A Unified Observational Theory for Solar Wind Columnar Turbulence

A. L. Berman
TDA Engineering Office

Solar wind columnar turbulence measurements are possibly the most important tool in probing the solar corona and solar wind, if for no other reason than their abundant availability. Fundamental to the extraction and utilization of the full informational content of such measurements is the confident understanding of the proper relationship between columnar turbulence and the most basic solar wind parameters – solar wind velocity and electron density.

This article reviews investigations of the primary forms of solar wind columnar turbulence, including Doppler phase fluctuation, spectral broadening, weak interplanetary scintillation, and electron density. Based on the totality of these measurements, a unified, self-consistent, observational hypothesis for solar wind columnar turbulence is proposed as follows:

(1) The dependence on radial distance r of solar wind mean electron density N_e is well represented by the power law model (with A and B fit coefficients):

$$N_e(r) \cong Ar^{-6} + Br^{-2.3}$$

(2) To first order, all primary measurements of solar wind columnar turbulence are well represented by the signal path integration of the electron density model from (1) above (R = signal path):

$$\text{Columnar turbulence} \propto \int N_e(r)\, dR$$

I. Introduction

Solar wind columnar turbulence measurements constitute perhaps the most important overall experimental tool in probing the solar corona and solar wind, if for no other reason than the sheer abundance of such measurements. As an example, the Deep Space Network (DSN) automatically computes and records Doppler phase fluctuation data whenever a spacecraft is being tracked for any purpose. The primary data types being considered here as columnar turbulence are spacecraft Doppler phase fluctuation, spacecraft carrier frequency spectral broadening, and signal intensity fluctuation ("interplanetary scintillation") of both coherent (spacecraft) and noncoherent (natural) sources. Interplanetary scintillation of natural sources is further restricted to the region of "weak" scintillation.

Careful measurement and modelling of solar wind columnar turbulence can be ascribed to at least three major considerations, as follows:

(1) The validation of the theoretical understanding of the mechanics of columnar turbulence generation.

(2) The removal (via calibration) of columnar turbulence, which to varying degrees obscures many other radio science experiments.

(3) The precise functional determination of more basic solar wind parameters, such as electron density and velocity, once the relationship between columnar turbulence and these parameters has become well understood.

This article reviews experimental measurements of electron density and the primary forms of solar wind columnar turbulence. Based on the totality of these observations, this article suggests a heuristic observational hypothesis for solar wind columnar turbulence, stated as follows:

(1) The dependence on radial distance r of solar wind mean electron density N_e is well represented by the power law model (with A and B fit coefficients):

$$N_e(r) \cong Ar^{-6} + Br^{-2.3}$$

(2) To first order, all primary measurements of solar wind columnar turbulence are well represented by the signal path integration of electron density (R = signal path):

$$\text{columnar turbulence} \propto \int N_e(r)\, dR$$

II. Electron Density

Electron density radial dependence in the inner corona (here to be defined as $r \leqslant 5r_o$, where r is radial distance and r_o is the solar radius) has been frequently investigated via white light (eclipse) photometry analysis and K coronameter measurements. Examples of white light photometry analysis are van de Hulst (Ref. 1), and Blackwell (in Ref. 2), while examples of K coronameter measurements are Saito (Ref. 3), Hansen, et al. (Ref. 4), and Saito, et al. (Ref. 5). In general, usage of these techniques has resulted in a consistent determination of inner corona electron density which is well represented by the power law radial function:

$$N_e(r) \cong Ar^{-6}$$

In the extended corona (here to be defined as $5r_o \leqslant r \leqslant$ 1AU), a greater variety of techniques have been utilized to determine the radial dependence of electron density. In addition to the techniques used for the inner corona, measurements in the extended corona have been made by radio interferometry (Muhleman, et al., Ref. 6), single frequency spacecraft range (Muhleman, et al., Refs. 7 and 8; Edenhofer, et al., Ref. 9), dual-frequency spacecraft range (Berman, et al., Ref. 10), pulsar time delay (Counselman, et al., Ref. 11, and Weisberg, et al., Ref. 12), and in situ spacecraft measurements (Ogilvie, et al., Ref. 13). The radial dependence (power law index) of electron density in the extended corona has been somewhat controversial, and estimates from various experiments have spanned the following rather large range of power law radial indices:

