# A Voltage-Controlled Optical Radio Frequency-Phase Shifter

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*The problem of stabilizing an optical-fiber link, in which an RF-modulated optical wave is used for frequency standard transmission, is investigated in this article. Higher reliability is expected if RF phase-shifting, phase error detection, etc., can be done directly on the modulated optical signal. In the following pages, a variable optical phase shifter for the above purpose is proposed, and its properties are illustrated.*

## I. Introduction

To transmit time and frequency standards with accuracy of parts in $10^{14}$, a feedback system must be used to stabilize the transmission path against external influence. The existing systems at the Deep Space Network use coaxial lines as the path, and the stabilization problem has been investigated by Lutes (Ref. 1). For stabilization, the signal at the receiving end of the path is transmitted, through the *same* path, back to the transmitting end, where its phase is compared with the standard reference source (a hydrogen maser). Any phase fluctuations at the end of the path are corrected by a voltage controlled phase-shifter inserted in the path. Reflections along the transmission line are highly undesirable for such reflections add signals travelling in opposite directions and vary their phases. This can be avoided if the forward and backward signals are of different frequencies, so that they can easily be isolated with filters. However, in this case, the dispersion of the line and the voltage-controlled phase-shifter $(VC\phi S)$ produce errors in phase correction, as is evident from the following simple calculations: In Fig. 1, the reference signal of $A$ is transmitted down the line, and the phase at the receiving end $B$ is $\beta_1 L + \phi$, where $L$ is the length of the line, $\beta_1$ the

forward propagation constant, $\phi$ the phase shift introduced by the $VC\phi S$. On the return path, the propagation constant is $\beta_2$ which may be different from $\beta_1$. The phase of the received signal $E_3$ is thus $(\beta_1 + \beta_2) L + 2\phi$. The phase of $E_3$ is to be compared with the reference $E_1$. Now, suppose that the length of the line is changed from $L$ to $L + \Delta L$, due to temperature change or any other means. Then, the phase of $E_3$ is changed, and the feedback system will produce a signal to change $\phi$ and compensate for the change in line length. The required change in the $VC\phi S$ is thus $\Delta\phi = - (\beta_1 + \beta_2) \Delta L$. With this change, however, the phase at $B$ is

$$\beta_1 (L + \Delta L) + \phi + \Delta\phi = \frac{\beta_1 - \beta_2}{2} \Delta L + \beta_1 L + \phi$$

and thus is a function of $\Delta L$, i.e., phase at $B$ is *not* stabilized unless $\beta_1 = \beta_2$. One can also see from this calculation why the forward and backward signal *must* propagate through the *same* line and *same* phase shifter in order to achieve phase stabilization at $B$.

In view of the above difficulties, an optical transmission line is proposed using radio frequency modulated optical signals transmitted through optical fibers for frequency and time standard distribution. Line dispersion of the modulated optical wave is virtually zero at all radio frequencies. However, the voltage-controlled phase shift ($VC\phi S$) cannot be performed directly on the optical signal, for a shift in the RF phase (5 MHz signal, say) of a quarter cycle means a change in path length of 16 m and a continuously variable optical delay line covering such a wide range is not available. Thus, one has to perform this operation at radio frequencies, and cannot avoid the dispersion of RF phase-shifters.

In the following paragraphs, a means of performing voltage-controlled phase shift directly on the modulated optical signal is described, and its characteristics are investigated. It is found that with this phase shifter, one can compensate, to a certain extent, any line dispersion, if it exists. Thus, one can use two different optical frequencies, instead of two different radio frequencies for forward and backward signals, even when optical dispersion exists. Shown in Fig. 2 is a proposed system, in which a 5 MHz standard signal is carried forward on $0.85\mu$ and back on $1.06\mu$ optical waves, corresponding to emissions of GaAs semiconductor laser and Nd:YAG laser, respectively.

