Carrier Tracking Loop Performance in the Presence of Strong CW Interference

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In a coherent data link, narrow-band radio frequency interference (RFI) near the carrier frequency can degrade the link performance by impacting the carrier tracking loop behavior and producing a partial or complete loss of coherence. If the RFI is strong enough, this effect can occur even though the frequency of the interference lies well beyond the carrier tracking loop bandwidth. In 1973, F. Bruno and A. Blanchard independently performed similar analyses of the response of a phase-locked loop (PLL) to a continuous wave (CW) interferer, and derived conditions under which the loop dropped carrier lock and tracked the interference instead. This paper compares the contributions of these two analysts, and extends Bruno's closed form approximation for the loop phase error. This result is applied in a subsequent article to the general problem of coherent detection of residual and suppressed carrier telemetry in the presence of strong CW interference.

I. Introduction

Because of increasing competition for the available RF spectrum, the DSN has become concerned about the disruptive potential of RFI on network operations. In an effort to define the RFI threat, the DSN is currently developing a sensitive wide-band RFI monitoring capability to detect and identify sources of RFI at the Goldstone complex (Ref. 1). This surveillance system will characterize the RF environment in which the DSN functions, but a parallel activity is needed to investigate the effects of different classes of RFI on components of a DSN receiver, and ultimately to determine the resulting degradation in link performance. One recent example of this is Low's simulation work at DSS 11 to measure the increase in telemetry error rate due to CW interference at odd harmonics of the data subcarrier (Ref. 2). As a further contribution to this effort, this paper reviews and extends some earlier analyses of the impact of a CW interferer on the performance of a carrier tracking loop.

One of the first investigations of this problem was the experimental study conducted in 1966 by Britt and Palmer at Langley Research Center on second-order PLL's (Ref. 3). They measured the loop phase error as a function of the interference-to-carrier power ratio (P_I/P_C), and determined limiting values of P_I/P_C for which carrier lock was lost, for CW interference within the loop passband. In 1972, Ziemer, at the University of Missouri, reported on a perturbation analysis of weak CW interference in Costas loops (Ref. 4). He restricted the interference levels to be small enough to allow the degraded carrier tracking loop to remain in its linear operating range. In their independent analytical treatment of PLL's, in 1973, Bruno (Ref. 5) of Hazeltine Corporation and Blanchard (Ref. 6) of Centre Spatial de Toulouse (France) eliminated this constraint, permitting P_I/P_C to become large enough for the loop to produce a steady-state static phase error well outside the linear region. Using the same mathematical approach, they solved for the loop phase error and derived loss of lock conditions that are principally valid for strong CW interference beyond the loop passband. However, whereas Blanchard left his solutions in implicit form, Bruno derived explicit approximations for the degraded loop phase error that are accurate over a small part of the lock region. In the rest of this paper, we will examine the contributions of these two analysts in some detail, and extend Bruno's closed form approximate phase error results to the entire lock region.

II. Analysis

Consider a PLL which is initially locked to a carrier with amplitude A and frequency ω_o. In the presence of a CW interferer at offset frequency $\Delta\omega$, with $\alpha^2 \equiv P_I/P_C$, the PLL input is

$$r(t) = \sqrt{2}A\,[\sin \omega_o t + \alpha \sin(\omega_o + \Delta\omega)t]\,. \tag{1}$$

Neglecting the $2\omega_o$ term, the loop error signal is

$$\begin{aligned} \epsilon(t) &= \sqrt{2}\,r(t)\cos[\omega_o t - \phi(t)] \\ &= A\,[(1 + \alpha\cos\Delta\omega t)\sin\phi(t) + \alpha\sin\Delta\omega t\cos\phi(t)] \end{aligned} \tag{2}$$

Using the operator $p = d/dt$, the loop phase error is constrained by

$$\phi(t) = -K_{VCO}\left[\frac{F(p)}{p}\right]\epsilon(t)\,, \tag{3}$$

where $F(s)$ is the loop filter and K_{VCO} is the gain of the voltage-controlled oscillator (VCO).

