Definition of Antenna Microwave Time Delay for VLBI Clock Synchronization

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This article presents derivations and definitions of antenna time delays above the phase calibrator injection points for VLBI time synchronization work. As nearly as possible, the symbols and nomenclature are made to be identical or similar to those used in ranging work. These definitions will be helpful in assessing the accuracy requirements of antenna time delay values to be provided to the VLBI clock sync project by the Microwave Phase Center Calibration work unit.

I. Introduction

Very Long Baseline Interferometry (VLBI) methods are used primarily in the DSN for improving knowledge of station locations, but have other applications as well. A recent publication that gives an overview of the VLBI activity for the DSN is *DSN Progress Report 42-46*, published August 1978. This issue contains tutorial VLBI papers and an extensive VLBI bibliography.

One of the VLBI activities at JPL and an important VLBI application is the clock sync project. This project involves determining how much the master frequency clock at one station is offset from the master clock at another station in the DSN. In order to verify that the VLBI method can be used for clock sync, it is necessary to know the time delays of the antenna feed (and transmission media) above the phase calibrator injection point at each of the two stations involved in the baseline measurement.

The VLBI processing, which includes modeling and a fringe phase stop method (Ref. 1), results in a term that has a time varying component (geometric delay) and a time invariant or constant term. The time varying delay is normally attributed entirely to phase delay changes produced by geometric baseline separation of the two antennas tracking the VLBI source and the changes in antenna pointing angle with time. The constant term is usually attributed to unknown clock errors and unknown delays of the transmission media, antenna, and the microwave subsystem above the phase calibrator injection points. The basic assumption is made that these latter delays are constant with time and antenna pointing angles. The constant term is ignored in VLBI baseline solutions for station location work, but the delays of the individual terms comprising the constant term need to be known for VLBI clock sync work.

The analysis of VLBI clock sync is probably well understood by those involved in this work, but other than an excellent article by Thomas (Ref. 2), these analyses are generally available only in internal JPL documents or unpublished notes.

It is the purpose of this article to present a fundamental VLBI time delay analysis from a microwave engineer's viewpoint and show the similarity to ranging system analysis. It is also the purpose of this article to present definitions of antenna time delay above the phase calibrator injection point so that responsibilities of the Microwave Phase Center Calibration work unit can be clarified and assessed.

II. Definitions of Symbols

The following is a list of symbols used in this article. As much as possible, the symbols and terminology are like those of ranging (Ref. 3) and those used by Fanselow (Ref. 4). Most of the symbols are indicated on Fig. 1. The i subscript refers to any station (e.g., $i = 1, 2$). Figure 1 should be regarded primarily as a time delay diagram. An attempt is made to depict both time and space points on the same diagram. It is assumed that both antennas track the VLBI radio source so that the direction of propagation of the plane wave is along a line that is parallel to the antenna axis of dish symmetry at each station.

τ_{Si} = time delay of the VLBI signal measured from the reference plane wave to the phase calibrator injection point via the transmission media, the Cassegrainian antenna path, and the microwave subsystem path at station i.

τ_{FSi} = optical path length delay from the reference plane wave to the station i location. It is assumed here that the antenna is transparent and the plane wave signal propagates at free space velocity to the station location.

$\Delta\tau_{FSi}$ = additional time delay (in excess of free space) due to the effects of solar plasma, ionosphere, and the troposphere.

τ_{Di} = time delay of the plane wave in free space to travel from the aperture of the antenna (edge of dish) to the plane containing the second (moving) axis of antenna rotation. It is assumed here that the antenna is transparent. The same definition is used in ranging work.

τ_{Ci} = time delay from the aperture of the antenna to the receive horn phase center as measured along the convoluted antenna optics path of the Cassegrainian antenna. The same definition is used in ranging work.

$\tau_{4,i}$ = time delay from the receive horn phase center through the horn assembly to the phase calibrator injection point located somewhere in the microwave subsystem. This same definition holds for ranging work if "phase calibrator" in the above is replaced by "translator."