The advantages of this system are numerous. Separation of forward and backward signals is easy with optical filters. Radio frequency mixers (phase detectors) and frequency dividers used in the present system can be avoided. The usual advantages of fiber optic links such as small size, light weight, insensitivity to electromagnetic interference, and ground loop problems hold for this system also.

## II. The Optical Phase-Shifter

Directly shifting the phase of microwave signals on an optical carrier can be achieved with the scheme shown in Fig. 3. The RF signal on an optical carrier is split with either a voltage-controlled optical directional coupler, or beam splitter with variable attenuations in each path. One path goes through a fixed-length delay line with the *same* dispersion characteristic as the transmission line. The signals are recombined with a beam splitter (adder). Here, one assumes that the delay line length is long compared with the coherence length of the optical carrier, so that addition of the signals takes place *incoherently*. Any optical interference effect is thus disregarded. The phase $\psi$ of the recombined signal is controlled by the splitting coefficient $A$; $A = 0$ corresponds to zero phase shift, and $A = 1$ corresponds to a phase shift of $\beta l$, where $l$

is the delay line length and $\beta$ the propagation constant at that optical frequency. Analytically, one has

$$\psi = \tan^{-1} \frac{A \sin \beta l}{A \cos \beta l + 1 - A} \qquad (1)$$

and the recombined signal is thus

$$1 + p \cos \psi \qquad (2)$$

where

$$p = [(A \sin \beta l)^2 + (A \cos \beta l + 1 - A)^2]^{1/2}. \qquad (3)$$

A plot of $\psi$ and $p$ vs $\beta l$, for different $A$'s, are shown in Figs. 4 and 5, where the average optical power is normalized to 1. One observes from Fig. 4 that $\psi$ is fairly linear in $\beta l$, but not quite. In Fig. 6, $\psi - A\beta l$ is plotted vs. $\beta l$ for different $A$'s, and one can see the deviation from linearity for $\beta l$ near $\pi/2$. For $A = 0, 1/2, 1, \psi$ is exactly linear in $\beta l$.

One would, however, like the linearity of $\psi$ in $\beta l$ to hold for all values of $A$, since linearity is necessary for compensation of the line dispersion. To see this, one goes back to Fig. 1 and assumes that the phase-shifter introduces different phase shifts $\psi_1$ and $\psi_2$ on the forward and backward signal, in proportion to their respective $\beta$'s. Thus

$$\psi_1 = \phi\beta_1$$

$$\psi_2 = \phi\beta_2$$

$$\Delta\psi_1 = \Delta\phi\beta_1 \qquad (4)$$

$$\Delta\psi_2 = \Delta\phi\beta_2$$

To hold the phase of $E_3$ constant, one has

$$-(\beta_1 + \beta_2)\Delta L = \Delta\psi_1 + \Delta\psi_2$$

$$= \Delta\phi(\beta_1 + \beta_2) \qquad (5)$$

Thus,

$$\Delta\phi = -\Delta L$$

and the phase at $B$ is $\beta_1(L + \Delta L) + \psi_1 + \Delta\psi_1 = \beta_1 L + \psi_1$ independent of $\Delta L$, i.e., stabilized regardless of the dispersion.

## III. Compensation of Line Dispersion

One would like to ascertain how much error is introduced due to the slight nonlinearity of $\psi$ vs. $\beta l$. To facilitate the analysis, we first develop an approximate formula for $\psi$ near $A = 1/2$. This is done in the Appendix, giving to first order in $\epsilon$

$$\psi = \frac{\beta l}{2} + 2\epsilon \tan \frac{\beta l}{2} \qquad (6)$$

where

$$\epsilon = A - \frac{1}{2}, \text{ and}$$

$l$ as before is the length of the delay arm of the phase shifter. The tangent term produces the undesirable nonlinearity. The phase shifts introduced to the forward and backward signal are thus, respectively,