Equations (2) and (3) cannot be solved analytically for $\phi(t)$ for all values of α and $\Delta\omega$. However, based on their experimental observations, Bruno and Blanchard both adopted the steady-state trial solution

$$\phi(t) = \lambda + \sigma\sin(\Delta\omega t + \nu)\,, \tag{4}$$

where the static phase error $\lambda[\epsilon - \pi/2, \pi/2]$. Note that Eq. (4) implies that the average VCO frequency is ω_o, reflecting the assumption that the loop remains locked to the carrier with an RFI-induced oscillation at the beat frequency $\Delta\omega$. Substituting this trial solution for $\phi(t)$ into Eqs. (2) and (3), and equating the dc, $\cos\Delta\omega t$, and $\sin\Delta\omega t$ coefficients on both sides of the resulting equation, Bruno derived the lock constraints

$$\sin\lambda = -\frac{\sigma^2\delta\cos\psi}{2J_0(\sigma)}\,. \tag{5}$$

$$\sin(\lambda - \nu) = -\frac{\sigma^2\delta\cos\psi}{2\alpha J_1(\sigma)}\,. \tag{6}$$

$$\left[\frac{\sigma\delta\sin\psi + 2J_1(\sigma)\cos\lambda}{J_0(\sigma) - J_2(\sigma)}\right]^2 + \left[\frac{\sigma^2\delta\cos\psi}{2J_1(\sigma)}\right]^2 = \alpha^2\,. \tag{7}$$

where ψ is the phase angle of $F(j\Delta\omega)$, δ is the normalized offset frequency

$$\delta \equiv \frac{\Delta\omega}{A\,K_{VCO}\,|F(j\Delta\omega)|}\,. \tag{8}$$

and ψ and δ are implicit functions of $\Delta\omega$ in Eqs. (5)-(7). The Bessel functions above result from expansions of $\sin[\sigma\sin(\Delta\omega t + \nu)]$ and $\cos[\sigma\sin(\Delta\omega t + \nu)]$. This trial solution is valid if components at $2\Delta\omega$ and higher can be ignored, which requires that σ be on the order of 1 rad or less so that terms of the form $J_n(\sigma)$ for $n > 2$ are negligible relative to their lower order counterparts. This does not severely limit the usefulness of these results since larger values of σ correspond to peak phase errors on the order of $\pi/2$ or higher, at which point it becomes questionable whether the loop can properly be characterized as being locked to the carrier. Since σ increases monotonically with α, the implication is that as the interference power rises, there is a brief transition region in which the loop is not locked to the carrier or the CW interferer, and the form of the phase error differs from Eq. (4) for large σ.

Blanchard adopted the same trial solution and again considered only components at dc and $\Delta\omega$, but he made several simplifying assumptions. He neglected $J_n(\sigma)$ for $n > 1$, restricted himself to second order loop filters of the form

$$F(s) = \frac{1 + \tau_2 s}{1 + \tau_1 s} \tag{9}$$

or

$$F(s) = \frac{1 + \tau_2 s}{\tau_1 s} \tag{10}$$

where $\tau_1 \gg \tau_2$, and assumed that $\Delta\omega \gg 1/\tau_2$ so that

$$|F(j\Delta\omega)| \approx \frac{\tau_2}{\tau_1} \tag{11}$$

$$\psi \approx 0.$$

Then Eq. (7) reduces to

$$\alpha^2 = \left[\frac{\sigma^2\delta}{2J_1(\sigma)}\right]^2 \left\{1 - 4\left[\frac{J_1(\sigma)}{J_0(\sigma)}\right]^4\right\} + \left[\frac{2J_1(\sigma)}{J_0(\sigma)}\right]^2 . \tag{12}$$

Actually, Blanchard's restrictions are not unreasonable. Most PLL's are currently implemented with second order filters, and for a given α, a large $\Delta\omega$ is consistent with a small σ, which is a necessary condition for the analysis above to be valid. What Bruno's results can accommodate that Blanchard's preclude are situations in which $\Delta\omega$ is too small for Eq. (11) to hold, yet α is low enough to yield an acceptably small value of σ.

