Effects of Asymmetric Passband Filtering on the Phase of the Costas Loop's Reconstructed Carrier

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The reconstructed carrier of a telemetry return signal is used in deriving the Doppler and range information in the radiometric systems. When suppressed carrier BPSK signalling with Costas loop demodulation is used, there are concerns on the amount of shift in the reconstructed carrier phase, when the received signal suffers asymmetric bandpass filtering through the various stages of the receiver. This paper quantifies this effect and concludes that the phase shifts due to asymmetric bandpass filtering on the Costas loop's reconstructed carrier can be slightly worse than those suffered by the residual carrier loop's reconstructed carrier. However, they are well within the error budgets of the radiometric system.

I. Introduction

Figure 1 illustrates the various stages of a Block IV receiver. Various bandpass filtering occurs in the chain of IF stages in the receiver before the signal is demodulated, either with a phase lock loop, when residual carrier signals are received, or with a Costas loop, when suppressed carrier signals are received. In either case, there will be asymmetric bandpass filtering on the received signal spectrum. The bandpass filters following the first mixer-amplifier will always have their center frequencies line up with the proper IF frequencies, when carrier tracking is achieved so that the VCO is in frequency lock with the received carrier. Nevertheless, filtering on the signal spectrum is still not exactly symmetrical with respect to the center frequency, due to limitations in bandpass filter designs. The dominant asymmetric filtering effect, however, comes from the maser amplifier, which precedes the first mixer-amplifier in the receiver IF chain. Since the maser amplifier is in front of the first mixer-amplifier, the received carrier does not necessarily line up with the center frequency of the maser when there are Doppler offsets. Because of these effects there will be phase shifts on the reconstructed carrier, and the exact amount will be Doppler and maser-characteristic dependent. It is true, however, that these phase shifts will occur on the residual carrier loop's reconstructed carrier, as well as on that of the Costas loop.

The reconstructed carrier phase is used in the radiometric system in deriving range and Doppler information. Consequently it is necessary that these phase shifts be maintained at reasonably small values over the range of Doppler offsets of interest. For the residual carrier loop in the existing Block III or Block IV receiver, the phase shifts on the reconstructed carrier due to Doppler shifts and passband filtering are well calibrated and are known to be well within the error budgets of the existing radiometric systems. When a Costas loop is used to demodulate suppressed carrier BPSK telemetry (Ref. 1), however, no known data are available. Further, since the

Costas loop utilizes the whole signal spectrum in reconstructing its carrier phase, it is unclear how severe the effect of asymmetric bandpass filtering will be on the phase shift of the reconstructed carrier in the presence of Doppler offsets.

An analytical approach is attempted in order to answer this question quantitatively in this paper. Measured results are also obtained (Ref. 2) which are in fair agreement with these analysis results. In our analyses, the dominant bandpass filtering is assumed to be the maser, which is approximated by a two-pole Tchebycheff characteristic with a 1-dB bandwidth approximately equal to 3.5 times the data spectrum. It is found that, in the case of the Costas loop, the Doppler offset has to be approximately on the order of one-third the filter bandwidth in order to create phase shifts on the order of 25 degrees. For a 30 MSPS data rate, this implies Doppler offsets of 30 MHz or more, which is much larger than the realistic Doppler offsets of practical interest. For realistic values of Doppler offsets around 1 MHz, the phase shift will be on the order of 1 degree.

II. Analysis

In the presence of Doppler offsets, the phase of the reconstructed carrier of a residual carrier tracking loop is basically determined by the phase delay characteristic of the maser amplifier at the detuned carrier frequency, relative to the center frequency of the maser. In the case of the Costas loop, since the entirety of the signal spectrum is used to reconstruct the carrier phase, this phase will depend upon the maser delay characteristics at the detuned carrier frequency as well as at all the harmonics in the data spectrum.