$$\psi_1 = \frac{\beta_1 l}{2} + 2\epsilon \tan -\frac{\beta_1 l}{2}$$

$$\qquad (7)$$

$$\psi_2 = \frac{\beta_2 l}{2} + 2\epsilon \tan \frac{\beta_2 l}{2}$$

Assume that initially $A$ is set to $1/2$ (i.e., $\epsilon = 0$). Now, a change in line length $\Delta L$ produces an error signal that changes $A$ and thus $\psi_1$ and $\psi_2$:

$$\Delta\psi_1 = 2\epsilon \tan \left(\frac{\beta_1 l}{2}\right)$$

$$\qquad (8)$$

$$\Delta\psi_2 = 2\epsilon \tan \left(\frac{\beta_2 l}{2}\right)$$

The amount of change in $A$ necessary for phase stabilization is given by Eq. (5):

$$\Delta\psi_1 + \Delta\psi_2 = -\Delta L (\beta_1 + \beta_2)$$

hence giving

$$\epsilon = -\frac{\Delta L}{2} \left( \frac{\beta_1 + \beta_2}{\tan \dfrac{\beta_1 l}{2} + \tan \dfrac{\beta_2 l}{2}} \right) \qquad (9)$$

With this change, the change in phase at the receiving end is

$$\delta = \beta_1 \Delta L + \Delta\psi_1$$

$$= \beta_1 \Delta L + 2\epsilon \tan \frac{\beta_1 l}{2} \qquad (10)$$

$$= \left[ \beta_1 - \frac{\left(\tan \dfrac{\beta_1 l}{2}\right) \Delta L (\beta_1 + \beta_2)}{\tan \dfrac{\beta_1 l}{2} + \tan \dfrac{\beta_2 l}{2}} \right] \Delta L$$

One would of course like $\delta$ to be zero, as would be the case if the tangent functions are replaced by linear functions (i.e., $\phi$-shifter linear in $\beta$), or $\beta_1 = \beta_2$ (i.e., no dispersion).

To proceed, we approximate

$$\tan \frac{\beta l}{2} = \frac{\beta l}{2} + \frac{1}{3} \left(\frac{\beta l}{2}\right)^3$$

For the range of interest, $\beta l < \pi/2$, the above expression is accurate to within five percent. Substitution in Eq. (10) gives

$$\delta = \left(\frac{\beta_1}{12}\right) \left(\beta_1^2 - \frac{\beta_1^3 + \beta_2^3}{\beta_1 + \beta_2}\right) l^2 \Delta L. \qquad (11)$$

Furthermore, dispersion data (Ref. 2)[1] gives the following: at carrier wavelength of $0.85\mu$, $\beta = \beta_1 = 2\pi/3984$ cm for 5 MHz modulating signal, and at $1.06\mu$, $\beta = \beta_2 = 2\pi/3989$

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[1] For multimode fibers far from cutoff, intrinsic material dispersion dominates waveguide dispersion.

cm. Hence, we can write $\beta_2 = \beta_1 + \delta\beta$, where $\delta\beta$ is small. Substitution into Eq. (11) gives, to first order in $\delta\beta$,

$$\delta = \left(\frac{\beta_1}{12}\right)\left(\beta_1^3 - \frac{2\beta_1^3 + 3\beta_1^2\,\delta\beta}{2\beta_1 + \delta\beta}\right)l^2\,\Delta L$$

$$\approx \frac{\beta_1^3}{12}\left[1 - \left(1 + \frac{3}{2}\cdot\frac{\delta\beta}{\beta_1}\right)\left(1 - \frac{\delta\beta}{2\beta_1}\right)\right]l^2\,\Delta L$$

$$\boxed{\delta \approx \frac{\beta_1^2\,l^2}{12}\,\delta\beta\Delta L} \tag{12}$$

As expected, this phase error is small when $\beta l$ is small, for then $\psi$ is more linear in $\beta l$.

