The Gravitational Wave Detection Experiment: Description and Anticipated Requirements

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One of the most exciting challenges facing gravitational theoreticians and experimenters in the remaining decades of this century will be the search for "gravitational waves" as predicted by Einstein's General Theory of Relativity. Proposals have been advanced to search for gravitational waves in ultraprecise two-way Doppler data. In such an experiment, the total measurement system includes the Deep Space Network Tracking System, the spacecraft, the intervening media, and the data processing system. Preliminary estimates of gravitational wave characteristics are used to define a baseline experiment, with a total measurement system fractional frequency fluctuation of 1×10^{-15}, *and a desirable experiment, with a total measurement system fractional frequency fluctuation of* 1×10^{-17}.

The experiment to detect gravitational waves in ultraprecise two-way Doppler data is described, as are the anticipated requirements for the Deep Space Network, the spacecraft, and the data processing system. The article concludes by describing the steps necessary to provide the capability to perform this experiment.

I. Introduction

One of the most exciting challenges facing gravitational theoreticians and experimenters in the remaining decades of the 20th century will be the attempted detection and measurement of "gravitational waves" as predicted by Einstein's General Theory of Relativity. Gravitational waves are spatial strains propagating at the speed of light which are (expected to be) generated by the violent collapse of stars, star clusters, galaxy cores, quasars, and seyfert galaxies into supermassive black holes (for instance, see Thorne and Braginsky, Ref. 1). Proposals have been advanced to use "ultraprecise" two-way Doppler data while tracking spacecraft at large distances (i.e., > 1 AU) to detect very-low-frequency ($\sim 10^{-3}$ Hz) gravitational waves (for instance, see Davies, Ref. 2, and Estabrook and Wahlquist, Ref. 3). Although such a scheme appears to be one of the more promising suggested, substantial improvements in current Deep Space Network (DSN) and spacecraft performance will have to be achieved, as well as provision of new capabilities, to bring to fruition the utilization of ultraprecise Doppler data in the search for gravitational waves. It is thus the purpose of this article to provide a brief description of the experiment to detect gravitational waves in ultraprecise Doppler data, and to establish anticipated requirements for the spacecraft, DSN, and subsequent data processing.

II. Definition of "Baseline" and "Desirable" Experiments

As already noted, gravitational waves are expected to be produced by the violent collapse of stellar bodies into supermassive black holes; these waves are expected to be evidenced as spatial strains propagating at the speed of light. Specifically, one expects (Refs. 1 and 3) that the waves will:

(1) Alter the distance between separated free masses.

(2) Produce a fractional frequency shift in ultraprecise Doppler data, on the order of the wave amplitude.

Thorne and Braginsky (Ref. 1) have estimated possible gravitational wave characteristics; these are presented in Fig. 1. As can be seen, the wave duration (τ) is proportional to the wave amplitude (h) and the expected ranges of durations and amplitudes[1] are:

$$40 \text{ seconds} < \tau < 40{,}000 \text{ seconds}$$

$$10^{-17} < h < 10^{-14}$$

In Section III it will be shown that the scheme to utilize ultraprecise Doppler data requires:

$$\tau < \text{Round-Trip-Light-Time (RTLT)}$$

Since the next decade of funded and proposed deep space missions have maximum distances of either Jupiter (Galileo Mission, Solar Polar Mission) or Saturn (Saturn Orbiter Mission), it is immediately apparent from Fig. 1 that one is constrained to search for waves in which the amplitude is $\sim 10^{-15}$ or less. It is also obvious that one would like to search for waves with amplitudes as low as 10^{-17} ($\tau = 40$ seconds). This then motivates the selection of a "baseline" experiment to look for the strongest expected waves, and a "desirable" experiment to search for a wide variety of waves. Using this rationale, and noting that there are stringent requirements throughout the entire measurement system, consisting of

(1) DSN Tracking System

(2) Intervening media

(3) Spacecraft

(4) Data processing

one proceeds to define the baseline experiment as follows:

(1) Search for waves with amplitude $h \gtrsim 10^{-15}$.

(2) Require total measurement system fractional frequency fluctuation ($\sigma(\Delta F/F)$) of 10^{-15} or less.[2]

(3) Require each identifiable independent element of the total measurement system to have a fractional frequency fluctuation of 3×10^{-16} or less.[3]

(4) Require averaging times (τ_a) between 50 and 5000 seconds.

