# A Circuit Model for Electromagnetic Properties of Waveguide Arcs

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*This is the third article in the series reporting the progress of a waveguide arc study undertaken by the Transmitter Group. In this article, a dielectric model of waveguide arcs is presented to relate measurable electromagnetic quantities to the physical parameters characterizing the breakdown process.*

In the course of studying waveguide arcs, the RF power that is reflected from or transmitted through or absorbed by the arcing plasma is often measurable (Refs. 1 and 2). Such measurable quantities reflect the macroscopic electromagnetic properties of the waveguide arcs as seen by a matched waveguide system. In this brief article, we outline an attempt to develop a "dielectric model" relating these measured quantities to the physical parameters characterizing the breakdown such as the electron density, collision frequency, and velocity distribution. We plan to use this model to correlate the arc properties determined by the electromagnetic and optical measurements.

For simplicity, we shall assume the arc plasma behaves like a dielectric with complex dielectric constant $\epsilon = \epsilon' - j\epsilon''$ ($j = \sqrt{-1}$) and has a cylindrical shape of diameter $d$ shunting the rectangular waveguide at the center of the broad wall. The imaginary part of the dielectric constant accounts for the losses taking place inside the dielectric. Figure 1 shows the equivalent circuit representation for such a dielectric rod shunting the waveguide (Ref. 3). $Z_a$ and $Z_b$ are given by:

$$\frac{Z_a}{R_0} = \frac{\sqrt{1-m^2}}{8m} \left\{ \left(\frac{4}{\alpha^2}+1\right) \frac{\epsilon''}{[(\epsilon'-1)^2+\epsilon''^2]} \right.$$

$$- \frac{\alpha^4 m^2 \epsilon''}{6} [6 + (2\epsilon' - 3)\alpha^2] - j \left[ \left(\frac{4}{\alpha^2}+1\right) \right.$$

$$\frac{(\epsilon'-1)}{[(\epsilon'-1)^2+\epsilon''^2]} - 2S_0 - \frac{\alpha^4 m^2}{12} [6(\epsilon'-1)$$

$$\left.\left. + \alpha^2(\epsilon'^2 - 3\epsilon' + 2 - \epsilon''^2)] \right] \right\} \tag{1}$$

$$\frac{Z_b}{R_0} = \frac{\alpha^4 m \sqrt{1-m^2}}{24} \{ \epsilon'' [6 + 2(2\epsilon' - 3)\alpha^2]$$

$$+ j [6(\epsilon'-1) + (2 - 3\epsilon')\alpha^2 + (\epsilon'^2 - \epsilon''^2)\alpha^2] \} \tag{2}$$

with

$R_0$ : real characteristic impedance of the waveguide system

$\lambda$ : free-space wavelength

$\lambda_c$ : cut-off wavelength

$a$ : inner dimension of the broad waveguide wall

$$\alpha \equiv \pi d/\lambda = \frac{\pi d}{2a}\,\frac{\lambda_c}{\lambda} = \left(\frac{\pi}{2}\right)\left(\frac{d}{a}\right)\left(\frac{1}{m}\right)$$

$$m \equiv \lambda/\lambda_c = \lambda/2_a$$

$$S_0 = \ln\left(\frac{4}{\pi}\frac{a}{d}\right) - 2 + 2\left\{\sum_{n=3,5,\ldots}^{\infty}\left[\frac{1}{n^2 - (1/m)^2}\right]^{1/2} - \frac{1}{n}\right\}$$

Assume the waveguide system is perfectly matched before arcing and neglect the effect of the arc movement. The impedance as seen by the source in the presence of arcing is given by

$$Z_{in} = Z_b + \frac{(R_0 + Z_b)\,Z_a}{Z_a + Z_b + R_0} \tag{3}$$

and the reflection coefficient is

$$\Gamma = \frac{Z_{in} - R_0}{Z_{in} + R_0} \tag{4}$$

After some lengthy calculation, the ratio of the reflected power $P_r$ to the incident power $P_{in}$ is given by

where

$$\frac{Z_a}{R_0} \equiv X_a + jY_a$$

$$\frac{Z_b}{R_0} \equiv X_b + jY_b$$

The remaining power is dissipated either in the arc plasma or in the termination. We shall identify them as the power absorbed by and transmitted through the arc respectively.

