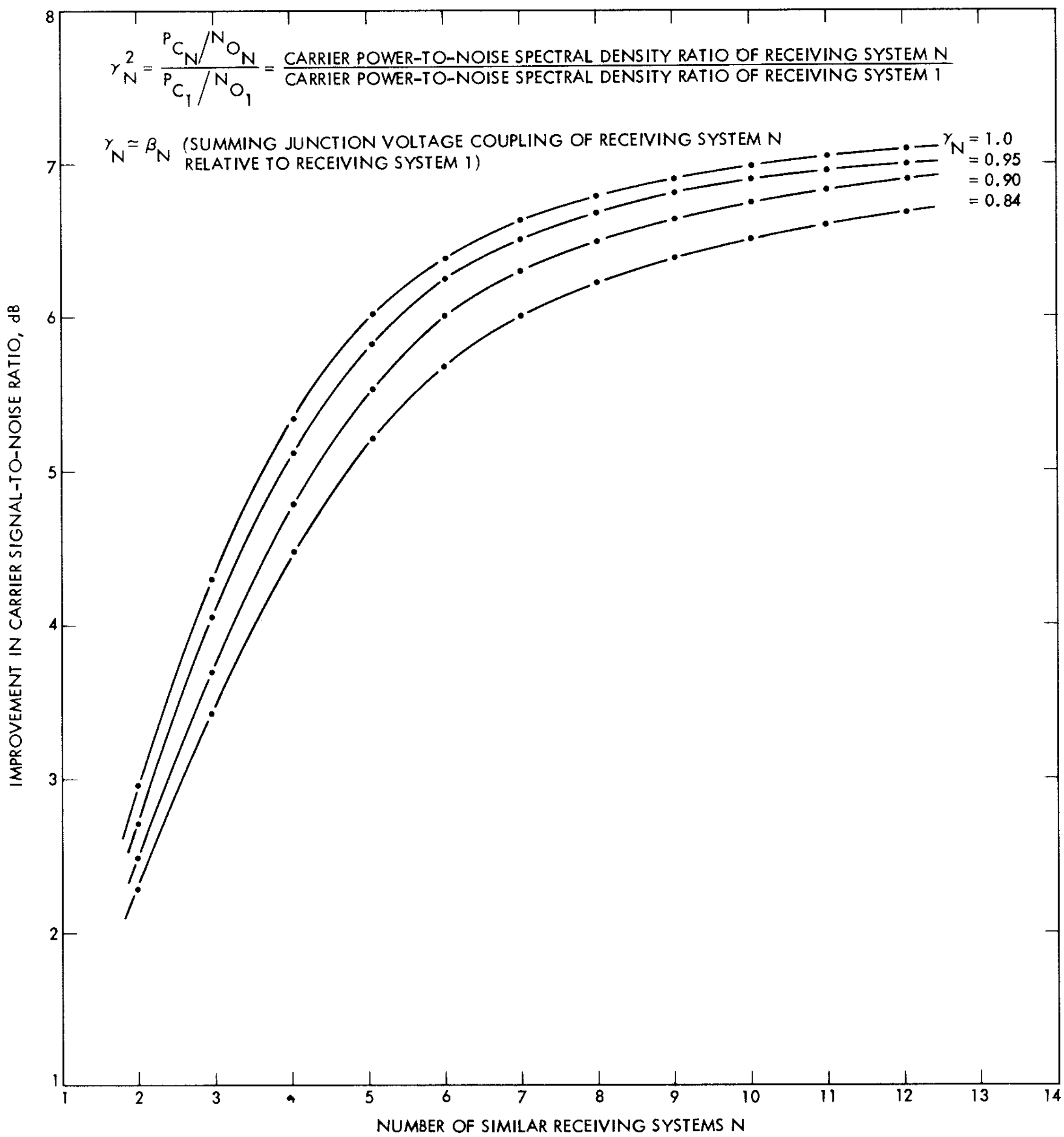

Fig. 2. Effect of summing junction voltage coupling on carrier signal-to-noise ratio improvement (receiver arraying for high-rate telemetry reception)

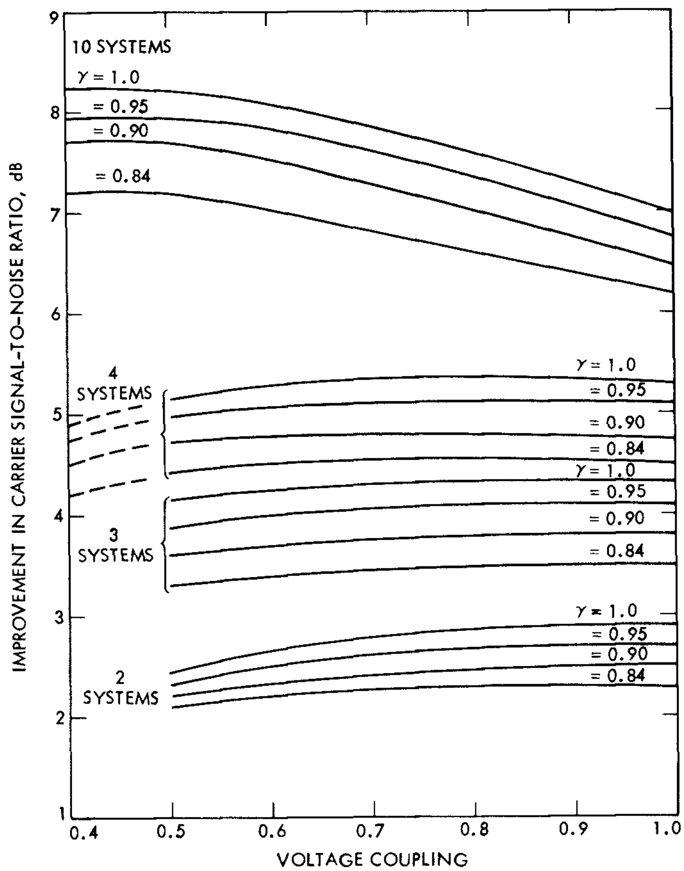

Fig. 3. Signal-to-noise ratio improvement for radio frequency carrier arraying for high-rate telemetry reception