Similarly, one defines a desirable experiment as follows:

(1) Search for waves with amplitudes $h \gtrsim 10^{-17}$.

(2) Require total measurement system fractional frequency fluctuation ($\sigma(\Delta F/F)$) of 10^{-17} or less.

(3) Require each identifiable independent element of the total measurement system to have a fractional frequency fluctuation of 3×10^{-18} or less.

(4) Require averaging times (τ_a) between 5 and 5000 seconds.

While it is considered technologically possible to achieve a baseline system in the 1980s, the desirable system may not be achievable until the 1990s, if then.

While fractional frequency fluctuation is most appropriate for the basic description of ultraprecise two-way Doppler (frequency) data, certain elements of the measurement system are more conveniently thought of in terms of phase fluctuation ($\sigma(\Delta\phi)$) or path length fluctuation ($\sigma(\Delta L)$). For the remainder of the article, the following equivalents will be assumed:

$$\sigma(\Delta\phi) = \sigma(\Delta F/F) \cdot d\phi/dt \cdot \tau_a$$

$$\sigma(\Delta L) = \sigma(\Delta F/F) \cdot c \cdot \tau_a$$

[1] It should be emphasized that these are *highly tentative* estimates.

[2] The symbol σ will be used to represent a "generalized" measure of fluctuation which ultimately may be determined to most appropriately be the Allan variance, RMS, etc.

[3] Since there are numerous independent elements, it is assumed appropriate to require each independent element to be specified at a level one-half order of magnitude *below* the total measurement system requirement.

where

ϕ = phase

F = frequency

t = time

τ_a = averaging time

L = length

c = speed of light

Some typical conversions are

(1) $\sigma(\Delta F/F) = 1 \times 10^{-15}$

τ_a	100 seconds	1000 seconds
$\sigma(\Delta\phi)$, X-band	0.3 deg	3.0 deg
$\sigma(\Delta L)$	0.03 mm	0.3 mm

(2) $\sigma(\Delta F/F) = 1 \times 10^{-17}$

τ_a	100 seconds	1000 seconds
$\sigma(\Delta\phi)$, X-band	0.003 deg	0.03 deg
$\sigma(\Delta L)$	0.0003 mm	0.003 mm

III. Experiment Description

The special feature which allows the usage of ultraprecise Doppler data for the possible detection of gravitational waves is a unique "three-pulse" signature which is a function of the spacecraft, Earth, and gravitational wave propagation direction geometry (Ref. 3). The pulses (fractional frequency shifts) result from effects which are conveniently described as follows:

(1) "Clock speed-up" (Earth only effect)

(2) "Buffeting" (equal Earth and spacecraft effect)

Consider a case where θ defines the angle between the gravitational wave propagation direction and the Earth-spacecraft line, as seen in Fig. 2. Clock speed-up is evidenced as pulses of maximum amplitude $-h$ and $+h$ at the time the wave impinges on Earth and a RTLT later, respectively. This is shown schematically in Fig. 3. Similarly, Earth buffeting is seen as pulses of maximum amplitude $+h \cos\theta$ when the wave impinges on Earth and a RTLT later. In addition, spacecraft buffeting produces a pulse of $-2h\cos\theta$ in the Doppler data a one-way-light-time (OWLT) after the wave strikes the spacecraft. Earth and spacecraft buffeting effects are schematically illustrated in Fig. 4. Finally, combining the clock speed-up and Earth and spacecraft buffeting effects, one obtains the unique three-pulse signature which is expected to be seen in ultraprecise two-way Doppler data. The characteristics of this signature are as follows:

(1) Pulse amplitudes

(a) $h(\cos\theta - 1)$

(b) $h(-2\cos\theta)$

(c) $h(\cos\theta + 1)$

(2) Pulse separation times (r = Earth-spacecraft distance)

(a) $\frac{r}{c}(1 + \cos\theta)$

(b) $\frac{r}{c}(1 - \cos\theta)$

Figure 5 schematically demonstrates the combination of clock speed-up and buffeting effects for θ = 60 degrees.

Finally, it is noted that the above exercise is for a single spacecraft measurement. Should dual spacecraft measurements be made, one could obtain the two-dimensional propagation direction of the gravitational wave – certainly a most valuable bit of additional information.

