Practical Limitations on Noiseless Optical Channel Capacity

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Even in noiseless optical channels one must take into account the fact that the time resolution available is finite. An optimization scheme under the constraint of a given information rate (in nats/second) and minimum time slot resolution is presented. It is shown that system efficiencies in excess of 10's of nats/photon will be extremely difficult to achieve due to fundamental time resolution limitations.

In a recent paper (Ref. 1) some aspects of the optical channel using an ideal binary quantum detector are considered. In particular, it was found that using a quantized PPM modulation scheme, for example, the channel capacity in nats/photon approaches infinity in the noiseless case (except for the unavoidable self-noise due to the Poisson statistics of the photons themselves). However, the final units that are relevant for a communication link design are nats/second. The purpose of this article is to show that due to physical limitations on the minimum time-resolution one can achieve, for any given capacity in nats/seconds there is an optimum point which maximizes the capacity in terms of nats/photon, thus minimizing the power requirement of the system.

Basically there are three types of time resolution bounds. The first and ultimate lower bound comes from the quantum mechanical uncertainty principle

$$\Delta E \cdot \Delta T \geqslant h \tag{1}$$

If the photon source is a laser, ΔE corresponds to the laser transition linewidth, i.e., $\Delta T \simeq 1/\Delta\nu$ where $\Delta\nu$ is the transition linewidth (largest $\Delta\nu$ observed is about 10^{13}, which corresponds to $\Delta T \sim 0.1$ picoseconds). In any case, it doesn't make much sense to consider for a photon of a certain wavelength ($\lambda = 2\pi\ c/\omega$) time scales that are shorter than the reciprocal of its frequency). Thus, we have the bound

$$\Delta T \gtrsim \frac{1}{\omega} \tag{2}$$

It is true that we actually count photoelectrons, not photons, and the arrival times of the former can be determined with practically arbitrary accuracy. However, time accuracies higher than the characteristic time scales of the physical processes associated with the photons (which are the actual information carriers) are meaningless.

A second, more practical bound comes from technological limitations, either due to the time response of the photon emitter, photodetector, and electronic circuitry or due to computational complexity. The third type of limitation comes from the finite bandwidth of the optical filter:

$$\Delta T \gtrsim \frac{2}{B_{opt}} \tag{3}$$

This limit usually lies between the first two mentioned above.

Now let us apply these limits to the optical communications problem. A model for the channel is shown in Fig. 1. We have a Q-ary alphabet where each letter corresponds to a pulse in a particular time slot out of the possible set of Q slots. Although not necessarily the obvious choice, this quantized PPM scheme has the advantage of being an orthogonal signaling in a channel without power and bandwidth constraints. This is also the scheme suggested in Ref. 2.

In any case it is clear that the best coding schemes call for concentrating the power in time slots which are as short as possible (i.e., performance is improved by increasing the peak to average power ratio). The transition probability is given by

$$\epsilon = e^{-N_s \Delta T} \tag{4}$$

Here N_s is the source intensity (in photons/second) and ΔT is the time slot. The capacity of the channel is

$$C_s = \left(1 - e^{-N_s \Delta T}\right) \ln Q \text{ nats/channel use} \tag{5}$$

In terms of nats/second we obtain

$$C_T = \frac{C_s}{T} = \frac{C_s}{Q\Delta T} \text{ nats/second} \tag{6}$$

or equivalently

$$C_T = \frac{1 - e^{-N_s \Delta T}}{\Delta T} \quad \frac{\ln Q}{Q} \text{ nats/second} \tag{7}$$

In terms of nats/photon we obtain

$$C_{\mathrm{ph}} = \frac{C_s}{\text{number of photons per channel use}} = \frac{C_s}{N_s \Delta T} \tag{8}$$

or equivalently

$$C_{\mathrm{ph}} = \frac{1 - e^{-N_s \Delta T}}{N_s \Delta T} \quad \ln Q \text{ nats/photon} \tag{9}$$

We will now calculate the channel performance in terms of nats/photon under constraints of given minimum time slot (ΔT) and fixed channel throughput rate (in nats/second). In this case $N_s = N_s(Q)$.