$$N_e(r) \cong Br^{-(2+\xi)}$$

$$0 \leqslant \xi \leqslant 0.5$$

In 1977, Berman (Ref. 14), reviewed the then available measurements of the radial dependence of electron density in the extended corona (Table 1 from Ref. 14). At that time the mean electron density power law exponent ($r^{-(2+\xi)}$) from the experiments listed in Table 1 of Ref. 14 was:

$$\xi = 0.298$$

Table 1 from Ref. 14 is updated here with the following additional new entries:

Source	Year	ξ	Type of Measurement
Ogilvie	1978	0.5	in situ density, Mariner 10
Saito	1978	0.14	K coronameter
Berman	1978	0.3	S minus X range, Viking

The average of the sixteen experiments in Table 1 (of this article) is:

$$\xi = 0.301$$

with a 1 standard deviation (1σ) of:

$$1\sigma = 0.13$$

Based on the contents of Table 1, it is concluded that:

$$N_e(r) \cong Ar^{-6} + Br^{-2.3}$$

continues as the best existing representation for mean electron density radial dependence in both the inner and extended corona. It should be stressed that confident knowledge of the average radial dependence of electron density is fundamental to the evaluation of turbulence models, since electron density is clearly the key parameter.

An additional important feature of the mean electron density model is the very sharp "break" at approximately $r \approx 4r_o$ between the Ar^{-6} and $Br^{-2.3}$ terms. Analysis of columnar turbulence measurements in the region surrounding $4r_o$ (signal closest approach distance) is most important in the process of validating columnar turbulence models.

III. Doppler Phase Fluctuation

In 1975, Berman and Rockwell (Ref. 17), analyzed two-way S-Band Doppler noise (or phase fluctuation, = ϕ) from the 1975 solar conjunctions of Pioneer 10, Pioneer 11, and Helios 1, in terms of a geometrical parameter = sin α/β, where α = Sun-Earth-probe angle and β = Earth-Sun-probe angle. A best fit to this parameter yielded:

$$\phi(\alpha,\beta) \approx K(\sin \alpha/\beta)^{-1.29}$$

Subsequently, Berman and Wackley (Ref. 18), showed that this same Doppler noise data provided a very good fit to the signal path integration of a nominal $Kr^{-2.3}$ electron density model, except in the region $a < 10r_o$, where a = signal closest approach distance. When a term Ar^{-6} was added, a good fit was achieved over the entire span of data.

In 1976, Berman and Wackley (Ref. 19), showed that the signal path integration of $Ar^{-6} + Br^{-2.3}$ provided a very good fit to the *1976* solar conjunction data of Helios 1 and 2, Pioneer 10 and Pioneer 11.

In 1976, Pioneer 11 underwent a solar conjunction in which the signal closest approach locus transversed very high (>80°) heliographic latitudes. Berman, et al. (Ref. 20), were able to show that Doppler noise dropped off sharply with increasing heliographic latitude, much as Saito (Ref. 3) and Counselman (Ref. 11) found for electron density.

In 1977, Berman, et al. (Ref. 15), analyzed a large volume (in excess of 800 pass-average data points) of two-way S-Band Doppler noise data accumulated during the Viking 1976 solar conjunction. A simultaneous, two parameter (coefficient and power law index) fit of these data to the signal path integrated electron density model $Br^{-(2+\xi)}$ was performed; a best fit was obtained for $\xi = 0.30$.