IV. Anticipated Gravitational Wave Detection Experiment Requirements

The major categories identified as constituents of the gravitational wave detection measurement system are:

(1) DSN Tracking System

(2) Intervening media

(3) Spacecraft

(4) Data processing

In general, requirements are necessary for multiple independent elements in each category. As already noted, each independently identifiable element will be "specified" at a performance level 1/2 order of magnitude better than the entire system requirement, under the assumption that they will combine in a fashion similar to root-sum-square. Each major category is discussed below.

A. DSN Tracking System

The central element of the DSN Tracking System is obviously the frequency standard of the DSS Frequency and Timing Subsystem. The performance of the new hydrogen masers currently being implemented is considered to be

$$\sigma(\Delta F/F) \approx 3 \times 10^{-15}$$

$$\tau_a > 100 \text{ seconds}$$

so that a decrease of about an order of magnitude will be required to achieve the baseline experiment defined in Section II:

$$\sigma(\Delta F/F) = 3 \times 10^{-16}$$

$$50 \text{ seconds} < \tau_a < 5000 \text{ seconds}$$

Numerous other DSN Tracking System elements will have to be evaluated as to their current performance, and the technology developed to obtain the desired frequency stability. Below are listed the various pertinent subsystems, with the appropriate performance parameter indicated in parentheses:

(1) DSS Frequency and Timing Subsystem (Frequency)
- (a) Hydrogen maser (or other frequency standard)
- (b) Frequency distribution

(2) DSS Receiver-Exciter Subsystem (Phase)
- (a) Closed-loop receiver
- (b) Doppler extractor
- (c) Exciter

(3) DSS Tracking Subsystem (Phase)

Metric Data Assembly

(4) Antenna Mechanical Subsystem (Length)

(5) Antenna Microwave Subsystem (Phase)

(6) Transmitter Subsystem (Phase)

(7) System Cabling (Frequency)

Table 1 summarizes the required performance for the above elements for both the baseline and desirable experiment cases.

B. Intervening Media

In passing between the Deep Space Station (DSS) and spacecraft, the signal passes through the following interactive media:

(1) Solar Wind (dispersive)

(2) Troposphere (neutral)

(3) Ionosphere (dispersive)

In the discussion to follow, the ionosphere, consisting of charged particles, will be considered jointly with the similar, but generally much larger effect of the Solar Wind.

(1) Solar Wind. The Solar Wind effect on the gravitational wave detection experiment has been explored in detail in a previous article (Ref. 4). The results from Ref. 4 are summarized as follows (for τ_a = 1000 seconds):

- (a) Current S-band (both uplink and downlink) Solar Wind limitation:

$$\sigma(\Delta F/F) = 3 \times 10^{-13}$$

- (b) Expected optimum X-band (uplink and downlink) Solar Wind limitation:

$$\sigma(\Delta F/F) = 1 \times 10^{-14}$$

It is a major conclusion of Ref. 4 that to achieve a Solar Wind fractional frequency fluctuation of $\sigma(\Delta F/F) = 3 \times 10^{-16}$, one would require simultaneous S- and X-band uplink and downlink frequency band capability. To achieve the desirable experiment level (3×10^{-18}), it is speculated that the spacecraft will require sophisticated new capabilities, such as an onboard frequency standard and a Doppler counter.

(2) "Wet" Troposphere. An extremely preliminary estimate (Ref. 5) for the wet troposphere at an elevation of 30 degrees (20 cm total signal retardation assumed) and an averaging time τ_a = 1000 seconds is:

$$\sigma(\Delta L/L) \approx 1 \times 10^{-14}$$

It is considered that to meet the baseline experiment requirements, water vapor radiometer calibration of the wet component of the troposphere will be required. For the desirable experiment level, new spacecraft capabilities are envisioned.

(3) "Dry" Troposphere. Again, an extremely preliminary estimate (Ref. 5) for the dry troposphere at an elevation of 30 degrees (400 cm total signal retardation assumed) and an averaging time τ_a = 1000 seconds is:

$$\sigma(\Delta L/L) \approx 7 \times 10^{-15}$$

It is considered that to meet the baseline experiment requirements, extremely accurate surface barometric pressure calibration of the dry component of the troposphere will be required.