A_i = point defined by the intersection of the axis of dish symmetry with the plane wave front that arrives at the second (moving) axis of rotation.

B_i = point defined by the intersection of the axis of dish symmetry with the plane wave front that arrives at the station i location. This point is shown in Fig. 2 for various types of antennas in the DSN.

R_i = equivalent point or location of the phase calibrator injection point on the antenna axis of symmetry using the *free space propagation distances* to locate this point (see Fig. 1). Point R_i is not the actual physical location of the phase calibrator on the antenna but is a convenient equivalent point useful in a time delay diagram analysis.

b_i = nominal antenna axis offset. It is the perpendicular distance between the primary and secondary antenna axes (see Fig. 2). This is equivalent to the term b used by Moyer (Ref. 5) and h by Fanselow (Ref. 4). The term h is not used here because it is used to denote zero delay device height in ranging work (Ref. 3).

v_c = speed of light in consistent length and time units.

τ_{Li} = time delay from point A_i to point R_i as measured in free space along the antenna axis of symmetry. This time delay, expressed mathematically in Eq. (17) of this article, shows that if

$$\tau_{Ci} + \tau_{4,i} < \tau_{Di}$$

then τ_{Li} will be a negative number and point R_i would actually be located above point A_i in Fig. 1. This fact will not alter the general case analysis presented in this article.

τ_{Vi} = antenna pointing angle dependent time delay from point A_i to point B_i as measured in free space along the antenna axis of symmetry.

τ_{Ai} = effective antenna-microwave time delay above the phase calibrator injection point.

$\Delta\tau_{CLK}$ = clock sync offset; it is the master clock time at station 2 minus the master clock time at station 1.

τ'_{BWS} = time delay resulting from bandwidth synthesis processing (see Ref. 2) of the data for two stations and two microwave frequencies that are separated by a frequency interval referred to as the "spanned bandwidth."

τ_{Ui} = cable delay between the actual phase calibrator output and the injection point in the microwave subsystem.

τ_m = modeled geometric delay

III. Derivation of Time Delay Relationships

A. Station Time Delays

From the Fig. 1 time delay diagram, it can be seen that the following absolute time delay relationships hold. All symbols are defined in the previous list of definitions.

$$\tau_{S1} = \tau_{FS1} + \Delta\tau_{FS1} - (\tau_{V1} + \tau_{D1}) + \tau_{C1} + \tau_{4,1} \tag{1}$$

$$\tau_{D1} + \tau_{L1} = \tau_{C1} + \tau_{4,1} \tag{2}$$

so from manipulation of Eq. (2)

$$\tau_{L1} = (\tau_{C1} + \tau_{4,1}) - \tau_{D1} \tag{3}$$

and it should be pointed out that τ_{L1} is the negative of $\tau_{\ell 1}$, which is a similar term used by Fanselow (Ref. 4). Defining the effective antenna-microwave delay as

$$\tau_{A1} = \tau_{L1} - \tau_{V1} \tag{4}$$

then substitution of Eqs. (3) and (4) into Eq. (1) gives

$$\tau_{S1} = \tau_{FS1} + \Delta\tau_{FS1} + \tau_{A1} \tag{5}$$

In a similar manner, it can be seen from Fig. 1 that the following relationship holds for Station 2:

$$\tau_{S2} = \tau_{FS2} + \Delta\tau_{FS2} - (\tau_{V2} + \tau_{D2}) + \tau_{C2} + \tau_{4,2} \tag{6}$$

$$\tau_{D2} + \tau_{L2} = \tau_{C2} + \tau_{4,2} \tag{7}$$

Manipulation of Eq. (7) gives

$$\tau_{L2} = (\tau_{C2} + \tau_{4,2}) - \tau_{D2} \tag{8}$$

Substitution of

$$\tau_{A2} = \tau_{L2} - \tau_{V2} \tag{9}$$

into Eq. (6) gives

$$\tau_{S2} = \tau_{FS2} + \Delta\tau_{FS2} + \tau_{A2} \tag{10}$$

Also, from Fig. 1, it can be seen that the following time delay relationship holds:

$$\tau_{FS2} = \tau_{FS1} + \tau_g \tag{11}$$

It is important to point out that all terms in Eq. (11) are free space or optical path delays and do not require corrections for additional delays occurring in the plasma, ionospheric or tropospheric media. Substitution of Eq. (11) into (10) gives

$$\tau_{S2} = \tau_{FS1} + \tau_g + \Delta\tau_{FS2} + \tau_{A2} \tag{12}$$

B. VLBI Observed Delay

Following the derivation and analysis of Thomas (see Ref. 2, Eq. 65), once the interferometer phase has been corrected with the phase calibrator tone phase, the bandwidth synthesis method will give the result:

$$\tau'_{BWS} = (\tau_{S2} - \tau_{S1}) + \Delta\tau_{CLK} - (\tau_{U2} - \tau_{U1}) - \tau_m \tag{13}$$

where $\Delta\tau_{CLK}$ is the synchronization clock offset of interest in this article and τ_{U1}, τ_{U2} are cable delays from the phase calibrator output to the injection point for Stations 1 and 2, respectively, and τ_m is the modeled geometric delay. Substitution of Eq. (5) and (12) into Eq. (13) gives the desired observed VLBI relationship of

$$\tau'_{BWS} = (\tau_g - \tau_m) + (\Delta\tau_{FS2} - \Delta\tau_{FS1}) + (\tau_{A2} - \tau_{A1}) - (\tau_{U2} - \tau_{U1}) + \Delta\tau_{CLK} \tag{14}$$

In the above equation, τ_m is a quantity that is known exactly. It is the modeled or predicted time varying delay based on assumed station locations. If the modeling is exact and all other terms in Eq. (14) are time invariant, then $\tau_g = \tau_m$ and τ'_{BWS} is also time invariant and one can assume that the modeling is correct as far as station location (geodetic) solutions are concerned. Clock sync-work involves the determination of the term $\Delta\tau_{CLK}$. As can be seen from Eq. (14) for the special case of $\tau_g \simeq \tau_m$ then manipulation of (14) gives

$$\Delta\tau_{CLK} \approx \tau'_{BWS} - (\Delta\tau_{FS2} - \Delta\tau_{FS1}) - (\tau_{A2} - \tau_{A1}) + (\tau_{U2} - \tau_{U1}) \tag{15}$$

The parameters needed for clock sync work then are $\Delta\tau_{FS2}$, $\Delta\tau_{FS1}$, of which major portions will be calibrated by techniques developed by the Water Vapor Radiometer work unit; τ_{A2}, τ_{A1}, whose techniques for measurement are to be furnished jointly by the Antenna Mechanical work unit and the Microwave Phase Center Calibration work unit and finally τ_{U2}, τ_{U1}, whose technique for measurement is the responsibility of the Phase Calibrator work unit. These are divisions of responsibilities that were unofficially agreed upon in Ref. 6. Since there is no obvious way to make direct differential measurements of the above required quantities, it is necessary to make absolute measurements of the individual delays and then difference them.

IV. Discussion of Antenna and Microwave Delays

In the preceding section, the terms required for VLBI clock sync project have been defined. In summary, the portion of time delays concerned with the antenna structure and microwave subsystem for any station i is the delay τ_{Ai}, which has been expressed mathematically in the preceding section as:

$$\tau_{Ai} = \tau_{Li} - \tau_{Vi} \tag{16}$$

where in more familiar ranging system terminology (Ref. 3)

$$\tau_{Li} = (\tau_{Ci} + \tau_{4,i}) - \tau_{Di} \tag{17}$$

and from Fanselow (Ref. 4)

$$\tau_{Vi} = \frac{b_i}{v_c}\left[f_{1i} \times (\text{geodetic elevation}) + f_{2i} \times (\text{declination}) + f_{3i} \times (\text{geodetic latitude})\right] \tag{18}$$

