Absolute Flux Density Calibrations: Receiver Saturation Effects

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The effect of receiver saturation is examined for a total power radiometer which uses an ambient load for calibration. Extension to other calibration schemes is indicated. The analysis shows that a monotonic receiver saturation characteristic could cause either positive or negative measurement errors, with polarity depending upon operating conditions. A realistic model of the receiver is made using a linear-cubic voltage transfer characteristic. The evaluation of measurement error for this model then provides a means for correcting radio source measurements. It also provides the means for assuring that this source of error is small in a particular situation.

I. Introduction

The Jet Propulsion Laboratory's Antenna Gain Calibration Program has demonstrated the feasibility of accurately determining the gain performance of large aperture antennas using radio metric measurements of natural radio sources (Ref. 1). One phase of this program examined systematic errors and their effects on the radio metric measurements. The measurement of low system operating temperature and ambient noise reference temperature demands operation of the radiometer receiving system over a fairly large dynamic range. This necessitated examination of receiver saturation. The saturation may be small, but it can have impact on radio source temperature measurements at the 1% error level.

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Consider a classical total power radiometer which goes through the sequence shown in Table 1. The parameters shown are defined as

T_{op} = System operating noise temperature (cold sky).

T_s = Increase in system operating noise temperature due to radio source.

T_{AMB} = System operating noise temperature when terminated in an ambient load.

G = System power gain through the receiver. This value is assumed to only be known nominally prior to use of T_{AMB} as a reference.

P_1, P_2, P_3 = Output powers of an ideal linear receiver as indicated by an ideal detector.

P'_1, P'_2, P'_3 = Observed output powers of a real receiver as indicated by an ideal detector.

First, consider an ideal linear receiver. Step 1 (Table 1) gives us GT_{op} when the antenna is pointed slightly away from the source. Step 2 gives us $G(T_{op} + T_s)$ when on source. Step 3 gives us a calibration value of GT_{AMB}. The value of T_{AMB} is known from physical temperature measurement of the load and other RF hardware calculations. Using a prime to denote estimated values, the estimated gain factor is

$$G' = \frac{P_3}{T_{AMB}} \tag{1}$$

However, since we are starting by assuming a linear receiver, we have $P_3 = GT_{AMB}$ and thus $G' = G$. Then the estimated value of T_s is measured in the classical manner as

$$T'_s = \frac{P_2 - P_1}{G} \tag{2}$$

and we clearly obtain $T'_s = T_s$.

II. Real Receiver Analysis

Let the real receiver be described by the compression factors C_1, C_2 and C_3 which have values between 0 and 1. These represent the reduction of receiver output due to saturation so that (see Table 1)

$$P'_1 = C_1P_1 < P_1 \tag{3}$$

$$P'_2 = C_2P_2 < P_2 \tag{4}$$

$$P'_3 = C_3P_3 < P_3 \tag{5}$$

We assume that the saturation is not in the detector, but in the intermediate frequency (IF) amplifier prior to the detector. To insure that this is the case, most accurate radiometers use a very accurate adjustable calibrated IF attenuator prior to the detector. The detector is used simply as an indicator and the IF attenuator is adjusted so the indicator is returned to a standard reading. This technique eliminates nonlinearity error in the detector because the detector always operates at the same level for each measurement step. The data is then the difference of the attenuation readings from the attenuator dial, for Steps 1, 2 and 3 in Table 1.

From (1) with P'_3 substituted for P_3 the measurement of gain for the real receiver is

$$G' = \frac{P'_3}{T_{AMB}} = C_3G \tag{6}$$

Then from (2) and (6), with $P_2 \rightarrow P'_2$, $P_1 \rightarrow P'_1$ and $G \rightarrow G'$, we have the measurement of source temperature as

$$T'_s = \frac{P'_2 - P'_1}{G'} = \frac{C_2}{C_3} T_s - \frac{(C_1 - C_2)}{C_3} T_{op} \tag{7}$$

The error in measurement is

$$\Delta T = T'_s - T_s = \left(\frac{C_2}{C_3} - 1\right) T_s - \frac{(C_1 - C_2)}{C_3} T_{op} \tag{8}$$

Observe that C_1, C_2 and C_3 are specific values of the function $C(GT_{IN})$ where T_{IN} is total system temperature, i.e.,

$$\begin{aligned} C_1 &= C(GT_{op}) \\ C_2 &= C[G(T_{op} + T_s)] \\ C_3 &= C(GT_{AMB}) \end{aligned} \tag{9}$$

We now define a *monotonic compression characteristic* for a receiver as one for which $C(x)$ is monotonically decreasing with x. Thus, if for example

$$T_{op} \leqslant T_{op} + T_s \leqslant T_{AMB} \tag{10}$$

then if a receiver had monotonic compression, it would have

$$1 \geqslant C_1 \geqslant C_2 \geqslant C_3 > 0 \tag{11}$$

i.e., C_1 has less compression and is closer to unity.