Square-wave data has the highest number of transitions of any data stream, and thus will suffer the most phase distortion due to filtering. Thus if the received signal is assumed to be square-wave modulated, the results obtained will be the worst case bound. Since a square-wave can be represented by the Fourier series,

$$d_{SQ}(t) = \sum_{k=0}^{\infty} a_{2k+1} \cos\left[\frac{2\pi(2k+1)t}{T}\right] \tag{1}$$

where the coefficients a_{2k+1} are given by

$$a_{2k+1} = \frac{4}{\pi}\frac{(-1)^k}{2k+1} \tag{2}$$

The received RF signal at carrier frequency ω_c can be written as:

$$\begin{aligned} S(t) &= d_{SQ}(t)\cos(\omega_c t + \theta) \\ &= \frac{1}{2}\sum_{k=0}^{\infty} a_{2k+1}\left\{\cos\left[\left(\omega_c - \frac{2\pi(2k+1)}{T}\right)t + \theta\right]\right. \\ &\quad \left. + \cos\left[\left(\omega_c + \frac{2\pi(2k+1)}{T}\right)t + \theta\right]\right\} \end{aligned} \tag{3}$$

where T is the symbol duration, and where θ is the carrier phase, to be tracked by the Costas loop. Let $G(\omega)$ be the transfer function of the maser characteristic. The filtered output with input $S(t)$ will be given by

$$\begin{aligned} \hat{S}(t) &= \frac{1}{2}\sum_{k=0}^{\infty} a_{2k+1}\left\{\alpha_{-(2k+1)}\cos\left[\left(\omega_c - \frac{2\pi(2k+1)}{T}\right)t\right.\right. \\ &\quad \left. + \theta + \beta_{-(2k+1)}\right] \\ &\quad \left. + \alpha_{(2k+1)}\cos\left[\left(\omega_c + \frac{2\pi(2k+1)}{T}\right)t + \theta + \beta_{2k+1}\right]\right\} \end{aligned} \tag{4}$$

where in Eq. (4) α_n and β_n denote the amplitude and phase responses of the bandpass characteristic $G(\omega)$, at $\omega = \omega_c + n2\pi/T$, respectively. In other words,

$$\begin{aligned} \alpha_n &\equiv G\left(\omega_c + \frac{2\pi n}{T}\right) \\ \beta_n &\equiv \arg\left[G\left(\omega_c + \frac{2\pi n}{T}\right)\right] \end{aligned} \tag{5}$$

and where arg(z) stands for the argument of the complex function z.

The operation of the Costas loop is identical in performance to that of the squaring loop, which squares the filtered signal $\hat{S}(t)$ and tracks the $2\omega_c$ component. Defining

$$A_n \equiv \begin{cases} a_n\alpha_n & \text{if } n > 0 \\ a_{-n}\alpha_n & \text{if } n < 0 \end{cases} \tag{6}$$

the signal $\hat{S}(t)$ can be written, from (4), as:

$$\hat{S}(t) = \frac{1}{2} \sum_{\substack{n=-\infty \\ n=\text{odd}}}^{\infty} A_n \cos \left[\left(\omega_c + \frac{2\pi n}{T}\right) t + \theta + \beta_n\right] \quad (7)$$

The squared signal $[\hat{S}(t)]^2$ is equal to

$$[\hat{S}(t)]^2 = \frac{1}{8} \sum_{\substack{\ell=-\infty \\ \ell,m=\text{odd}}}^{\infty} \sum_{m=-\infty}^{\infty} A_\ell A_m$$

$$\left\{\cos \left[\left(2\omega_c + \frac{2\pi(\ell+m)}{T}\right) t + 2\theta + \beta_\ell + \beta_m\right]\right.$$

$$\left. + \cos\left[\frac{2\pi(\ell - m)}{T} t + \beta_\ell - \beta_m\right]\right\} \quad (8)$$