In 1978, Berman (Refs. 21 and 22) showed that the combined set of all inner corona Doppler noise measurements obtained from Helios 1, Helios 2, and Viking, exhibited a very sharp break at approximately $a \approx 3r_o$, exactly as does the signal path integration of the composite $Ar^{-6} + Br^{-2.3}$ electron density model.

Finally, in 1978, Berman, et al. (Ref. 10), directly compared concurrent measurements of Viking two-way S-Band Doppler noise and Viking dual frequency (S-Band minus X-Band) range measurements; correspondence of these two parameters in Ref. 10 is seen to be excellent. Because of the tremendous volume of Viking Doppler noise and dual frequency range data utilized, and the very wide radial data span ($2r_o$ - $160r_o$), it is here considered that this final comparison is incontrovertible evidence that to first order, Doppler phase fluctuation is well represented by the signal path integration of electron density.

IV. Spectral Broadening

In 1978, Rockwell (Ref. 23), analyzed spacecraft carrier frequency spectral broadening (SB) between 2 and 20 Solar radii. (The data were primarily Helios data and had been originally provided by R. Woo.) Rockwell was able to demonstrate a good fit of these spectral broadening data to a signal path integration of $Ar^{-6} + Br^{-2.3}$. However, when Rockwell performed a simultaneous, two parameter fit to the data, he obtained a best fit for $Ar^{-6} + Br^{-2.77}$, which is a significantly steeper radial dependence than usually determined for the extended corona term. However, the most important feature of these spectral broadening data is that they clearly exhibit the sharp break at $a \approx 3r_o$ (see, for instance, Berman, Ref. 22, Fig. 3), just as do the inner corona Doppler phase fluctuation data.

Based on the fact that spectral broadening data follow the sharp (integrated) density break in the transition from the inner to the extended corona, it is here concluded that spectral broadening data to first order are reasonably well represented by the signal path integration of electron density.

V. Interplanetary Scintillation

Recently, Berman (Ref. 24), has suggested that the bulk of previously published determinations of the radial (power law) dependence of weak interplanetary scintillation data (the scintillation index) are reasonably consistent with the signal path integration of a power law electron density model of the form:

$$N_e(r) \cong r^{-(2+\xi)}$$

$$0.3 \leqslant \xi \leqslant 0.4$$

In addition, Chang (Ref. 25), has demonstrated proportionality between interplanetary scintillation of a coherent source and integrated electron density via concurrent measurement of both using the Pioneer 9 spacecraft. Similar correlation between natural source weak interplanetary scintillation and in situ density measurements has been determined by Erskine, et al. (Ref. 26), and Houminer, et al. (Ref. 27). Based on the totality of these observations, it seems reasonable to infer that to first order, weak interplanetary scintillation is reasonably well represented by the signal path integration of electron density.

It is appropriate to note that this heuristic interpretation of weak interplanetary scintillation conflicts with the currently popular explanation for the observed radial dependence of the scintillation index. Greatly simplified, such an explanation allows:

$$m^2 (a) \propto \int \sigma_{n_e}^2 (r) dr$$

where:

$$\sigma_{n_e} (r) = \text{electron density fluctuation}$$

$$\approx K r^{-\beta/2}$$

$$\beta \approx 4.1$$

so that:

$$m(a) \propto a^{-\left(\frac{\beta-1}{2}\right)} \approx a^{-1.55}$$

Recent papers which favor such an interpretation are Armstrong, et al. (Ref. 28), and Coles, et al. (Ref. 29).

However, the hypothesis advanced in this section is consistent with the earlier work of Little (Ref. 30), and implies the existence of a linear transverse fluctuation scale.

VI. Discussion and Summary

This article has reviewed the relationship between signal path integration of electron density and observations of various forms of solar wind columnar turbulence. The salient points are briefly recapped as follows:

(1) Electron density N_e.

(a) $N_e(r) \cong Ar^{-6} + Br^{-2.3}$ has been well established as a mean model by a wide variety of experimental techniques.