The first conclusion will now be made. The error in (8) can be positive or negative. To show this consider two special cases, satisfying (10) and (11).

(1) Let $T_{op} + T_s = T_{AMB}$. Then $C_2 = C_3$ and from (8) ΔT is *negative* since $C_1 > C_2$.

(2) Assume compression occurs only for T_{AMB}. Then $1 = C_1 = C_2 > C_3 > 0$ and from (8) ΔT is *positive.*

For any radiometer we will usually have $T_{op} < T_{AMB}$. Assuming T_{op} is less than T_{AMB}, two operating conditions exist. One condition is in (10). It and the other one possible are tabulated in Table 2. The error of the first was shown to possibly be positive or negative. For the second in Table 2, the error is always negative, under the monotonic assumption in (11). The breakpoint between positive and negative errors is difficult to find by simply setting (8) equal to zero, since we have not yet developed an analytic function for (9). The main result of this section is in (8). This is useful when the compression factors are known because we can then calculate the measurement error ΔT.

III. Modeling of Compression

The result above is useful only when C_1, C_2 and C_3 are each known. This is usually not the case. Usually a radiometer user knows only that he is operating some amount (backoff) below a reference compression point, e.g., the −0.1 dB point. In this section we consider a linear-cubic model and can calculate the measurement error when given only the backoff value.

The voltage input-output relationship of an amplifier is often modeled with the linear-cubic model[2]:

$$v_{out} = v_{in} - v_{in}^3 \tag{12}$$

This model is widely used for intermodulation product calculation, and seems appropriate here as well. We assume a bandpass filter to follow the amplifier prior to the detector. Thus, second and fourth order terms could be included in (12) but their output would be rejected by the filter and have no effect. The gain is unity for small v_{in}. Saturation increases for larger v_{in}. We assume that we will operate far below the point at which (12) has its maximum value and starts decreasing for very large v_{in}.

[2]A more general form for a real amplifier which incorporates a gain parameter α and a saturation parameter β is

$$v_{out} = \alpha v_{in} - \beta v_{in}^3$$

With no loss of generality in what is to follow, α and β can be taken to be unity as in (12).

An amplifier is usually characterized for sine wave inputs and outputs. Let $v_{in} = A \cos \omega t$, where ω is the IF center frequency. Substitution of this in (12), and manipulation gives

$$v_{out} = \left[A - \left(\frac{3A^3}{4}\right)\right] \cos \omega t - \frac{A^3}{4} \cos 3\omega t \tag{13}$$

The second term is rejected by the bandpass filter so the envelope voltage amplitude of the output sine wave is

$$e = A - \frac{3A^3}{4} \tag{14}$$

We now choose a standard reference compression point, C_r. Commonly, this is expressed in decibels so C_r (in dB) $= 10 \log_{10} C_r$, e.g., −0.1 dB from $C_r = 0.97724$. Since C_r is a power ratio, $C_r^{1/2}$ then is the factor by which e in (14) is reduced below the amplitude A obtained with a linear receiver. We wish to find the input envelope $A = A_o$ where this compression occurs. Thus

$$A_o - \frac{3A_o^3}{4} = A_o C_r^{1/2} \tag{15}$$

and hence

$$A_o^2 = \frac{4}{3}(1 - C_r^{1/2}) \tag{16}$$

Since power comes from the envelope as $A_o^2/2$, we have the input power giving compression C_r for a sine wave is

$$P_s = \frac{2}{3}(1 - C_r^{1/2}) \tag{17}$$

We now examine compression for a noise input. For Gaussian noise the envelope A has the Rayleigh probability density

$$p(A) = \frac{A}{P_n} \exp\left(-A^2/2P_n\right), A \geqslant 0 \tag{18}$$

where P_n = average input power of the noise. The output envelope after the filter is in (14) so the output power (observed by the detector) is

$$P_o = \overline{e^2}/2 = \frac{1}{2}\int_0^\infty \left(A - \frac{3A^3}{4}\right)^2 p(A)\, dA$$