Media calibration capabilities required for the baseline and desirable experiment are summarized in Table 2.

C. Spacecraft

Similarly to the DSN Tracking System, individual critical path elements within the Spacecraft Radio Subsystem, such as the receiver, transponder, transmitter, need to be specified at $\sigma(\Delta F/F)$ levels of 3×10^{-16} and 3×10^{-18} for the baseline and desirable experiments, respectively.

D. Data Processing

Data processing numerical accuracies will need to be commensurate with the $\sigma(\Delta F/F)$ levels of 3×10^{-16} and 3×10^{-18} for the baseline and desirable experiments, respectively. Translated into units of cycles, the appropriate requirements for data processing numerical accuracies are as follows:

(1) Baseline Experiment

(a) S-band – 3×10^{-5} cycle · $(\tau_a/50)$

(b) X-band – 1×10^{-4} cycle · $(\tau_a/50)$

(2) Desirable Experiment

(a) S-band – 3×10^{-8} cycle · $(\tau_a/5)$

(b) X-band – 1×10^{-7} cycle · $(\tau_a/5)$

V. Discussion and Summary

In order to progress towards a gravitational wave detection capability in regard to the proposed usage of ultraprecise Doppler data, a series of actions will need to be undertaken, as follows:

(1) Development of gravitational wave detection experiment "drivers," such as:

(a) $\sigma(\Delta F/F) = 3 \times 10^{-16}$ frequency standard

(b) X-band uplink capability

(2) Identification of all critical path elements in the Spacecraft and DSN Tracking System, including evaluation of current frequency stability performance and development of the appropriate technology to achieve a fractional frequency fluctuation of 3×10^{-16}.

(3) Additional study of the fractional length fluctuation of the troposphere, and development of the appropriate tools necessary to calibrate the troposphere to $\sigma(\Delta L/L) = 3 \times 10^{-16}$.

Prior to obtaining a total measurement system of $\sigma(\Delta F/F) = 1 \times 10^{-15}$, it is suggested that "demonstrations" in the early 1980s might be possible at about the $\sigma(\Delta F/F) \approx 1 \times 10^{-14}$ level. Significant features of such a demonstration would be:

(1) $\sigma(\Delta F/F) \approx 3 \times 10^{-15}$ frequency standard

(2) X-band uplink and downlink

Possible missions for such a demonstration would be the Solar Polar and the Galileo missions.

References

1. Thorne, K. S., and Braginsky, V. B., "Gravitational-Wave Bursts from Nuclei of Distant Galaxies and Quasars: Proposal for Detection Using Doppler Tracking of Interplanetary Spacecraft," *The Astrophysical Journal*, Volume 204: L1-L6, February 15, 1976.

2. Davies, R. W., "Issues in Gravitational Wave Detection With Space Missions," *Transactions of the International Conference on Gravitational Waves and Radiations,* Paris, 1973.

3. Estabrook, F. B., and Wahlquist, H. D., "Response of Doppler Spacecraft Tracking to Gravitational Radiation," *General Relativity and Gravitation*, Volume 6, No. 5, 1975.

4. Berman, A. L., "Solar Wind Density Fluctuation and The Experiment to Detect Gravitational Waves in Ultraprecise Doppler Data," in *The Deep Space Network Progress Report 42-44*, Jet Propulsion Laboratory, Pasadena, California, 15 April 1978.