Note that Eq. (5) is meaningless when the magnitude of the right-hand side exceeds 1. Consequently, Bruno and Blanchard both adopted $|\lambda| = \pi/2$ to be the limiting condition for the loop to be locked to the carrier. They later confirmed this result experimentally. So, for a given $\Delta\omega$, the loop remains in lock for $|\lambda| \leq \pi/2$, which translates into $\sigma \leq \sigma_o$ or $\alpha \leq \alpha_o$. Equation (5) defines σ_o as a function of δ:

$$\frac{\sigma_o^2}{J_0(\sigma_o)} = \frac{2}{|\delta \cos\psi|}, \tag{13}$$

and, using $\sigma = \sigma_o$ with $|\lambda| = \pi/2$ in Eq. (7) yields α_o:

$$\alpha_o^2 = \left[\frac{J_0(\sigma_o)}{J_1(\sigma_o)}\right]^2 \left\{1 + \left(\frac{2\tan\psi}{\sigma_o}\right)^2 \left[\frac{J_1(\sigma_o)}{J_0(\sigma_o) - J_2(\sigma_o)}\right]^2\right\} \tag{14}$$

In Appendix A, it is shown that the implicit dependence of α_o on δ in Eqs. (13) and (14) can be replaced by the simple explicit relationship

$$\alpha_o^2 \cong 2\left|\frac{\delta}{\cos\psi}\right| \tag{15}$$

provided

$$\frac{\sigma_o^2}{2}\left|\sin^2\psi - \frac{\sigma_o^2}{48}\right| << 1^1 \tag{16}$$

In particular, when $\Delta\omega$ is much larger than the PLL bandwidth so that $\psi \approx 0$, Eq. (15) is accurate for $\sigma_o^4 << 96$, which is everywhere Eqs. (13) and (14) are valid.

For given loop parameters and values of δ and $\alpha \leq \alpha_o$, it is difficult to compute σ using Eqs. (7) or (12). While Blanchard left his solution in this implicit form, Bruno simplified the computation by substituting

$$J_0(\sigma) - J_2(\sigma) \cong 1 - \frac{3}{8}\sigma^2 \text{ and } 2J_1\frac{(\sigma)}{\sigma} \cong 1$$

in Eq. (7), resulting in the perturbation expressions,

$$\sigma^2 \cong \frac{\alpha^2}{\delta^2 + 2\delta\sin\psi\cos\lambda + \cos^2\lambda + \frac{3}{4}\alpha^2} \tag{17}$$

for small σ. However, Eq. (17) is still complicated by the dependence of $\cos\lambda$ on σ via Eq. (5). To circumvent this, Bruno restricted his approximation to the region $\sigma \ll \sigma_o$ or $|\sin\lambda| \ll 1$, allowing Eq. (17) to trivially simplify to

$$\sigma^2 \cong \frac{\alpha^2}{\delta^2 + 2\delta\sin\psi + 1 + \frac{3}{4}\alpha^2}. \tag{18}$$

In Appendix B, it will be proved that for $\sigma^2 \ll \sqrt{2}$ and any value of $|\lambda| \leq \pi/2$, σ^2 can be accurately computed from the explicit approximation

$$\sigma^2 \cong \frac{\alpha^2}{\delta^2 + 2\delta\sin\psi + 1} \tag{19}$$

In particular, if $\sigma_o^2 \ll \sqrt{2}$, Eq. (19) is valid over the entire PLL lock region. Completing the solution for λ and ν for small σ, Eqs. (5) and (6) reduce to

$$\sin\lambda \cong -\frac{\sigma^2\delta\cos\psi}{2}, \tag{20}$$

$$\sin(\lambda - \nu) \cong -\frac{\sigma\delta\cos\psi}{\alpha}. \tag{21}$$

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[1] Bruno derived Eq. (15) (as did Blanchard with $\psi = 0$) for small σ_o (or large α_o); however, the constraint of Eq. (16) under which Eq. 15 is valid, is our contribution.