$$= P_n - 6P_n^2 + \frac{27}{2} p_n^3 \tag{19}$$

The compression factor for noise C_n, is implicitly defined as $P_o = C_n P_n$ to give

$$C_n = \frac{P_o}{P_n} = 1 - 6P_n + \frac{27}{2} p_n^2 \tag{20}$$

Now let the input noise power P_n be below the sine wave compression point P_s by the backoff $B < 1$:

$$P_n = BP_s \tag{21}$$

We then have, by (21), (20) and (17)

$$C_n = 1 - 4B(1 - C_r^{1/2}) + 6B^2(1 - C_r^{1/2})^2 \tag{22}$$

We have already presumed small saturation, so C_r is very close to unity. Thus $(1 - C_r^{1/2})$ is $<<1$ and since $B < 1$, the third term in (22) is second order in $B(1 - C_r^{1/2})$ and is negligible with respect to the second term. Thus

$$C_n \simeq 1 - 4B(1 - C_r^{1/2}) \tag{23}$$

This is the main result of this section.

An important question may now be answered. At some reference level the compression for a sine wave input is C_r. At what backoff from this reference level will a noise input have the same compression? We start with (23) and set $C_n = C_r = C$. Since $B(1 - C^{1/2}) << 1$, we use $(1 + x)^{1/2} \simeq 1 + x/2$ to get from (23)

$$C^{1/2} = 1 - 2B(1 - C^{1/2}) \tag{24}$$

Solving for B gives $B = 1/2$. Thus we conclude that a noise input must be backed off a factor of 2(= 3 dB) from a sinusoidal input to have the same compression. This is reasonable since the noise input has large excursions which are compressed more than a sinusoidal input.

IV. Application to Radiometer Error

In (8) we have the error in measurement of T_s in terms of the compression factors C_1, C_2 and C_3. In (23) we now have an expression for these factors under the assumption of a linear-cubic receiver model.

We first characterize the receiver for a sinusoidal input by establishing the reference level at which a compression of C_r occurs, say −0.1 dB. This reference level can be determined by use of a signal generator and precision attenuators. For use as a radiometer, with a noise input, this same compression occurs for a reference temperature T_r. This temperature represents an input noise power reduced by a backoff of 1/2 with respect to the reference sinusoid. Thus for total input temperature T_{in}, backoff is given by

$$B = \frac{T_{\text{in}}}{2T_r} \tag{25}$$

From (23) we thus have

$$C_n = 1 - 2\frac{T_{\text{in}}}{T_r}(1 - C_r^{1/2}) \tag{26}$$

This gives the compression vs input temperature T_{in} under the condition that a compression of C_r occurs at reference temperature T_r. Square-rooting (26) and using the argument prior to (24) easily shows that $C_n = C_r$ when $T_{\text{in}} = T_r$.

For simplicity, define

$$\delta = 2(1 - C_r^{1/2})/T_r \tag{27}$$

Then the compression factors needed for (8) are of the form in (9):

$$C_1 = 1 - \delta T_{\text{op}}$$

$$C_2 = 1 - \delta(T_{\text{op}} + T_s)$$

$$C_3 = 1 - \delta T_{\text{AMB}} \tag{28}$$

Substitution into (8) gives

$$\frac{\Delta T}{T_s} = \frac{2(1 - C_r^{1/2})(T_{AMB} - 2T_{op} - T_s)}{T_r[1 - 2(1 - C_r^{1/2})T_{AMB}/T_r]} \tag{29}$$

Since $(1 - C_r^{1/2})$ will be small, and assuming T_{AMB} is not much larger than T_r, we can drop the second term in the denominator bracket to give

$$\frac{\Delta T}{T_s} \simeq 2(1 - C_r^{1/2}) \frac{T_{AMB} - 2T_{op} - T_s}{T_r} \tag{30}$$

This is the main result of this section. In this form it can be used to calculate correction factors for measurements.

Several observations can be made:

(1) The fractional error $\Delta T/T_s$, linearly decreases as T_s increases.

(2) The error goes to zero when

$$T_s = T_{AMB} - 2T_{op} \tag{31}$$

(3) The error is positive or negative depending upon whether T_s is below or above the value in (31).