**Numerical Estimation:** The present coaxial system uses a 300-m line operating at 5 MHz. Under field conditions, a fluctuation in line length of 30 cm is measured without line stabilization, and is reduced to $\sim 0.02$ cm when stabilized. Thus, the maximum phase error expected in the stabilized system is $\sim 3 \times 10^{-5}$ radians, limiting factors being line dispersion, phase shifter dispersion, and component stability. Now, *assume* that without stabilization, the optical fiber line experiences the same fluctuation of 30 cm, equivalent to $2\pi\,(30/4000)$ rads phase fluctuation. Then, taking $\beta l = 2 \times 2\pi$ $(30/4000)$ rads, $\delta\beta = 2 \times 10^{-6}$ cm$^{-1(2)}$, $\Delta L = 30$ cm, one estimates from Eq. (12) that $\delta \approx 4 \times 10^{-8}$ rads — three orders of magnitude lower than the present value.

**Noise:** What may hinder the system from approaching the above performance is the stability of the components (laser diodes, detectors) and noise. It is known that when laser diodes are coupled to fibers, reflections from fiber end faces cause the lasers to fluctuate (Ref. 3). This noise, the origins of which are not yet well understood, is large by any standard for a communication link. It could be suppressed by inserting an optical isolator between the fiber and the laser, or by anti-reflection coating the fiber end face which, as a bonus, would also increase the coupling efficiency. The stability of the components shall be investigated experimentally and if found unsatisfactory, feedback stabilization of individual components will be applied.

## IV. Conclusion

Higher stability in frequency and time standard distribution is expected by using optical fibers and the above-mentioned optical phase-shifter. The simplicity of an optical system using separate optical wavelengths for forward and backward transmissions should render it more reliable. Further increase in performance can be obtained if we cascade a series of optical phase-shifters each of which has a small tunable range (small $\beta l$), for it was shown that the phase error increases as $(\beta l)^2$. Inherent to this phase-shifter, however, is a loss in average optical power (down 3 dB when the two arms are recombined with a beam splitter) and modulation depth (Fig. 5). This, together with the inevitable loss in optical equipment connections, does not recommend the cascade to go beyond two or three stages. One thus has to optimize the signal-power/phase-error tradeoff.

Further advantage can be made of the optical transmission link by processing signals directly in optics. For example, the phase error detector, as shown in Fig. 2, can be performed with a scheme shown in Fig. 7. The mixing process can be readily done with an optical modulator, thus avoiding RF equipment that is vulnerable to electromagnetic interference. Moreover, automatic gain control can be done with something as simple as an optical attenuator without affecting the RF phase, while it would be difficult at RF.

# Acknowledgment

# References

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3. Hirota, O. and Suematsn, Y., "Noise Properties of Injection Lasers Due to Reflected Waves," *IEEE J. Quantum Electron.,* Vol. QE-15, pg. 142.
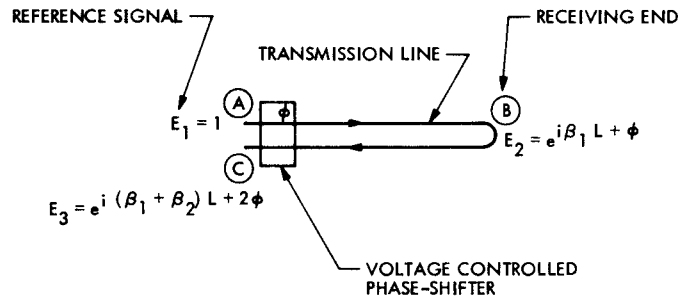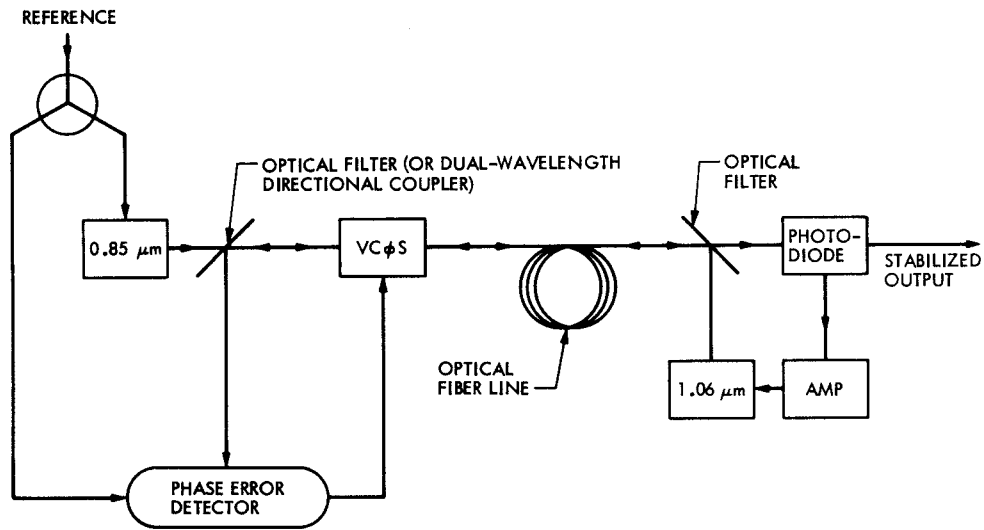
Fig. 1. Simplified diagram of a phase-stabilized transmission line