III. Example

To illustrate these results, consider a loop filter of the form of Eq. (9), representative of a PLL in a DSN receiver. For example, suppose

$$\tau_1 = 2 \text{ sec}$$

$$\tau_2 = \frac{1}{8} \text{ sec}$$

$$A K_{VCO} = 1000 \text{ sec}^{-1} \tag{22}$$

for which the loop noise bandwidth at threshold and at the specified operating point are given by

$$2B_{LO} \cong \frac{3}{2\tau_2} = 12 \text{ Hz},$$

$$B_L \cong \frac{1 + \dfrac{A K_{VCO} \tau_2^2}{\tau_1}}{4\tau_2} = 17.6 \text{ Hz}. \tag{23}$$

Applying Eq. (13) to the given loop parameters, we find that $\sigma_o \lessapprox 1$ rad for $\Delta\omega/2\pi \gtrapprox B_L$: this is the region for which the analysis above is valid. Also, for this range of $\Delta\omega$, the constraint of Eq. (16) is satisfied, indicating that Eq. (15) can be used to compute α_o^2, the limiting interference-to-carrier power for which the PLL maintains carrier lock. The loop lock region is illustrated in Figs. 1 and 2. Using Eqs. (5) and (7) for $\Delta\omega/2\pi$ near B_L, and the simpler approximations of Eqs. (19) and (20) for $\Delta\omega/2\pi \gg B_L$, profiles of α^2 and σ were also computed for $|\lambda| < \pi/2$ and presented in these figures. As expected, the farther the interference lies outside the loop bandwidth, the larger the value of P_I/P_C required to pull the loop out of lock, and the smaller its effect on the loop behavior as measured by σ_o. Also, $\alpha_o^2 \gg 1$ over much of the lock region for which the analysis applies, which supports the strong CW interference restriction in the title of this paper. Of course, as mentioned earlier, for offset frequencies within the loop bandwidth corresponding to $\sigma_o \gtrapprox 1$ rad, the loop can be pulled out of lock for $P_I \sim P_C$, and Eq. (4) simply does not represent the form of the phase error in this region.

Figure 1 shows that for a given value of $\Delta\omega$, the loop phase error is not significantly degraded until α^2 approaches the limit α_o^2. This behavior is illustrated in more detail in Fig. 3 for $\Delta\omega/2\pi = 1000$ Hz, using Eqs. (19) and (20).

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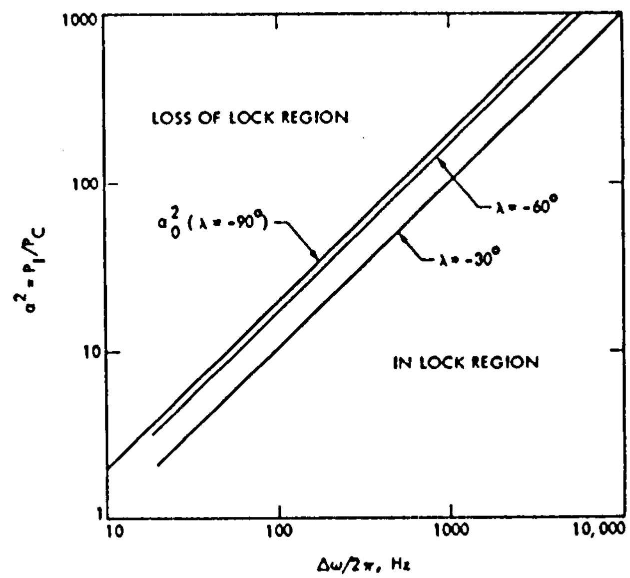

Fig. 1. Limiting interference-to-carrier power levels for loss of lock when $F(s) = \dfrac{1 + s/8}{1 + 2s}$ and $A\,K_{VCO} = 1000\ s^{-1}$