and values of b_i, f_{1i}, f_{2i}, and f_{3i} are given in Table 1 for various stations in the DSN. The parameter v_c is the speed of light in consistent time and length units. It should be remembered that even though the symbols of τ are similar for ranging, they are not exactly the same because τ in VLBI is defined for a measurement using a spanned bandwidth of about 40 MHz, while for ranging the equivalent spanned bandwidth is about 1 MHz. It should also be pointed out that it has been assumed that the antenna delay terms in Eq. (17) are constants with time during the period of observation of a VLBI radio source. This is an erroneous assumption for DSN antennas because subreflector defocussing and antenna structural deformation due to gravity loading and multipath cause the quantity $(\tau_{Ci} - \tau_{Di})$ to change as a function of elevation angle and therefore also as a function of time. In a recent publication (Ref. 7), it was shown that computer methods can be used to compute the quantity $(\tau_{Ci} - \tau_{Di})$ for defocussed subreflector cases. It is the objective of the Microwave Phase Center Calibration work unit and Antenna Mechanical Calibration work unit to provide some data on the effects of multipath and structural deformations on the variations of $(\tau_{Ci} - \tau_{Di})$ as a function of elevation angle.

The equations presented in this article are applicable for the general case. Some interesting special case derivations, such as for dish-mounted phase calibrators, Z-corrections for VLBI, and changes of reference plane for antenna optic delays, are presented in Appendices A and B.

V. Summary and Conclusion

The time delay terms for microwave subsystem and antenna delays above the phase calibrator injection point have been defined. These definitions are helpful in forming a basis for assessing the quantities and accuracies that can be achieved for VLBI clock sync project.

Nominal time delay values for τ_{Ai} have been obtained for the current S-band configurations for DSS 14 and DSS 13. These quantities and measurement techniques will be reported in a subsequent DSN article.

References

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7. Cha, A. G., Rusch, W. V. T., and Otoshi, T. Y., "Microwave Delay Characteristics of Cassegrainian Antennas," *IEEE Trans. on Antennas and Propagation*, Vol. AP-26, No. 6, pp. 860-865, Nov., 1978.

Table 1. Algorithm for computation of τ_{Vi} (from Ref. 4)

$$\tau_{Vi} = \frac{b_i}{v_c} \cos \left[f_{1i} \times (\text{geodetic elevation}) + f_{2i} \times \text{declination} + f_{3i} \times (\text{geodetic latitude}) \right]$$

i	DSS	Nominal b, m	f_1	f_2	f_3
1	11	6.706	0.0	1.0	0.0
2	12	6.706[a]	0.0	1.0	0.0
3	13	0.9144	1.0	0.0	0.0
4	14	0.000	0.0	0.0	0.0
5	41	6.706	0.0	1.0	0.0
6	42	6.706	0.0	1.0	0.0
7	43	0.000	0.0	0.0	0.0
8	44	1.2192	(To be determined)		
9	51	6.706	0.0	1.0	0.0
10	61	6.706	0.0	1.0	0.0
11	62	6.706	0.0	1.0	0.0
12	63	0.000	0.0	0.0	0.0

[a] Still valid after recent 34-m-diameter update.