(2) Doppler Phase Fluctuation Data ϕ.

(a) ϕ is well represented by the signal path integration of $Ar^{-6} + Br^{-2.3}$

(b) ϕ falls off with increasing heliographic latitude as does N_e.

(c) A simultaneous 2 parameter fit of a very large volume of ϕ to the signal path integration of $Br^{-(2+\xi)}$ yields $\xi = 0.30$.

(d) ϕ exhibits a sharp break at $a \approx 3r_0$, exactly as does signal path integrated electron density.

(e) ϕ shows excellent correlation with concurrent measurements of signal path integrated electron density over a wide span of signal closest approach distances.

(3) Spectral Broadening Data (SB).

(a) SB is well represented by $Ar^{-6} + Br^{-2.3}$.

(b) SB exhibits a sharp break at $a \approx 3r_0$ as does Doppler phase fluctuation data and signal path integrated electron density.

(4) Scintillation Index m.

(a) m is well represented by the signal path integration of a power law electron density model $N_e(r) \approx Br^{-(2+\xi)}; 0.3 \leqslant \xi \leqslant 0.4$.

(b) Proportionality between m and signal path integrated electron density has been demonstrated via concurrent spacecraft measurements.

This article concludes that the above evidence provides a persuasive and cohesive picture in which all primary observations of solar wind columnar turbulence are seen to be well represented to first order by the signal path integration of a well established electron density model.

Finally, it is noted that this heuristic observational hypothesis conflicts with theoretical interpretations currently in favor. Nonetheless, earlier investigations predicted exactly such results as hypothesized in this article, based on the premise of a linear transverse fluctuation scale. Examples are Little (Ref. 30), Hollweg (Refs. 31 and 32), and Hollweg, et al. (Ref. 33).

References

1. van de Hulst, H. C., "The Electron Density of the Solar Corona," *Bulletin of the Astronomical Institutes of The Netherlands,* Volume XI, Number 410, February 2, 1950.

2. Anderson, J. D., Esposito, P. B., Martin, W. L., Thornton, C. L., and Muhleman, D. O., "Experimental Test of General Relativity Using Time Delay Data from Mariner 6 and Mariner 7," in *The Astrophysical Journal,* Volume 200, August 15, 1975.

3. Saito, K., "A Non-Spherical Axisymmetric Model of The Solar K Corona of The Minimum Type," *Annals of the Tokyo Astronomical Observatory,* University of Tokyo, Second Series, Volume XII, Number 2, Mitaka, Tokyo, 1970.

4. Hansen, R. T., Garcia, C. J., Hansen, S. F., and Loomis, H. G., "Brightness Variations of the White Light Corona During the Years 1964-1967," in *Solar Physics 7,* 1969.

5. Saito, K., Poland, A. I., and Munro, R. H., "A Study of The Background Corona Near Solar Minimum," in *Solar Physics 55,* 1977.

6. Muhleman, D. O., Ekers, R. D., and Formalont, E. B., "Radio Interferometric Test of The General Relativistic Light Bending Near The Sun," *Physical Review Letters,* Volume 24, Number 24, 15 June 1970.

7. Muhleman, D. O., Anderson, J. D., Esposito, P. B., and Martin, W. L., "Radio Propagation Measurements of The Solar Corona and Gravitational Field: Applications to Mariner 6 and 7," in *Proceedings of the Conference on Experimental Tests of Gravitation Theories, Technical Memorandum 33-499,* edited by Davies, R. W., published by the Jet Propulsion Laboratory, Pasadena, California, 1 November 1971.

8. Muhleman, D. O., Esposito, P. B., and Anderson, J. D., "The Electron Density Profile of the Outer Corona and the Interplanetary Medium from Mariner 6 and Mariner 7 Time Delay Measurements," *The Astrophysical Journal,* 211, February 1, 1977.