A special case is of considerable value. Let T_{op} and T_s be very small compared to T_{AMB}. Then

$$\frac{\Delta T}{T_s} = 2(1 - C_r^{1/2}) \frac{T_{AMB}}{T_r} \tag{32}$$

Further, as long as

$$T_s + 2T_{op} \leqslant T_{AMB} \tag{33}$$

the result in (30) is bounded by (32). Thus, we obtain the useful result which can be used to provide an upper bound on saturation effects (condition 33 must apply):

$$\frac{\Delta T}{T_s} \leqslant 2(1 - C_r^{1/2}) \frac{T_{AMB}}{T_r} \tag{34}$$

V. Example

To illustrate the above calculations, consider the following example parameters representative of a high performance radiometer using a maser:

$$T_{op} = 15 \text{ K}$$

$$T_s = 100 \text{ K}$$

$$T_{AMB} = 300 \text{ K} \tag{35}$$

Let the receiver be set so that a temperature $T_r = 400$ K gives a compression of −0.1 dB ($C_r = 0.97724$). The error is, from (30), at the 1% level:

$$\frac{\Delta T}{T_s} = 0.0097 \tag{36}$$

The maximum error for any T_s satisfying (33) with $T_s \leqslant 270$, is, from (34)

$$\frac{\Delta T}{T_s} \leqslant 0.0172$$

VI. System Constraint for Small Error

Let us now find the constraint on the receiver compression so that the system error will be small. Assume that (33) holds and let us use the bound in (34) so that our constraint will hold for any value of T_s in (33). Let us take a value of $\Delta T/T_s = E_{MAX}$ as the worst case error we can accept. Then for equality in (34) we have the tradeoff in T_{AMB}/T_r vs C_r:

$$E_{MAX} = 2(1 - C_r^{1/2}) \frac{T_{AMB}}{T_r} \tag{37}$$

Examples of this tradeoff are shown in Table 3 for a value of $E_{MAX} = 0.003$. This Table shows that −0.026 dB compression at a value of T_r which is 3 dB above T_{AMB} guarantees that $\Delta T/T_s$ is below 0.003. This is also true if there is −0.131 dB compression when T_r is 10 dB above T_{AMB}. Clearly there is an infinite number of pairs of values satisfying (34). The usefulness of (34) is that only one value of compression need be specified and this can be done for a nearly arbitrary value of T_r.

In many radiometers, the system calibration is done by use of a calibrated active noise source reference (gas discharge tube or noise diode) which adds a precise amount of noise to the system input. This is as opposed to the use of an ambient load for calibration as done above. With small changes, it is straightforward to apply the technique above to the case of an active reference. The key point is that Step 3 in Table 1 now has "off source–noise reference on" and temperature $T_{op} + T_C$, where T_C is the calibrated noise reference contribution. Also the gain estimate in (6) becomes

$$G' = \frac{P_3' - P_1'}{T_C}$$

The remainder of the analysis closely follows the above and is omitted for brevity.

VII. Conclusions

An analysis has been made of measurement error in a radiometer due to receiver saturation. The general result is in (8). A linear-cubic model of receiver saturation was then assumed and an explicit result was obtained in (30).

The measurement error is characterized by only one measurement of compression which can be made at nearly any reference temperature. The work here is useful for calculating correction factors, or at the very least, verifying that error is insignificant for a particular situation.

Reference

1. Freiley, A. J., P. D. Batelaan and D. A. Bathker, *Absolute Flux Density Calibrations of Radio Sources: 2.3 GHz*, Technical Memorandum 33-806, Jet Propulsion Laboratory, Pasadena, California, December 1, 1977 (available from authors upon request).

Table 1. Summary of measurement

Step	Description	Temperature	Output power of ideal receiver	Output power of real receiver
1	Off source (cold sky)	T_{op}	$P_1 = GT_{op}$	$P_1' = C_1 GT_{op}$
2	On source	$T_{op} + T_s$	$P_2 = G(T_{op} + T_s)$	$P_2' = C_2 G(T_{op} + T_s)$
3	Ambient load	T_{AMB}	$P_3 = GT_{AMB}$	$P_3' = C_3 GT_{AMB}$

Table 2. Error polarity

Condition	Error, ΔT
$T_{op} + T_s < T_{AMB}$	Positive/negative
$T_{op} + T_s > T_{AMB}$	Negative

Table 3. Saturation tradeoff

Case	T_{AMB}/T_r	Worst case compression
A	0.5 (−3.0 dB)	0.99401 (−0.026 dB)
B	0.1 (−10.0 dB)	0.97023 (−0.131 dB)