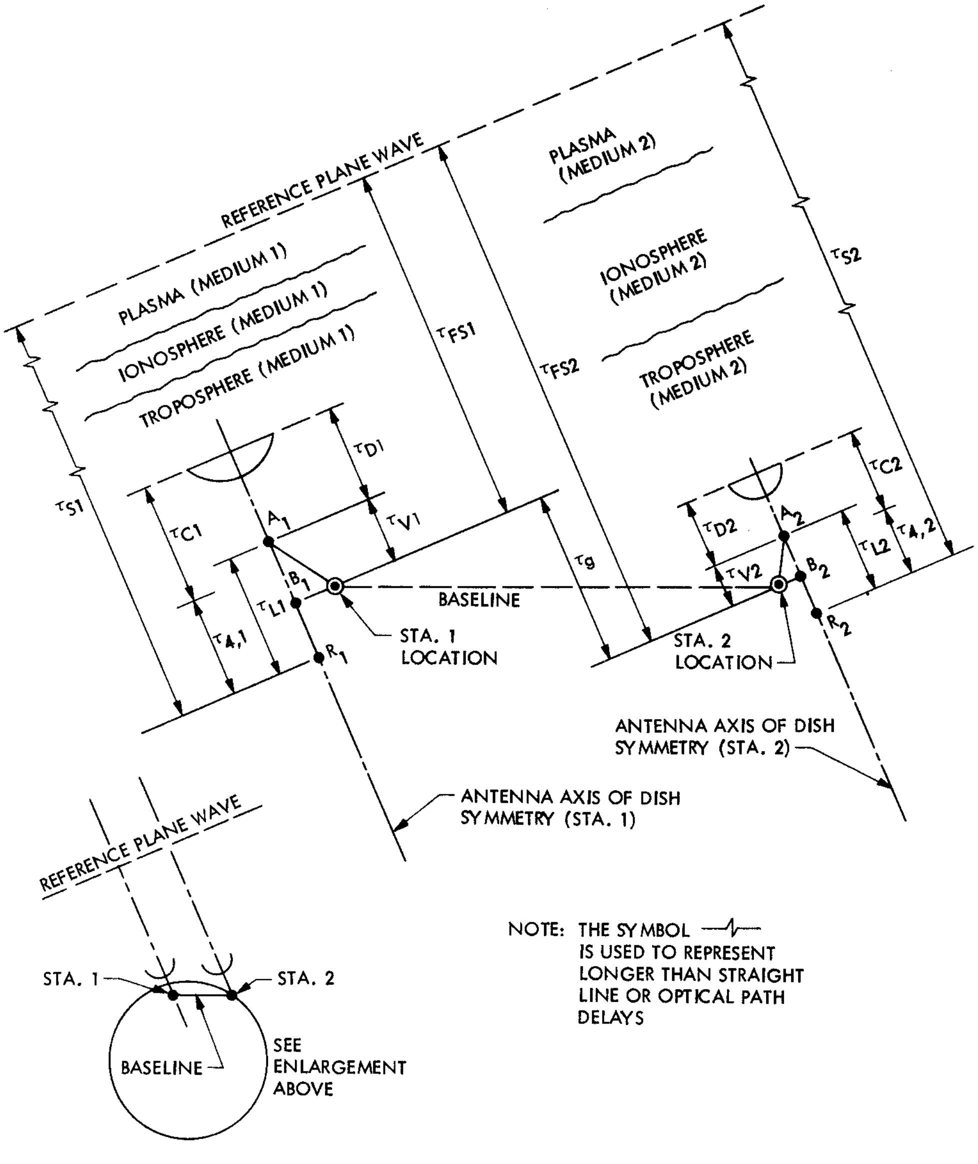


Fig. 1. Time delay diagram for two station VLBI measurement

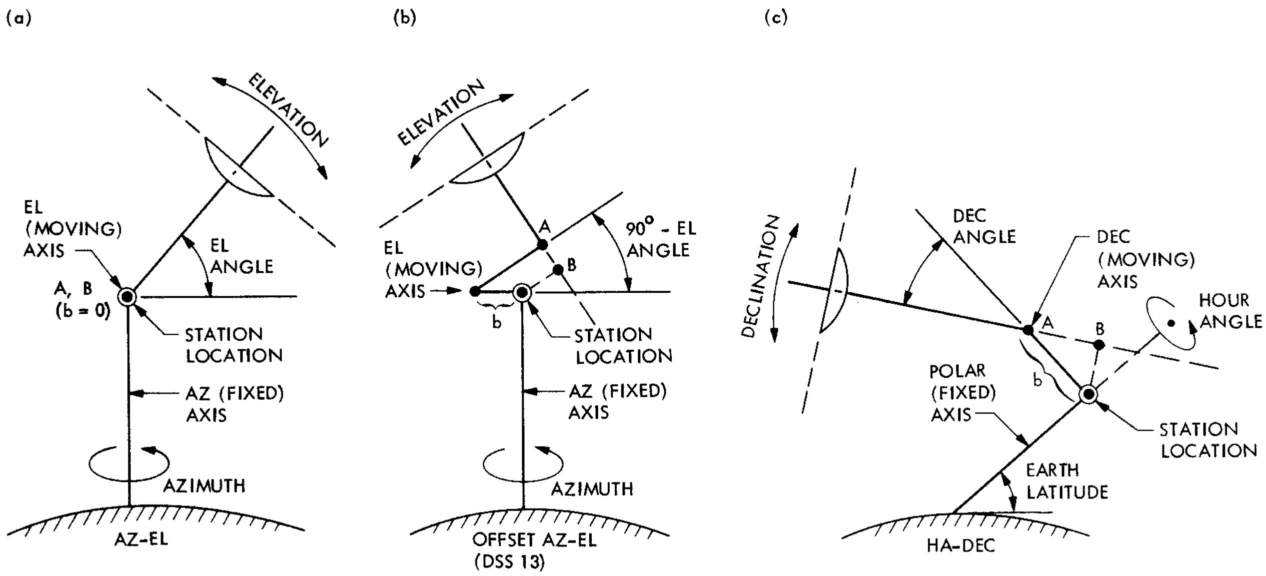


Fig. 2. Station locations and points A and B for various types of DSN antennas (a) AZ-EL, (b) offset AZ-EL, and (c) HA-DEC

Appendix A
Equivalent Z-Correction for VLBI

I. Dish-Mounted Phase Calibrator

Figure A-1 shows a dish-mounted phase calibrator similar to a dish-mounted Zero Delay Device (ZDD) described in Ref. 3. From the time delay diagram in Fig. A-1, it can be seen that

$$\tau_{Si} = \tau_{FSi} + \Delta\tau_{FSi} - (\tau_{Vi} + \tau_{hi}) \qquad \text{(A-1)}$$

and τ_{hi} here is the free space delay from the point R_i (projection of point where the phase calibrator is mounted on the dish surface) to point A_i on the antenna. Manipulation of Eq. (A-1) gives

$$\tau_{FSi} + \Delta\tau_{FSi} = \tau_{Si} + (\tau_{Vi} + \tau_{hi}) \qquad \text{(A-2)}$$