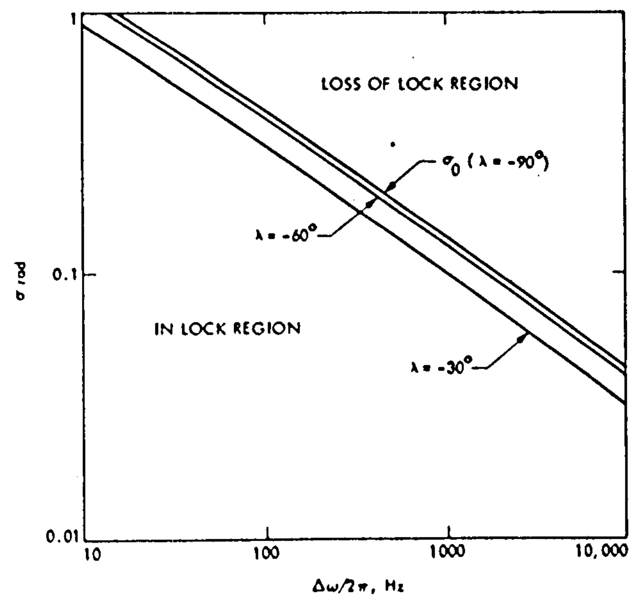

Fig. 2. Corresponding limiting values of σ (phase error beat frequency amplitude)

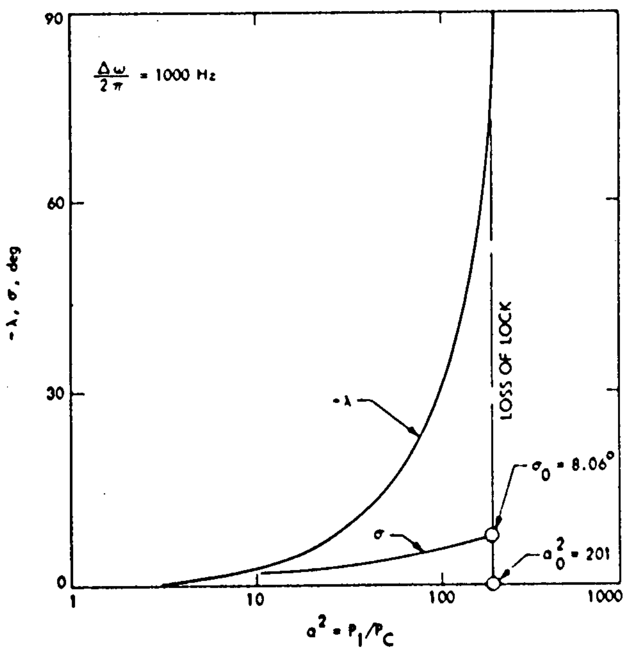

Fig. 3. Lock parameters of previous loop at $\Delta\omega/2\pi$ = 1000 Hz

Appendix A

Derivation of Lock Constraint Approximation

Equations (13) and (14) can be combined to yield the expression

$$\frac{\alpha_o^2}{2\left|\dfrac{\delta}{\cos\psi}\right|} = J_0(\sigma_o)\left\{\left[\frac{\cos\psi}{2J_1\dfrac{\sigma_o}{\sigma_o}}\right]^2 + \left[\frac{\sin\psi}{J_0(\sigma_o) - J_2(\sigma_o)}\right]^2\right\}. \tag{A-1}$$

But to order σ_o^4, the Bessel functions above may be approximated by the power series

$$J_0(\sigma_o) \cong 1 - \frac{\sigma_o^2}{4} + \frac{\sigma_o^2}{64}.$$

$$2J_1\frac{\sigma_o}{\sigma_\varphi} \cong 1 - \frac{\sigma_o^2}{8} + \frac{\sigma_o^4}{192}.$$

$$J_0(\sigma_o) - J_2(\sigma_o) \cong 1 - \frac{3\sigma_o^2}{8} + \frac{\sigma_o^4}{48}. \tag{A-2}$$

Computing the inverse square of the last two polynomials, we find that

$$\left[2J_1\frac{\sigma_o}{\sigma_o}\right]^{-2} \cong 1 + \frac{\sigma_o^2}{4} + \frac{7\sigma_o^4}{192}.$$

$$[J_0(\sigma_o) - J_2(\sigma_o)]^{-2} \cong 1 + \frac{3\sigma_o^2}{4} + \frac{73\sigma_o^4}{192}. \tag{A-3}$$

Substituting these expressions into Eq. (1) yields the result

$$\frac{\alpha_o^2}{2\left|\dfrac{\delta}{\cos\psi}\right|} \cong 1 + \frac{\sigma_o^2}{2}\sin^2\psi\left(1 + \frac{7\sigma_o^2}{16}\right) - \frac{\sigma_o^4}{96} \tag{A-4}$$

Therefore the lowest order in σ_o, Eq. (15) is valid provided the constraint of Eq. (16) is satisfied. The σ_o^4 term is retained in Eq. (16) because it dominates when $\psi = 0$.

