A Reexamination of the Radial Dependence of Weak Interplanetary Scintillation

A. L. Berman

TDA Engineering Office

Recent investigations of weak interplanetary scintillation have found strong correlation between the scintillation index m and both in-situ and integrated electron density. A similar measure of solar wind columnar turbulence, doppler phase fluctuation, has been shown to be in excellent correspondence with the signal path integration of a specific, well-established mean electron density model – $N_e(r) \cong Kr^{-2.3}$, where N_e = electron density, K = constant and r = radial distance. It is thus natural to heuristically inquire whether m might not also correlate with the signal path integration of such an electron density model.

This article reexamines the power law radial dependence of m for weak interplanetary scintillation data published by various investigators during the last decade. The data are found to be consistent with the signal path integration of a power law electron density model of the form:

$$N_e(r) \cong Kr^{-(2+\xi)}; \quad 0.3 \lesssim \xi \lesssim 0.4$$

I. Introduction

Recently Erskine et al. (Ref. 1) have determined strong correlation between interplanetary scintillation (the scintillation index m) and near-earth, in-situ, solar wind electron density measurements. These findings confirm results obtained earlier by Chang (Ref. 2), wherein a linear relationship between integrated electron density and coherent signal intensity scintillation was determined via concurrent measurements of both using the Pioneer 9 spacecraft. Findings similar to Erskine were obtained for interplanetary scintillation by Houminer, et al. (Ref. 3).

The radial dependence of electron density in the extended corona (here defined to be $r \geqslant 5r_0$, where r = radial distance and r_0 = solar radius) has been reviewed by Berman (Ref. 4), and is found by a variety of techniques to be well represented in the mean by the power law model $Kr^{-2.3}$. In addition, Berman (Ref. 5) and Berman, et al. (Refs. 6 and 7) have found excellent agreement between extended corona doppler phase fluctuation data obtained from the Helios, Pioneer, and Viking spacecraft and a signal path integrated electron density model of the form $Kr^{-2.3}$. Given the findings of Erskine, Chang, Houminer, and Berman, it is natural to inquire whether previous estimates of the radial dependence of (weak) inter-

planetary scintillation are consistent with the signal path integration of a power law electron density model of radial index approximately -2.3.

This article reviews radial dependence measurements obtained from weak interplanetary scintillation data over the last decade, and relates these measurements to the appropriate signal path integration of a power law source function. The determined radial dependence of weak interplanetary scintillation data is found to be in reasonable agreement with other experimental measurements of electron density and columnar turbulence, under the assumption of proportionality between weak interplanetary scintillation and signal path integrated electron density.

II. Interplanetary Scintillation Measurements

Interplanetary scintillation refers to relatively high frequency ($\gtrsim 1$ Hz) fluctuations in received signal amplitude. The scintillation index is defined (Ref. 1) as:

$$m = \left(\int P(\nu) d\nu \right)^{1/2} = \Delta I / \langle I \rangle$$

where

m = scintillation index

P = power spectrum of signal intensity fluctuations

ν = fluctuation frequency

ΔI = rms intensity fluctuation

$\langle I \rangle$ = average source intensity

For natural (noncoherent) signal sources, the scintillation index correlates with columnar turbulence under conditions of low turbulence ("weak" scintillation). As columnar turbulence increases and the scintillation index approaches 1, saturation occurs, and further increases in columnar turbulence produce a sharp decrease in the scintillation index. Therefore, usage of the scintillation index in measuring the geometrical dependence of columnar turbulence is restricted to regions of weak scintillation (as determined by source frequency and elongation angle). There are no such restrictions with coherent (spacecraft) sources as saturation of the scintillation index does not occur.

Much of the analysis of interplanetary scintillation has indicated a total power law radial dependence of $m(a) \cong Ka^{-1.5}$, where K = constant and a = closest approach distance. Berman (Ref. 8) has shown that for correlation with

signal path integrated source functions, usage of a single geometrical parameter (e.g., a) can lead to significant distortions in modeling and interpretation. In the following section, previously obtained scintillation radial dependencies will be analyzed and mapped back to the signal path integration of an appropriate power law source function.

III. Scintillation Index Radial Dependence

Following the method of Ref. 8 under the assumption that the radial dependence of the scintillation index is proportional to the signal path integration of a $r^{-(2+\xi)}$ electron density model, one obtains

$$m(a,\beta) \cong K\beta a^{-(1+\xi)}$$

where

K = constant

β = Earth-Sun-source angle

a = closest approach distance

However, all previously published determinations of the radial dependence of the scintillation index have been made as a sole power law function of closest approach distance (i.e., "total" radial dependence as in Ref. 8):

$$m(a) \cong Ka^x$$

again following the method of Ref. 8, $m(a, \beta)$ is rewritten:

$$m(a,\beta) \cong Ka^{-(1+\xi)+\ln\beta/\ln a}$$

and approximated as

$$m(a) \approx Ka^{-(1+\xi)+\ln(\beta_1/\beta_2)/\ln(a_1/a_2)}$$

where the subscripts 1 and 2 define the applicable (radial) span of data. As was noted in Ref. 8:

(1) The term $\ln(\beta_1/\beta_2)/\ln(a_1/a_2)$ can only be considered as an approximate indication of the possible (power law index) distortion in using a single geometrical parameter.