Comparison to the Round Trip Delay Time Equations in Ref. 3 shows that the equivalent Z-correction for VLBI for this dish-mounted case is

$$Z_{VLBI} = \tau_{Vi} + \tau_{hi} \qquad \text{(A-3)}$$

which is very similar to the ranging Z-correction for dish-mounted ZDD except it is only one-half the value. This is because VLBI is a downlink measurement only, while ranging is a round trip time delay measurement.

II. General Case

For the general case configuration shown in Fig. 1, where the phase calibrator is located internal to the antenna system, the time delay from the VLBI source to the phase calibrator injection point was derived as

$$\tau_{Si} = \tau_{FSi} + \Delta\tau_{FSi} + (\tau_{Li} - \tau_{Vi}) \qquad \text{(A-4)}$$

where in this general case

$$\tau_{Li} = (\tau_{Ci} + \tau_{4,i}) - \tau_{Di} \qquad \text{(A-5)}$$

Manipulation of Eq. (A-5) gives

$$\tau_{FSi} + \Delta\tau_{FSi} = \tau_{Si} + Z_{VLBI} \qquad \text{(A-6)}$$

where

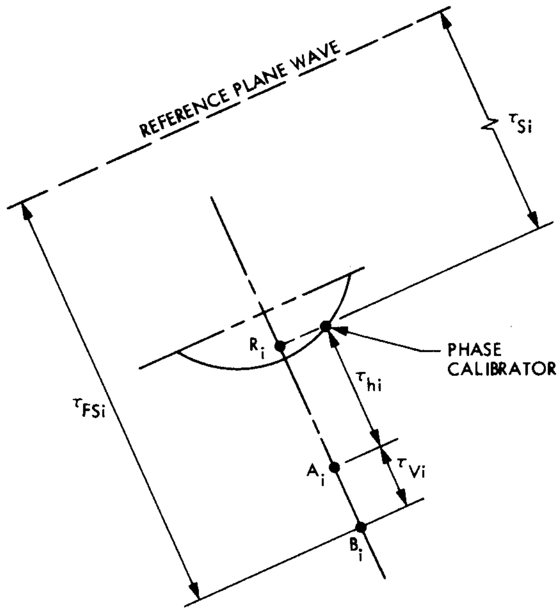
$$Z_{VLBI} = \tau_{Vi} - \tau_{Li} = -\tau_{Ai}$$