Appendix B

Derivation of Extended Range Approximation for σ^2

For $\sigma^2 \ll 8/3$, the Bessel function expressions of Eq. (A-2) simplify to

$$J_0(\sigma) \cong 2J_1\frac{\sigma}{\sigma} \cong J_0(\sigma) - J_2(\sigma) \cong 1 \tag{B-1}$$

Substituting Eq. (B-1) into Eqs. (5) and (7) yields the approximations

$$\sin\lambda \cong -\frac{\sigma^2\delta\cos\psi}{2}, \tag{B-2}$$

$$\sigma^2 \cong \frac{\alpha^2}{\delta^2 + 2\delta\sin\psi\cos\lambda + \cos^2\lambda} \tag{B-3}$$

We want to prove that in the lock region,

$$|\sin\lambda| \leq 1, \tag{B-4}$$

Eq. (B-3) can be accurately approximated by Eq. (19), provided $\sigma^2 \ll \sqrt{2}$.

When $|\delta\cos\psi| \leq \sqrt{2}$, we have

$$|\sin\lambda| = \frac{\sigma^2}{2}|\delta\cos\psi| \leq \frac{\sigma^2}{\sqrt{2}} \ll 1 \tag{B-5}$$

so that $\cos\lambda \cong 1$ and the desired result follows trivially.

Now consider the region $|\delta\cos\psi| \geq \sqrt{2}$. To demonstrate that Eqs. (B-3) and (19) are equivalent, we will prove that the difference between the two denominators is negligible. That is, we want to show that

$$\Delta \equiv |(2\delta\sin\psi + 1) - (2\delta\sin\psi\cos\lambda + \cos^2\lambda)|$$

$$= \sin^2\lambda\left|1 + \frac{2\delta\sin\psi}{1+\cos\lambda}\right| \ll \delta^2 + 2\delta\sin\psi + 1. \tag{B-6}$$

To verify Eq. (B-6), it is sufficient to prove that

$$\sin^2\lambda \ll \min_{1\leq\eta\leq 2}\left[\frac{\delta^2 + 2\delta\sin\psi + 1}{|1 + \eta\delta\sin\psi|}\right] \tag{B-7}$$

since $\lambda\epsilon[-\pi/2, \pi/2]$. But for $\sigma^2 \ll \sqrt{2}$,

$$|\sin\lambda| \ll |\delta\cos\psi|/\sqrt{2}, \tag{B-8}$$

and, using Eq. (B-4), this implies that

$$\sin^2\lambda \ll |\delta\cos\psi|/\sqrt{2}. \tag{B-9}$$

So Eq. (B-7) will follow if we can show that

$$\beta \equiv \min_{1\leq\eta\leq 2}\left[\frac{\sqrt{2}(\delta^2 + 2\delta\sin\psi + 1)}{|\delta\cos\psi||1 + \eta\delta\sin\psi|}\right] \geq 1 \tag{B-10}$$

From Fig. B-1, it is evident that β may be written in the form

$$\beta = \begin{cases} \dfrac{\sqrt{2}(\delta^2 + 2\delta\sin\psi + 1)}{|\delta\cos\psi|(1 + 2\delta\sin\psi)} & ; 0 \leq \delta\sin\psi < \infty \\[2ex] \dfrac{\sqrt{2}(\delta^2 + 2\delta\sin\psi + 1)}{|\delta\cos\psi|(1 + \delta\sin\psi)} & ; -\dfrac{2}{3} \leq \delta\sin\psi \leq 0 \\[2ex] \dfrac{\sqrt{2}(\delta^2 + 2\delta\sin\psi + 1)}{|\delta\cos\psi|(-1 - 2\delta\sin\psi)} & ; -\infty < \delta\sin\psi \leq -\dfrac{2}{3}. \end{cases} \tag{B-11}$$

It can be shown that there is no loss of generality in restricting δ and ψ to the range $-\pi/2 \leq \psi \leq \pi/2$ and $\delta \geq \sqrt{2}/\cos\psi$, since other values of these parameters yield a value of β from this restricted range.