Fig. 2. Schematic diagram of a stabilized line using dual wavelength two-way transmission
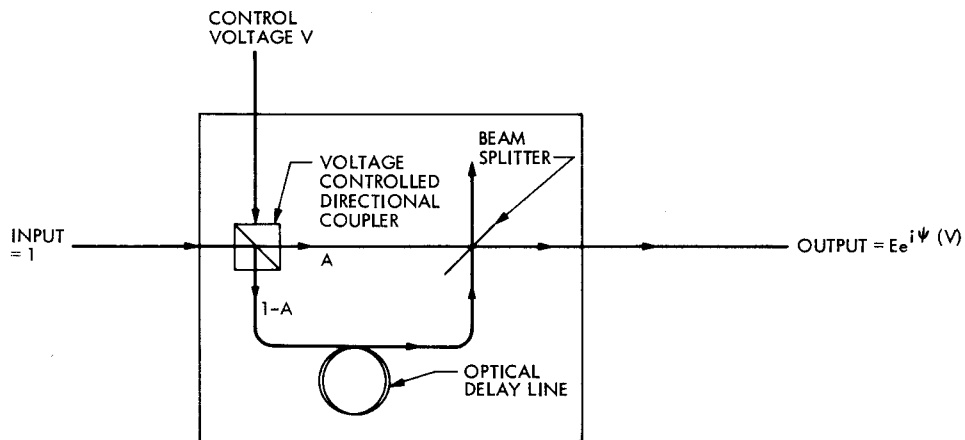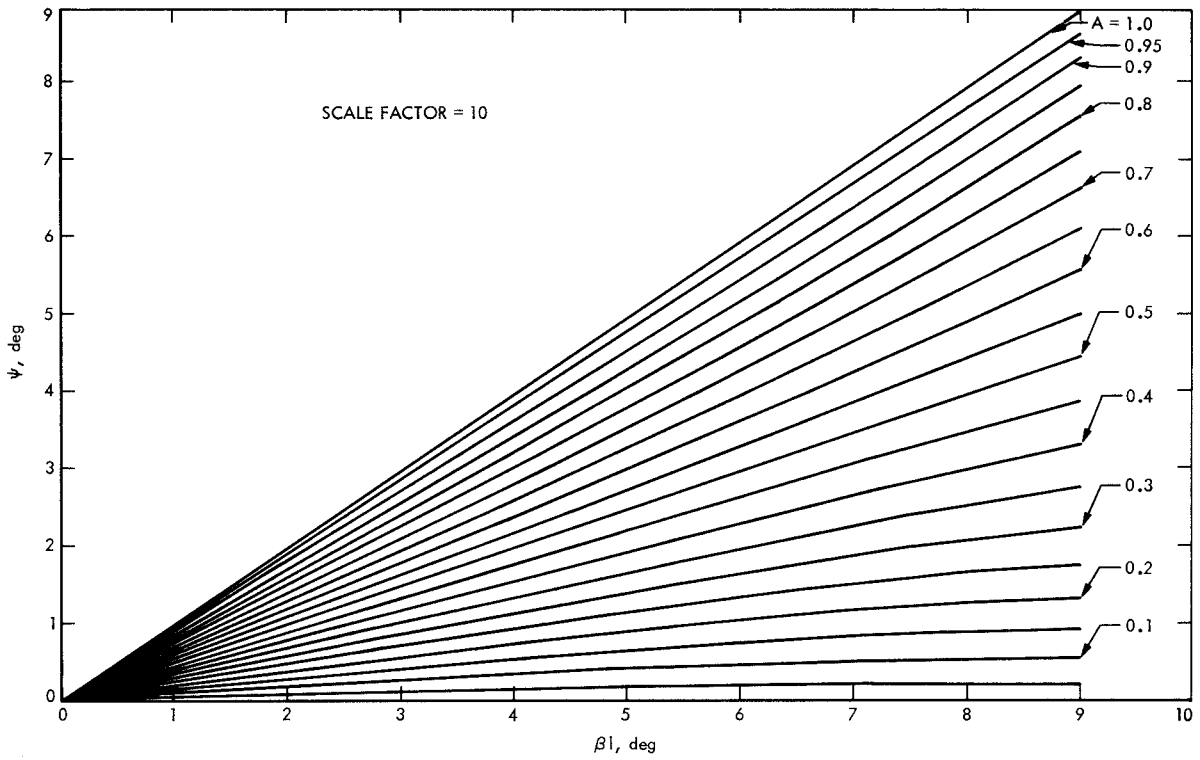