$$= \tau_{Vi} + \tau_{Di} - (\tau_{Ci} + \tau_{4,i}) \qquad \text{(A-7)}$$

and for 64-m antennas

$$\tau_{Vi} = 0$$

So we have exact agreement with the ranging Z-corrections if all uplink terms in the ranging Z-corrections were eliminated.



NOTE: POINT R_i IS THE PROJECTION OF THE PHASE CALIBRATOR ONTO THE ANTENNA AXIS OF SYMMETRY

Fig. A-1. Dish-mounted phase calibrator

Appendix B

Change of Reference for Antenna Optics Delay to an Arbitrary Plane in Front of the Dish Aperture Plane

For the derivations presented in the main part of this article, the common reference plane for the definitions of τ_C and τ_D was arbitrarily chosen to be the dish aperture plane (i.e., the plane that touches the edge of the dish). This reference plane could have been defined to be any plane in front of the antenna as shown in Fig. B-1 so that

$$\tau'_{Di} = \tau_{Di} + \tau_{Ki} \tag{B-1}$$

$$\tau'_{Ci} = \tau_{Ci} + \tau_{Ki} \tag{B-2}$$

where τ_{Ki} is the delay from the new reference plane to the dish aperture plane. The value of τ_{Ki} may or may not necessarily be the equivalent free space delay because of the effects of quadripod scattering, subreflector edge diffraction, and multipath effects. This is a problem area that is currently being investigated by this author under the Microwave Phase Center Calibration work unit. The term τ_{Ki} can be made large to correspond to the far field criteria $2D^2/\lambda$ or can be made much smaller to correspond to the distance to the paraboloid focal plane (a plane perpendicular to the antenna axis of symmetry passing through the paraboloid focus). The latter reference plane was used by Cha et al. (Ref. 7) in his definition of antenna delay. To derive the first component of antenna delay for the geometry of Fig. B-1, it is necessary only to use Eq. (B-1) and (B-2) in Eq. (17) to obtain

$$\tau_{Li} = (\tau'_{Ci} + \tau_{4,1}) - \tau'_{Di} \tag{B-3}$$

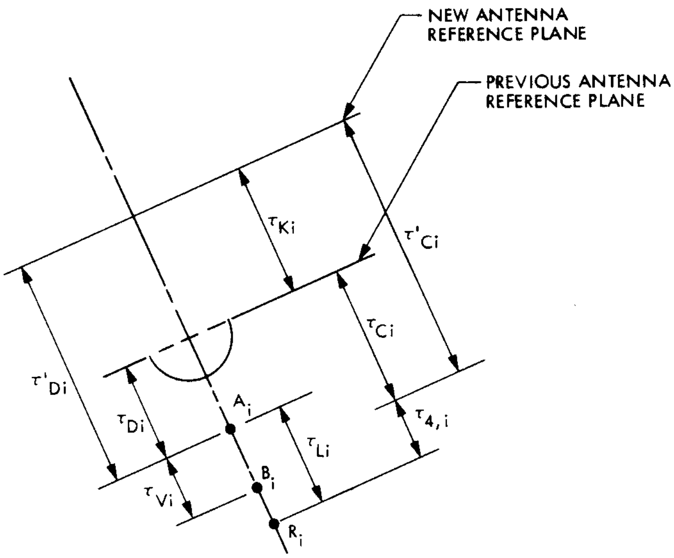


Fig. B-1. Geometry for change of reference plane for τ_{Ci} and τ_{Di}