9. Edenhofer, P., Esposito, P. B., Hansen, R. T., Lueneburg, E., Hansen, S. F., Martin, W. L., and Zygielbaum, A. I., "Time Delay Occultation Data of the Helios Spacecrafts and Preliminary Analysis for Probing the Solar Corona," in the *Journal of Geophysics 42,* 1977.

10. Berman, A. L., Wackley, J. A., and Hietzke, W. H., "The First Direct Comparison of Doppler Phase Fluctuation to Integrated Electron Density Over an Extensive Span of Signal Closest Approach Distance – A Brief Report," IOM ALB-78-35, March 10, 1978 (JPL Internal Document).

11. Counselman III, C. C., and Rankin, J. M., "Density of the Solar Corona from Occultations of NPO532," *The Astrophysical Journal,* Volume 175, August 1, 1972.

12. Weisberg, J. M., Rankin, J. M., Payne, R. R., and Counselman III, C. C., "Further Changes in the Distribution of Density and Radio Scattering in the Solar Corona," *The Astrophysical Journal,* Volume 209, October 1, 1976.

13. Ogilvie, K. W., and Scudder, J. D., "The Radial Gradients and Collisional Properties of Solar Wind Electrons," in *The Journal of Geophysical Research,* Volume 83, Number A8, August 1, 1978.

14. Berman, A. L., "Electron Density in the Extended Corona: Two Views," in *The Deep Space Network Progress Report 42-41,* Jet Propulsion Laboratory, Pasadena, California, October 15, 1977.

15. Berman, A. L., Wackley, J. A., Rockwell, S. T., and Kwan, M., "Viking Doppler Noise used to Determine the Radial Dependence of Electron Density in the Extended Corona," in *The Deep Space Network Progress Report 42-38,* Jet Propulsion Laboratory, Pasadena, California, 15 April 1977.

16. Newkirk, G., "Structure of the Solar Corona," in *The Annual Review of Astronomy and Astrophysics,* Vol. 5, 1967.

17. Berman, A. L., and Rockwell, S. T., "Analysis and Prediction of Doppler Noise During Solar Conjunctions", in *The Deep Space Network Progress Report 42-30,* Jet Propulsion Laboratory, Pasadena, California, 15 December 1975.

18. Berman, A. L., and Wackley, J. A., "Doppler Noise Considered as a Function of the Signal Path Integration of Electron Density," in *The Deep Space Network Progress Report 42-33,* Jet Propulsion Laboratory, Pasadena, California, 15 June 1976.

19. Berman, A. L., Wackley, J. A., and Rockwell, S. T., "The 1976 Helios and Pioneer Solar Conjunctions – Continuing Corroboration of the Link Between Doppler Noise and Integrated Signal Path Electron Density," in *The Deep Space Network Progress Report 42-36,* Jet Propulsion Laboratory, Pasadena, California, 15 December 1976.

20. Berman, A. L., Wackley, J. A., Rockwell, S. T., and Yee, J. G., "The Pioneer 11 1976 Solar Conjunction: A Unique Opportunity to Explore the Heliographic Latitudinal Variations of the Solar Corona," in *The Deep Space Network Progress Report 42-35,* Jet Propulsion Laboratory, Pasadena, California, 15 October 1976.

21. Berman, A. L., "Electron Density and Doppler RMS Phase Fluctuation in the Inner Corona," in *The Deep Space Network Progress Report 42-44,* Jet Propulsion Laboratory, Pasadena, California, 15 April 1978.

22. Berman, A. L., "Solar Wind Turbulence Models Evaluated Via Observations of Doppler RMS Phase Fluctuation and Spectral Broadening in The Inner Corona," in *The Deep Space Network Progress Report 42-44,* Jet Propulsion Laboratory, Pasadena, California, 15 April 1978.