The component at $2\omega_c$ in $[\hat{S}(t)]^2$, which will be tracked by the squaring loop, is observed from Eq. (8) to be:

$$y(t) = \frac{1}{4} \sum_{\ell=1}^{\infty} A_\ell A_{-\ell} \cos [2\omega_c t + 2\theta + \beta_\ell + \beta_{-\ell}] \quad (9)$$

which, after some simple algebra, can also be written in the following form:

$$y(t) = \frac{1}{4} v \cos [2(\omega_c t + \theta) + 2\psi] \quad (10)$$

where

$$v = \sqrt{\left(\sum_{k=0}^{\infty} C_{2k+1} \cos [\beta_{2k+1} + \beta_{-(2k+1)}]\right)^2 + \left(\sum_{k=0}^{\infty} C_{2k+1} \sin [\beta_{2k+1} + \beta_{-(2k+1)}]\right)^2} \quad (11)$$

$$\psi = \frac{1}{2} \tan^{-1} \left\{\frac{\sum_{k=0}^{\infty} C_{2k+1} \sin [\beta_{2k+1} + \beta_{-(2k+1)}]}{\sum_{k=0}^{\infty} C_{2k+1} \cos [\beta_{2k+1} + \beta_{-(2k+1)}]}\right\} \quad (12)$$

and where for $k = 0, 1, 2, \ldots$,

$$C_{2k+1} = A_{2k+1} A_{-(2k+1)} = (a_{2k+1})^2 \alpha_{2k+1} \alpha_{-(2k+1)} \quad (13)$$

The phase shift in the Costas loop's reconstructed carrier for square-wave modulating signals undergoing bandpass filtering is then equal to Ψ, as given in Eq. (12).[1]

It is instructive to notice that, when bandpass filtering is absent, i.e., when $\alpha_n = 1$ and $\beta_n = 0$, v in Eq. (11) becomes

$$v = \sum_{k=0}^{\infty} \left(\frac{4}{\pi} \frac{1}{2k+1}\right)^2 = 2 \quad (14)$$

and $\Psi = 0$. Hence the error signal $y(t)$ becomes

$$y(t) = \frac{1}{2} \cos 2(\omega_c t + \theta) \quad (15)$$

which is clearly the $2\omega_c$ component in the squared signal $[\hat{S}(t)]^2$ when the filtering effect is negligible.

In general, when the filter phase response is odd symmetric with respect to the center frequency, so that

$$\beta_{2k+1} = -\beta_{-2k+1} \quad (16)$$

then it is clearly true from Eq. (12) that $\Psi = 0$, i.e., there will be no phase shift. For a general bandpass characteristic such that Eq. (16) does not necessarily hold, the phase shift Ψ is given by Eq. (12).

[1] The value ψ was used rather than 2ψ since the Doppler system tracks the carrier at ω_c rather than $2\omega_c$.

III. Numerical Results

Currently the X-band maser passband – 1 dB bandwidth in the Block III or Block IV receivers is about 40 MHz. To receive multimegabit telemetry with data rates up to 30 MBPS at the DSN in the 1980's it is imperative to increase the maser bandwidth. It is assumed here that the future maser will have an equivalent baseband bandwidth of about 100 MHz, in order to receive data rates up to 30 MBPS. Without available data on the future maser, the best one can do at the present is to assume a certain bandpass characteristic for it and evaluate the phase shift effect on the reconstructed carrier, in the presence of Doppler offsets. This has been performed experimentally (Ref. 2). The experiment was actually performed at IF, but was intended to simulate the effect of maser filtering. A two-pole Tchebycheff bandpass characteristic was used to simulate the maser bandpass characteristic. The circuit diagram of this bandpass filter is illustrated in Fig. 2. It has center frequency at the 10 MHz IF and a 1-dB bandwidth of 350 KHz. This has then a bandwidth roughly equal to three times the data rate of 100 KBPS, which is one of the data rates used in the tests reported in Ref. 2. The amplitude and phase responses of this bandpass characteristic are plotted in Figs. 3 and 4 respectively. The phase shift in the reconstructed carrier in the residual carrier loop case, in the presence of Doppler, is basically equal to the phase response of this filter at the detuned frequency. For the case of Costas loop demodulation, the phase shift Ψ can be computed from Eq. (12), with the information available in Figs. 3 and 4. The computed result is illustrated in Fig. 5. It is observed that a Doppler offset of 100 kHz can only produce a phase shift of 25 degrees. The Doppler offset of 100 kHz corresponds to an offset of one data rate, which corresponds to one-third of the maser 1-dB bandwidth assumed. When these results are translated to the future maser with a 1-dB bandwidth of 100 MHz (i.e., 3 times the maximum data rate of 30 MSPS), it can be predicted that a phase shift of 25 degrees in the Costas loop's reconstructed carrier will result when there is a Doppler offset of 30 MHz. This, of course, assumes that the two-pole Tchebycheff characteristic is a good approximation to the maser passband. Also shown in Fig. 5 is the phase shift that the residual carrier loop will suffer in the presence of Doppler offsets, for the same passband characteristics assumed. It can be seen from Fig. 5 that the phase shift in the Costas loop's reconstructed carrier is only slightly worse than that of the residual carrier loop.