Fig. 3. An optical phase shifter

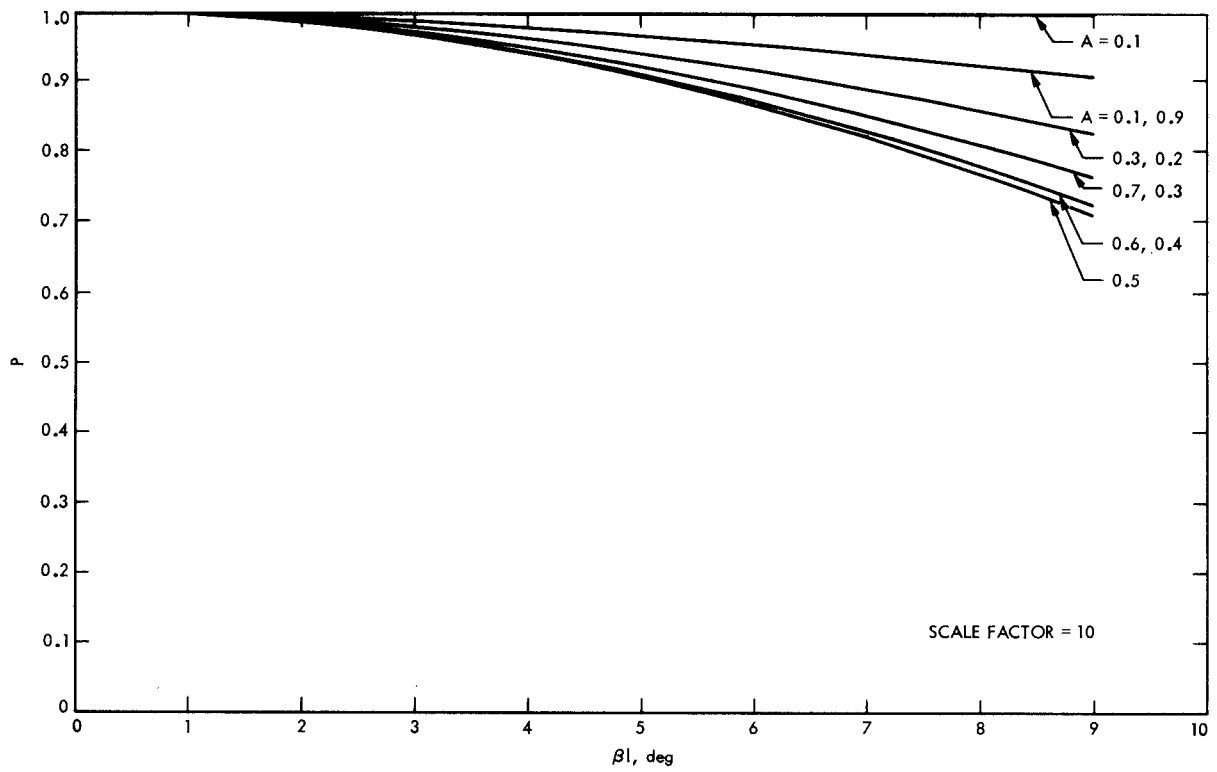**Fig. 4. The RF phase shift ψ vs. βℓ for various splitting coefficients A**