In Fig. 2, results of C_{ph} vs. Q are shown for throughput rates of 10^4 and 10^6 nats/second. Values of ΔT correspond to the uncertainty principle $\Delta T = 10^{-15}$ sec ($= 1/\omega$ for $\lambda \simeq 1.8$ μm) and for current technological limitations of $\Delta T \simeq 10^{-9}$ sec. It is clearly seen that each curve has an optimum operating point which minimizes the energy consumption of the system (i.e., a point where C_{ph} is maximum). For example, the $C_T = 10^6$ nats/second point is $Q \simeq 2.10^3$. At this point $C_{\mathrm{ph}} \simeq 6.5$ nats/photon.

Note also that with a huge improvement in ΔT six orders of magnitude (from 10^{-9} seconds to 10^{-15} seconds) we get a much smaller increase (a factor of two to three) in C_{ph}. This shows that pushing the technological limits (i.e., decreasing ΔT) would not gain very much in terms of C_{ph}.

It is important to note that for fixed C_T and ΔT, as we change Q, N_s (transmitter intensity) also changes, as can be seen from Eq. (7). For $Q > Q_{\mathrm{opt}}$, N_s approaches infinity. Furthermore, the optimum operating point occurs for $Q\Delta T \simeq 10/C_T$. This implies a PPM block interval (needed for computation of peak to average power ratio) of about 10 bits at all C_T's.

References

1. Pierce, J. R., "Optical Channels – Practical Limits with Photon Counting," *IEEE Trans. Comm.*, COM-26 (1978), pp. 1819-1821.

2. Gagliardi, R. M. and Karp, S., *Optical Communications*, Wiley, New York, 1976.

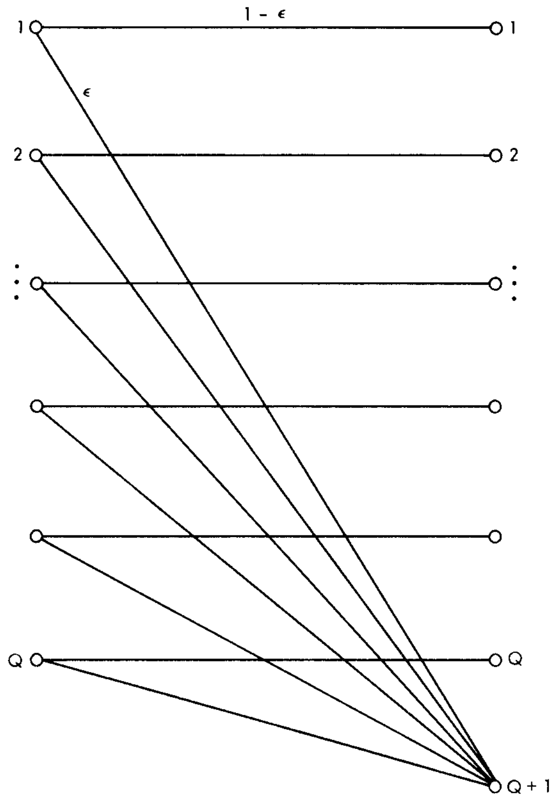

Fig. 1. *Q*-ary PPM channel

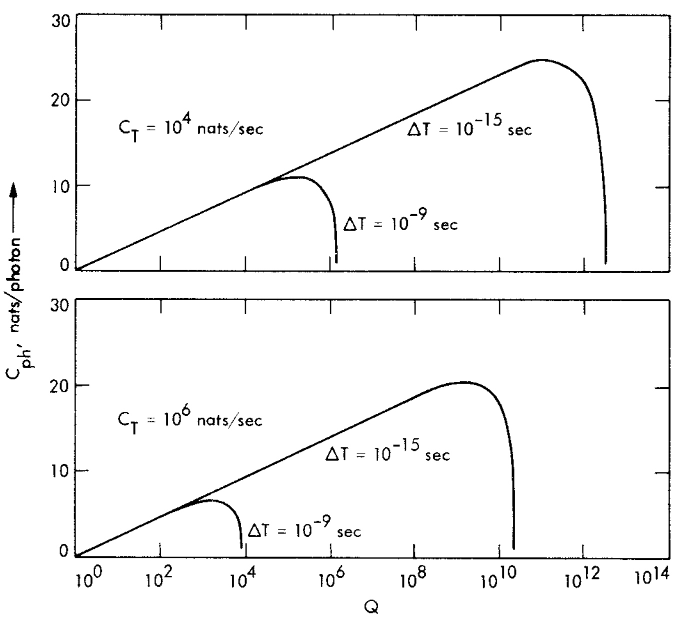

Fig. 2. Capacity of noiseless optical channel