To carry out the calculation further, we need to know the waveguide dimension, operating frequency, arc plasma size, and its dielectric constant. The arc size ($d/a$) is estimated to be between 0.1 to 0.2 judging from the arc tracks left behind in some reported S- and X-band experiments (Refs. 1 and 2). The effect on $Z_a$, $Z_b$ due to the uncertainty of the $d/a$ value is not completely negligible; nevertheless, the arc size can be considered relatively certain. The most important and unfortunately the least known parameter in evaluating Eqs. (1) and (2) is the dielectric constant of the arc plasma as seen below.

Assume local thermodynamic equilibrium (LTE) exists inside the arc plasma (this is usually satisfied for gas pressure larger than 1 atmosphere and probably valid for the waveguide arc also); the collision theory of gases (e.g., Ref. 4) gives the expression of plasma dielectric constant as follows:

$$\epsilon = 1 + \frac{ne^2}{m\omega\epsilon_0}\cdot\frac{4\pi}{3}\cdot\int\frac{1}{\omega - j\nu_c(u)}\cdot\frac{df_0(u,T)}{du}\cdot u^3\,du \tag{6}$$

where $\omega$ is the angular frequency of the electromagnetic wave, $n$ is the electron density, $e$ is the electronic charge, $m$ is the electronic mass; $\epsilon_0$ is the vacuum permittivity; $\nu_c(u)$ is the velocity dependent electron collision frequency and $f_0(u,T)$ is the unperturbed electron velocity distribution function. Because the state of the waveguide arc is not well understood, Eq. (6) is hard to evaluate. However, if we can assume a velocity-independent collision frequency, Eq. (6) can be reduced to a much simpler form as follows:

$$\epsilon = 1 - \frac{ne^2}{m\omega\epsilon_0}\cdot\frac{1}{\omega - j\nu_c} \equiv 1 - \frac{\omega_p^2}{\omega(\omega - j\nu_c)} \tag{7}$$

$$\frac{P_r}{P_{in}} = |\Gamma|^2 = \frac{[1 + Y_b(2Y_a + Y_b) - X_b(2X_a + X_b)]^2 + 4\,[(X_a + X_b)Y_b + X_b Y_a]^2}{[1 + Y_b(2Y_a + Y_b) - X_b(2X_a + X_b) - 2(X_a + X_b + 1)]^2 + 4\,[(X_a + X_b)Y_b + X_b Y_a + Y_a + Y_b]^2} \tag{5}$$

where $\omega_p^2 = ne^2/m\epsilon_0$ is the plasma frequency. Two extreme cases can be readily identified, namely that $\nu_c << \omega$ and $\nu_c >> \omega$. For $\nu_c << \omega$, the collision process can be totally neglected and $\epsilon = 1 - \omega_p^2/\omega^2$, i.e., an ideal collisionless plasma. For $\omega < \omega_p$, the dielectric constant is a negative real number resulting in an imaginary propagation constant (evanescent mode). On the other hand, $\omega > \omega_p$ results in a real propagation constant (propagation mode). For $\nu_c >> \omega$, the collisions are so frequent that they dominate the dielectric constant and $\epsilon = 1 - j\omega_p^2/\omega\nu_c$, resulting in substantial dissipation in the plasma. It is possible that as the arc strikes and forms, the condition may evolve from one realm to the other.

Since we are still in the process of characterizing waveguide arcs, no quantitative evaluation will be carried out at the present time. Further study along this line will be presented and discussed in the future as soon as the physical characterization of an arc plasma is feasible.

## References


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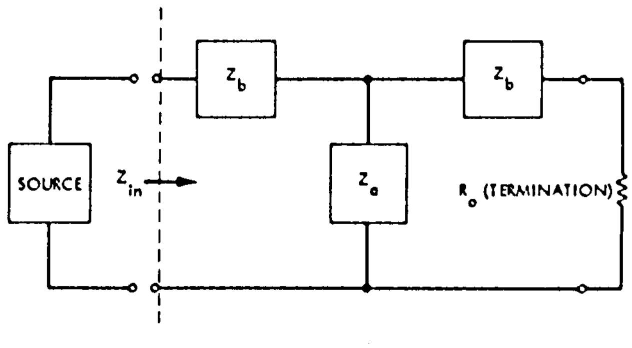


**Fig. 1. Equivalent Circuit Representation for Dielectric Shunted Waveguide System**