A 30 MHz Doppler offset is much larger than most realistic values. For a realistic Doppler offset of 1 MHz, which equals 1/30 of the maximum data rate and therefore is equal to about 1/100 of the (future) maser 2-dB bandwidth, the phase shift in the reconstructed carrier of the Costas (or residual carrier) loop is on the order of 1 degree (see Fig. 5). This is much smaller than that which is allowed by the radiometric system's error budget.

IV. Conclusions

Assuming the maser characteristic to be approximately equal to that of a two-pole Tchebycheff bandpass characteristic, with a 1-dB bandwidth at 3-5 times the data rate, the phase shifts suffered by the Costas loop's reconstructed carrier have been quantified, as functions of Doppler offsets. These offsets are then compared with the corresponding ones of the residual carrier loop's reconstructed carrier. It is concluded that, even though the corresponding phase shifts of the Costas loop are slightly worse than that of the residual carrier loop, they are both well within the error budgets of the radiometric system.

References

1. Lesh, J., "Tracking Loop and Modulation Format Considerations for High Rate Telemetry", *DSN Progress Report 42-44*, Jet Propulsion Laboratory, Pasadena, Calif., Jan. – Feb., 1978.

2. Reasoner, R., Stevens, G., and Woo, K. T., "Costas Loop Demodulation of Suppressed Carrier BPSK Signals in the DSN Environment – Experimental Results Obtained at TDL" (this issue of the *DSN Progress Report*).

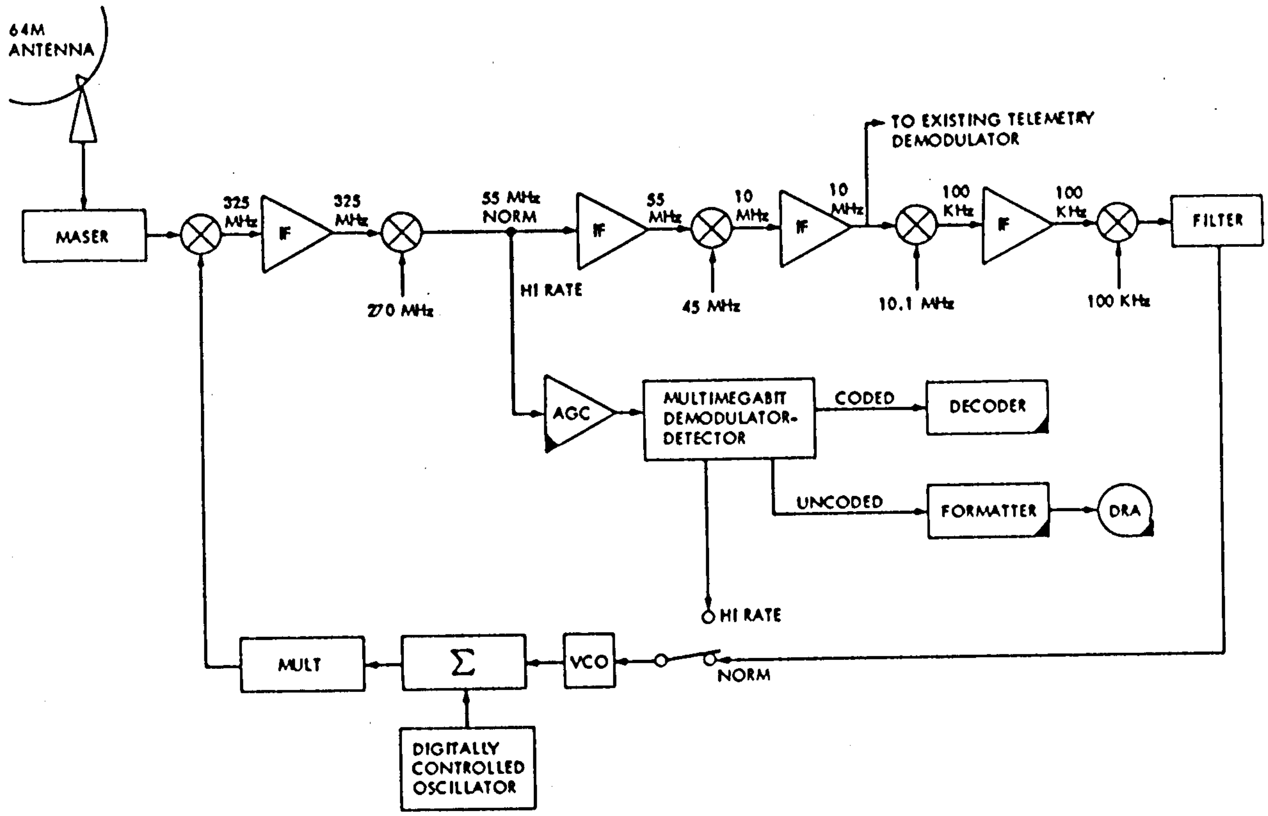
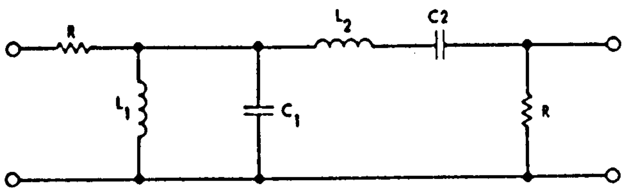