**Fig. 5. The RF output power p vs. βℓ for various splitting coefficients A**
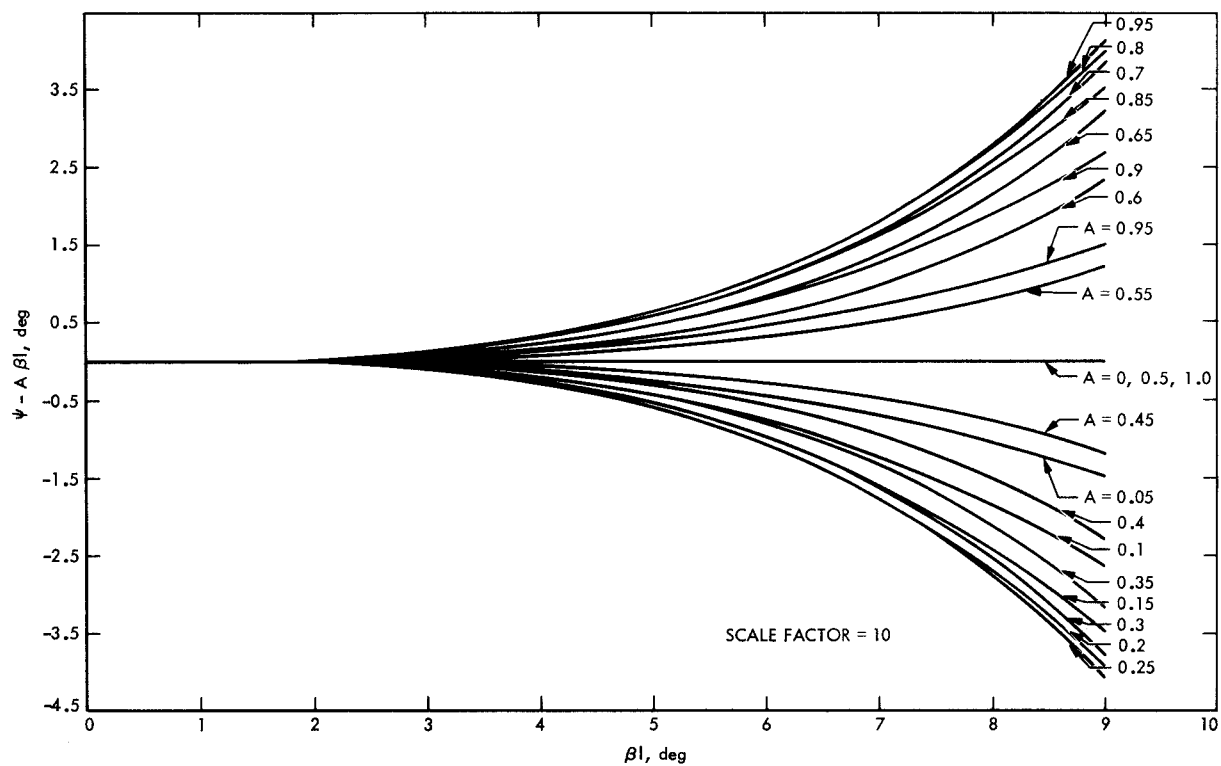
Fig. 6. Deviations from nonlinearity $\psi$-A$\beta\ell$ vs. $\beta\ell$ for various splitting coefficients A
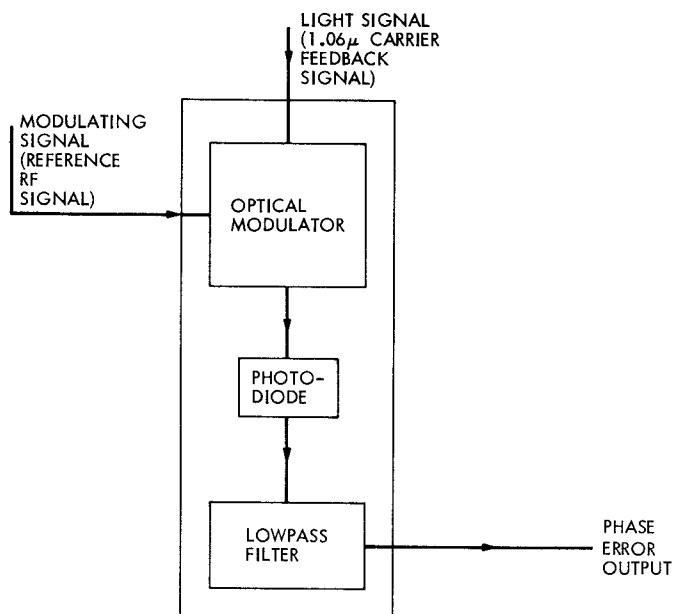