5. Private communication, S. D. Slobin.

Table 1. Required performance summary

Element	Baseline Experiment Requirement	Desirable Experiment Requirement
Total system, $\sigma(\Delta F/F)$	1×10^{-15}	1×10^{-17}
DSN Tracking System, $\sigma(\Delta F/F)$	$< 1 \times 10^{-15}$	$< 1 \times 10^{-17}$
Averaging time, seconds	$50 < \tau_a < 5000$	$5 < \tau_a < 5000$
Frequency and Timing Subsystem		
Frequency standard (H_2 maser), $\sigma(\Delta F/F)$	3×10^{-16}	3×10^{-18}
Frequency distribution, $\sigma(\Delta F/F)$	3×10^{-16}	3×10^{-18}
Receiver-Exciter Subsystem		
Closed-loop receiver, $\sigma(\Delta\phi)$, X-band	$0.05° \cdot \{\tau_a/50\}$	$0.00005° \cdot \{\tau_a/5\}$
Doppler extractor, $\sigma(\Delta\phi)$, X-band	$0.05° \cdot \{\tau_a/50\}$	$0.00005° \cdot \{\tau_a/5\}$
Exciter, $\sigma(\Delta\phi)$, X-band	$0.05° \cdot \{\tau_a/50\}$	$0.00005° \cdot \{\tau_a/5\}$
Antenna Mechanical Subsystem, $\sigma(\Delta L)$	0.005 mm $\cdot \{\tau_a/50\}$	0.000005 mm $\cdot \{\tau_a/5\}$
Antenna Microwave Subsystem, $\sigma(\Delta\phi)$, X-band	$0.05° \cdot \{\tau_a/50\}$	$0.00005° \cdot \{\tau_a/5\}$
Transmitter Subsystem, $\sigma(\Delta\phi)$, X-band	$0.05° \cdot \{\tau_a/50\}$	$0.00005° \cdot \{\tau_a/5\}$
System cabling, $\sigma(\Delta F/F)$	3×10^{-16}	3×10^{-18}
Data processing software		
Numerical accuracy, X-band	1×10^{-4} cycle $\cdot \{\tau_a/50\}$	1×10^{-7} cycle $\cdot \{\tau_a/5\}$
Spacecraft		
Radio Subsystem, $\sigma(\Delta F/F)$	3×10^{-16}	3×10^{-18}

Table 2. Required capabilities summary

Media Effect	Baseline Experiment Requirement	Desirable Experiment Requirement
Solar Wind		
• DSN	Receive and transmit simultaneous S- and X-band	Multiple one-way and two-way Doppler links
• Spacecraft	Receive and transmit simultaneous S- and X-band	On-board frequency standard and Doppler counter; Multiple one-way and two-way Doppler links
Wet troposphere		
• DSN	Water vapor radiometer	Multiple one-way and two-way Doppler links
• Spacecraft		On-board frequency standard and Doppler counter; Multiple one-way and two-way Doppler links
Dry troposphere		
• DSN	Surface barometric pressure	Multiple one-way and two-way Doppler links
• Spacecraft		On-board frequency standard and Doppler counter; Multiple one-way and two-way Doppler links