Fig. 1. Block diagram of the system under consideration



$R = 50\Omega$, $C_1 = 4500$ pf, $C_2 = 11.5$ pf

$L_1 = 0.056\ \mu H$, $L_2 = 22\ \mu H$

$f_0 = 10$ MHz, 3-dB BW = 0.5 MHz

Fig. 2. Block diagram of the two-pole Tchebycheff filter used to simulate the maser characteristic

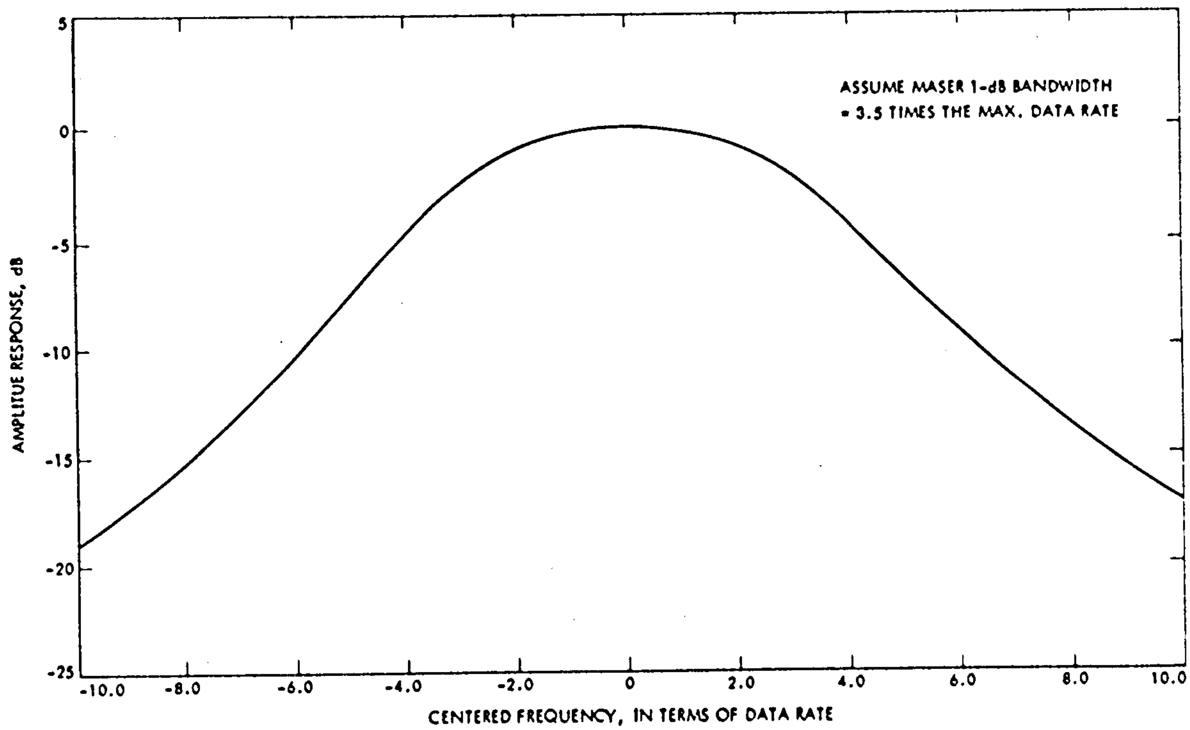


Fig. 3. Amplitude response of the BPF used to simulate the maser characteristic

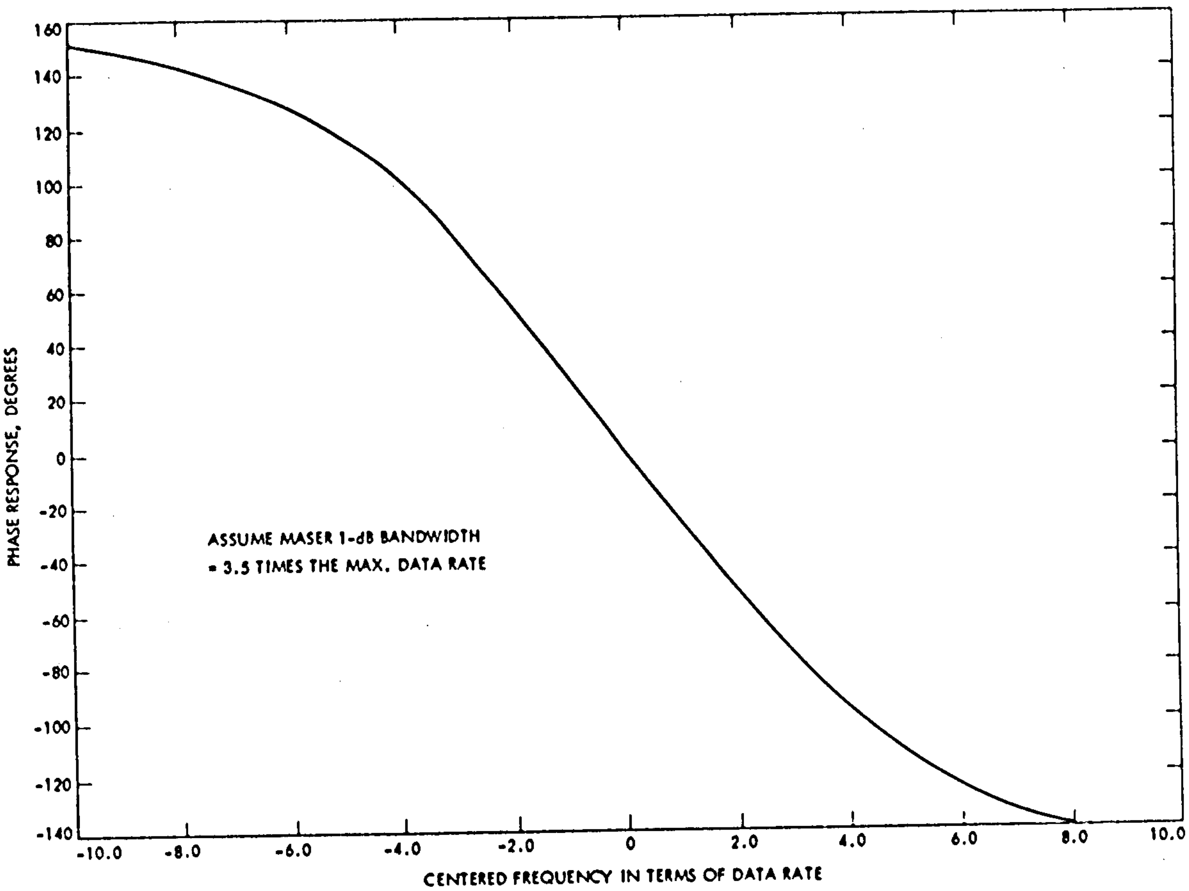