Now, using elementary calculus, for a given value of ψ in this range, β in Eq. (B-11) can be minimized over δ within its range: the results are presented in Table B-1. As shown in the plot of $\beta(\delta_{min})$ vs. ψ in Fig. B-2, β has a minimum value of 1 which occurs at $\psi = 54.74°$ and $\delta = \delta_{min} = 2.45$. This completes our proof.

Table B-1. Variation of minimum value of β with ψ

Range of ψ	$\min\limits_{\delta > \sqrt{2}/\cos\psi} (\beta) = \beta(\delta_{min})$	δ_{min}
$30° < \psi < 90°$	$(\sqrt{2}\sin\psi\cos\psi)^{-1}$	∞
$8.65° < \psi < 30°$	$2\sqrt{2}\,(1-\sin\psi)/\cos\psi$	$(1-2\sin\psi)^{-1}$
$0 < \psi < 8.65°$	$\dfrac{2\cos^2\psi + 4\sqrt{2}\cos\psi\sin\psi + 4}{2\cos^2\psi + 4\sqrt{2}\cos\psi\sin\psi}$	$\sqrt{2}/\cos\psi$
$-11.77° < \psi < 0$	$\dfrac{2\cos^2\psi + 4\sqrt{2}\cos\psi\sin\psi + 4}{2\cos^2\psi + 2\sqrt{2}\cos\psi\sin\psi}$	$\sqrt{2}/\cos\psi$
$-30° < \psi < -11.77°$	$-(\sqrt{2}\sin\psi\cos\psi)^{-1}$	∞
$-47.60° < \psi < -30°$	$2\sqrt{2}\,(1+\sin\psi)/\cos\psi$	$-(1+2\sin\psi)^{-1}$
$-90° < \psi < -47.60°$	$\dfrac{2\cos^2\psi + 4\sqrt{2}\cos\psi\sin\psi + 4}{-2\cos^2\psi - 4\sqrt{2}\cos\psi\sin\psi}$	$\sqrt{2}/\cos\psi$

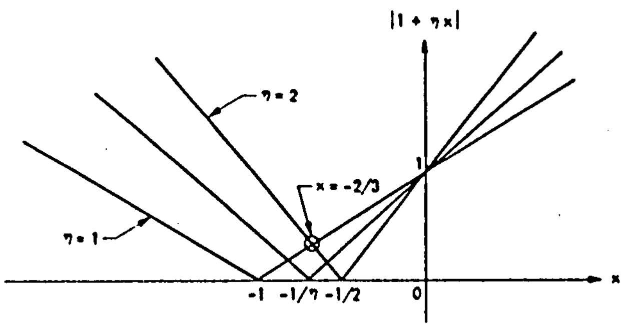

Fig. B-1. Upperbound on | 1 + η x | for 1 ≤ η ≤ 2

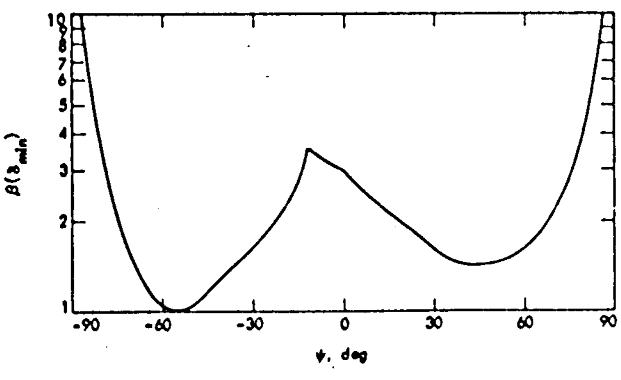

Fig. B-2. Behavior of $\min\limits_{\delta > \sqrt{2}/\cos\psi} (\beta)$ as a function of ψ