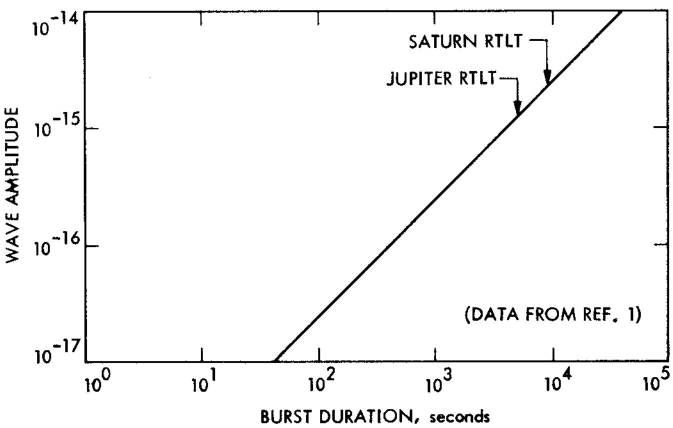

Fig. 1. Relationship between gravitational wave amplitude and burst duration

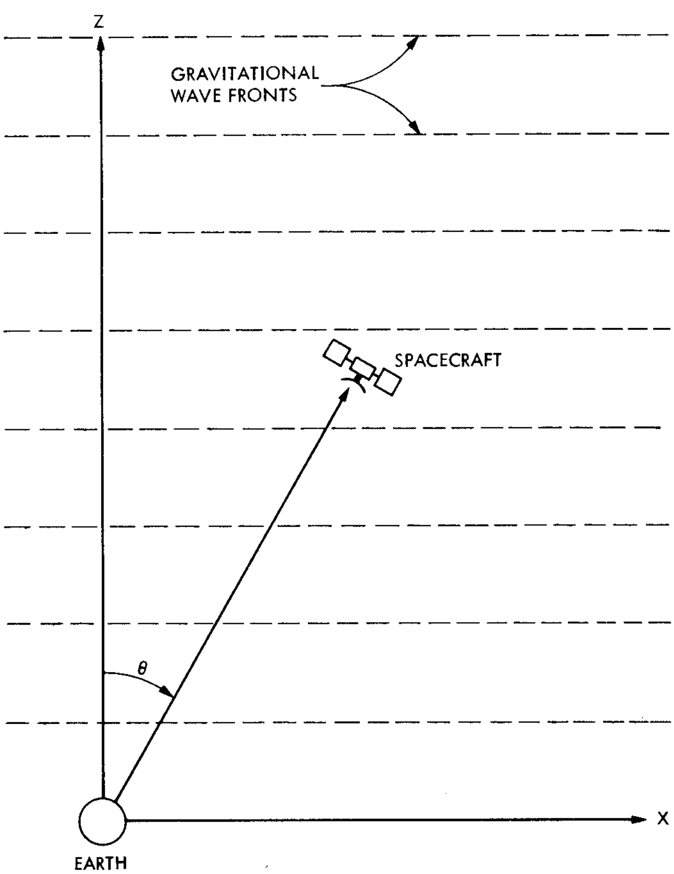

Fig. 2. Gravitational wave geometry

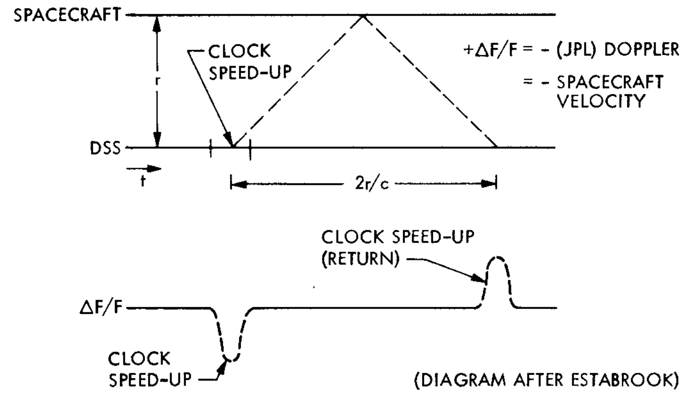

Fig. 3. Clock speed-up signature in ultraprecise Doppler data

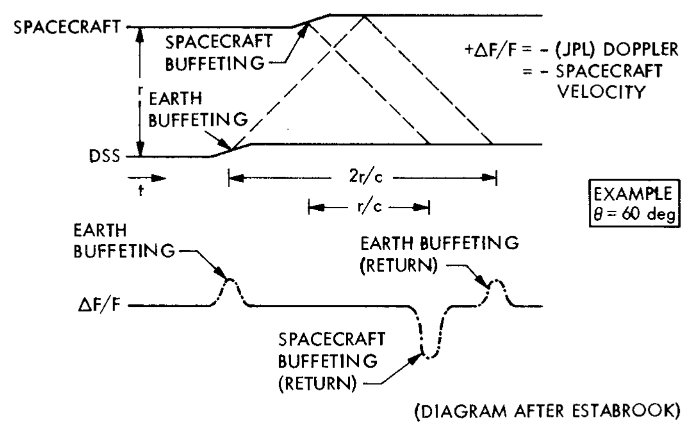

Fig. 4. Earth and spacecraft buffeting signature in ultraprecise Doppler data

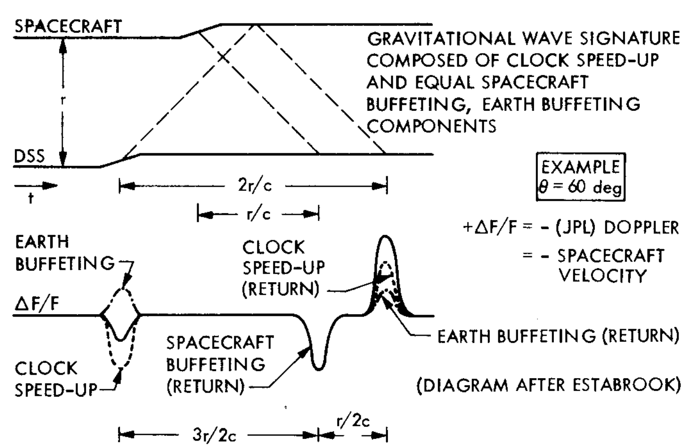

Fig. 5. Unique gravitational wave signature in ultraprecise Doppler data