Fig. 4. Phase response of the BPF used to simulate the maser characteristic

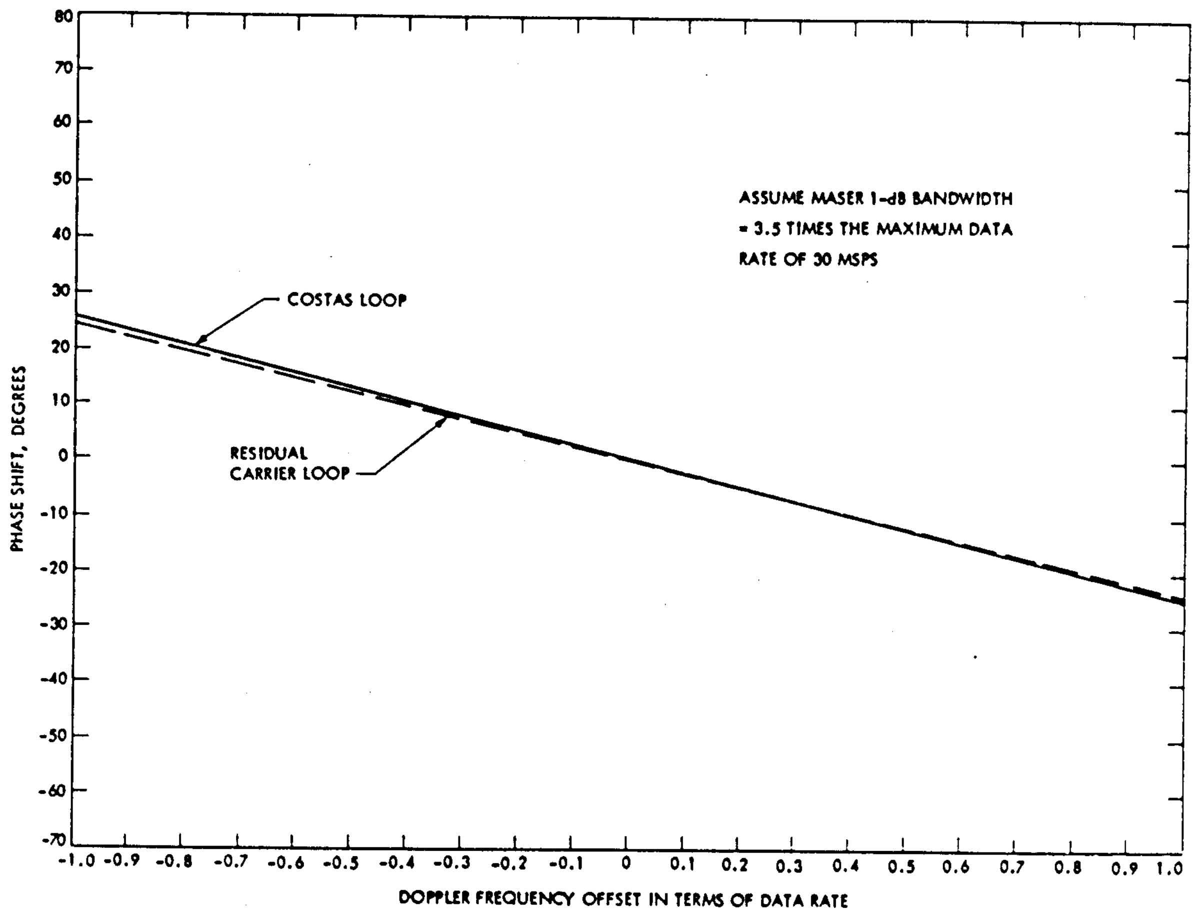


Fig. 5. Phase shift in reconstructed carrier due to Doppler offsets and the assumed receiver front end phase delay characteristic