Fig. 7. Phase-error detector

# Appendix

Equation 6 is derived below.

We have

$$\psi = \tan^{-1}\left(\frac{A \sin \beta l}{A \cos \beta l + 1 - A}\right).$$

For $A$ near $1/2$, $A = 1/2 + \epsilon$, so

$$\psi = \tan^{-1}\left(\frac{\sin \beta l + 2\epsilon \sin \beta l}{\cos \beta l + 1 + 2\epsilon (\cos \beta l - 1)}\right)$$

$$= \tan^{-1}\left\{\frac{\sin \beta l}{1 + \cos \beta l}\left[1 + 2\epsilon\left(1 - \frac{\cos \beta l - 1}{\cos \beta l + 1}\right)\right]\right\}$$

$$= \tan^{-1}\left[\left(\tan \frac{\beta L}{2}\right)\left(1 + \frac{4\epsilon}{1 + \cos \beta L}\right)\right].$$

Using the expansion

$$\tan^{-1}(x + \epsilon) = (x + \epsilon) - \frac{(x + \epsilon)^3}{3} + \frac{(x + \epsilon)^5}{5} + \cdots$$

$$= \left(x - \frac{x^3}{3} + \frac{x^5}{5} \ldots\right)$$

$$+ \epsilon(1 - x^2 + x^4 - x^6 + \cdots) + 0(\epsilon^2)$$

$$= \tan^{-1} x + \frac{\epsilon}{1 + x^2} + \cdots$$

one obtains

$$\psi = \frac{\beta l}{2} + \frac{4\epsilon \tan \frac{\beta l}{2}}{(1 + \cos \beta l)} \times \frac{1}{1 + \tan^2 \frac{\beta l}{2}}$$

$$= \frac{\beta l}{2} + 2\epsilon \tan \frac{\beta l}{2}$$

This formula is accurate to within seven percent for $\epsilon$ as large as $\pm 0.25$.