23. Rockwell, S. T., "An Empirical Spectral Bandwidth Model for Superior Conjunction," in *The Deep Space Network Progress Report 42-43,* Jet Propulsion Laboratory, Pasadena, California, 15 February 1978.

24. Berman, A. L., "A Reexamination of the Radial Dependence of Weak Interplanetary Scintillation," in *The Deep Space Network Progress Report 42-50,* Jet Propulsion Laboratory, Pasadena, California, 15 April 1979 (this volume).

25. Chang, H., *Analysis of Dual-Frequency Observations of Interplanetary Scintillations Taken by the Pioneer 9 Spacecraft,* Doctoral Dissertation, Department of Electrical Engineering, Stanford University, May 1976.

26. Erskine, F. T., Cronyn, W. M., Shawhan, S. D., Roelof, E. C., and Gotwols, B. L., "Interplanetary Scintillation at Large Elongation Angles: Response to Solar Wind Density Structure," in *The Journal of Geophysical Research,* Volume 83, Number A9, September 1, 1978.

27. Houminer, Z., and Hewish, A., "Correlation of Interplanetary Scintillation and Spacecraft Plasma Density Measurements," in *Planetary and Space Science,* Volume 22, Number 6, June 1974.

28. Armstrong, J. W., and Coles, W. A., "Interplanetary Scintillations of PSR 0531+21 at 74 MHz," in *The Astrophysical Journal 220,* 15 February 1978.

29. Coles, W. A., and Harmon, J. K., "Interplanetary Scintillation Measurements of The

Electron Density Power Spectrum in The Solar Wind," in *The Journal of Geophysical Research,* Volume 83, Number A4, April 1, 1978.

30. Little, L. T., "Small Scale Plasma Irregularities in The Interplanetary Medium," in *Astronomy and Astrophysics 10,* 1971.

31. Hollweg, J. V., "A Statistical Ray Analysis of the Scattering of Radio Waves by the Solar Corona," in *The Astronomical Journal,* Volume 73, Number 10, Part 1, December 1968.

32. Hollweg, J. V., "Angular Broadening of Radio Sources by Solar Wind Turbulence," in *The Journal of Geophysical Research, Space Physics,* Volume 75, Number 19, July 1, 1970.

33. Hollweg, J. V., and Harrington, J. V., "Properties of Solar Wind Turbulence Deduced by Radio Astronomical Measurements," in *The Journal of Geophysical Research, Space Physics,* Volume 73, Number 23, December 1, 1968.

Table 1. Electron density measurements of the form $Br^{-(2+\xi)}$ in the solar corona

Source	Reference	Year	ξ	Type of measurement
Ogilvie	13	1978	0.5	In Situ Density, Mariner 10
Saito	5	1978	0.14	K-Coronagraph
Berman	10	1978	0.3	S Minus X Range, Viking
Edenhofer	9	1977	0.2	S-Band Range, Helios 2
Berman	15	1977	0.30	S-Band Doppler Noise, Viking
Muhleman	8	1977	0.05	S-Band Range, Mariner 6
Muhleman	8	1977	0.08	S-Band Range, Mariner 7
Weisberg	12	1976	0.3[a]	Pulsar Time Delay
Counselman	11	1972	0.4[b]	Pulsar Time Delay
Muhleman	7	1971	0.41	S-Band Range, Mariner 6
Muhleman	6	1970	0.33	Radio Interferometry
Saito	3	1970	0.5	Photometry
Newkirk	16	1967	0.34[c]	Compilation of Techniques
Blackwell	2	1967	0.33	Solar Eclipse
Blackwell	2	1967	0.33	Solar Eclipse
Blackwell	2	1966	0.3	Solar Eclipse

[a] One of several solutions; this solution is in best agreement with average in situ density values at 1 AU.

[b] One of several solutions; this solution included heliographic latitude.

[c] Computed between N_e $(10r_o)$ and average in situ value (7.5 cm^{-3}) at 1 AU.