(2) The distortion becomes most pronounced for data wherein β approaches $90°$.

Table 1 presents the results of analyzing five published determinations of the radial dependence of the scintillation

index. The average total radial dependence of these five cases is (reestimated as):

$$m(a) \cong a^{-1.50}$$

The average correction $\ln(\beta_1/\beta_2)/\ln(a_1/a_2)$ to account for variations of β is:

$$\ln(\beta_1/\beta_2)/\ln(a_1/a_2) \cong -0.16$$

so that in the aggregate these measurements would be consistent with the signal path integration of a mean electron density model of the form $Kr^{-2.34}$. Even should one allow that the distortion has been *overstated* by as large a factor as 2, the data would still be consistent with a mean electron density model of the form $Kr^{-2.42}$. It is therefore here concluded that the bulk of published interplanetary scintillation data *is* consistent with the signal path integration of an electron density model N_e in the extended corona of the form:

$$N_e(r) \cong Kr^{-(2+\xi)}; \quad 0.3 \leqslant \xi \leqslant 0.4$$

It is interesting to compare these results to a theoretical derivation of the expected radial dependence of the scintillation index given by Woo (Ref. 14) as follows:

$$m(a) \cong Ka^{0.5}\sigma_{n_e}(a)$$

where σ_{n_e} = rms electron density fluctuation. From Neugebauer (Ref. 15), one assumes:

$$\sigma_{n_e}(a) \propto N_e(a)$$

and from Ref. 4 one has:

$$N_e(a) \propto a^{-2.3}$$

so that one expects from the Woo expression:

$$m(a) \cong a^{0.5}a^{-2.3} = a^{-1.8}$$

or a value clearly inconsistent with the aggregate of published values as seen in Table 1. Looked at from the opposite point of view, the Woo expression and $m \propto a^{-1.5}$ would require (under the assumption that $\sigma_{n_e}(a) \propto N_e(a)$) an electron density radial dependence of $r^{-2.0}$. Such a radial index is less negative than any index reported in the thirteen electron density experiments documented in Table I of Ref. 4.

IV. Discussion and Summary

Previous work has demonstrated that electron density in the extended corona ($r \geqslant 5r_0$) is well represented by the radial function $Kr^{-2.3}$ and that doppler phase fluctuation data are in excellent agreement with the signal path integration of $Kr^{-2.3}$. In this article, it has been shown that previously published weak interplanetary scintillation data, which display a scintillation index *total* radial dependence of approximately $a^{-1.5}$, are consistent with the signal path integration of a power law electron density model of the form $Kr^{-2.3}$ to $Kr^{-2.4}$. These results, in combination with previous doppler phase fluctuation results, suggest that most manifestations of columnar solar wind turbulence are or will be found to be proportional to signal path integrated electron density.

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Table 1. Previously published weak interplanetary scintillation index data and indicated source function power law index

Source	Year	Original published total radial dependence	Recalculated total radial dependence from data[a]	Data limits (of closest approach distance), AU	$\dfrac{\ln(\beta_1/\beta_2)}{\ln(a_1/a_2)}$	Indicated source function power law index
Hewish & Symonds (Ref. 9)	1969	−1.6	−1.57	0.12−1.0	−0.31	−2.26
Readhead (Ref. 10)	1971	−1.55	Same	0.05−0.83	−0.13	−2.42
Hewish (Ref. 11)	1971	−1.50	−1.52	0.04−0.9	−0.14	−2.38
Rickett[b] (Ref. 12)	1973	−1.6	−1.43		−0.15	−2.28
(2695 MHz)			−1.48	0.042−0.39	−0.06	−2.42
(611 MHz)			−1.42	0.12−0.48	−0.10	−2.32
(178 MHz)			−1.38	0.32−0.83	−0.28	−2.10
Coles[c] (Ref. 13)	1974		−1.42		−0.08	−2.34
(2695 MHz)			−1.50	0.045−0.15	−0.03	−2.47
(611 MHz)			−1.34	0.15−0.55	−0.12	−2.22
Average		−1.56	−1.50		−0.16	−2.34

[a]When possible, the total radial index was remeasured from published data (the fit line) and used where appropriate.

[b]These data were the measured fit lines in Fig. 3 (from Ref. 12) and include the 2695-, 611-, and 178-MHz data; the 81.5-MHz data were not used because the measured fit line was approximately −2.0. When the four slopes are averaged, a value close to the Ricketts value −1.6 is achieved.

[c]These data were the measured fit lines in Fig. 1 (from Ref. 13) and include the 2695- and 611-MHz data.