

Optimal Periodic Binary Codes of Lengths 28 to 64

S. Tyler and R. Keston

Quality Assurance — Ground Data Systems Section

Computer searches were performed to find repeated binary phase coded waveforms with optimal periodic autocorrelation functions. The best results for lengths 28 to 64 are given. These codes have extensive applications in radar and communications.

I. Introduction

Repeated binary phase coded waveforms with 100 percent duty factors form an important class of signals utilized extensively in radar and communications (Ref. 1).

Some codes with good periodic autocorrelation functions are known. For lengths of 27 and less, the best codes are fairly well known. For lengths of $2^n - 1$, maximal length P-N codes have optimal autocorrelation functions. However, for most lengths there is no practical algorithm for obtaining the best code (Ref. 2).

This article gives codes of lengths 28 to 64. Some of these codes are "optimal," others merely the best the authors have been able to find so far. These codes often represent a marked improvement over what has been reported previously in the available literature. Codes of these lengths are not merely useful in themselves; they can also be combined to give good codes of longer lengths. For instance, our best result for length 52 was obtained by copying the optimal code of length 13 four times and inverting one of the copies.

For codes of lengths above about 40, it is not practical to search exhaustively for the best code (Ref. 3). Smaller searches must be done. In this study, initial guesses were used and then modified until no further modification produced a superior code. Hopefully, methods for determining initial guesses and

for modifying these will be improved as experience is gained in this field.

A periodic binary code can have several good features. In this study, we look at just two:

- (1) The peak sidelobe in the autocorrelation function is small.
- (2) The sum of the squares of the sidelobes in the autocorrelation function is small.

II. Examples

Consider a code of length 7:

+ + + + - + -

To get the elements of the autocorrelation function:

+ + + + - + - is the original code.

- + + + + - + is the code shifted one position.

- + + + - - - is the arithmetic product for each position.

-1 is the sum of these products; it is the first sidelobe element of the autocorrelation function.

Shifting by 2:

++++--

+ - + + + -

+ - + + + + +3 is the sum.

Shifting by 3:

++++--

+ - + + + -

- + - + - + -1 is the next element of the autocorrelation function.

Shifts by 4, 5, 6 are equivalent to those of 3, 2, and 1.

The first element of the autocorrelation function is the main lobe. It corresponds to shifting by zero positions. The other elements are the sidelobes (the main lobe is not a sidelobe).

Thus the autocorrelation function of (+ + + - + -)

is (7 -1 3 -1 -1 3 -1).

Here P = peak sidelobe magnitude = 3

S = sum of squares of sidelobes = 22

Two simple relationships manifest themselves:

- (1) Each element of the autocorrelation function when taken modulo 4, is equal to the length of the code modulo 4.
- (2) The sum of the elements of the autocorrelation function equals the square of the sum of the elements of the code (Ref. 4).

An "optimal" code: +++--+- would have autocorrelation function

7, -1, -1, -1, -1, -1, -1

where

P = 1

S = 6

For length 8, the "optimal" code is

++++--

with autocorrelation function

8, 0, 0, 0, -4, 0, 0, 0

Here P = 4

S = 16.

Two other results are used in determining the optimality of codes:

- (1) No code of length greater than 4 has P = 0.
- (2) No code has length 1 modulo 4, length greater than 13, and P = 1 (Ref. 5).

III. Results

Table 2 shows the best codes for each lengths 28 to 64. Table 1, showing the results for lengths 3 to 27, is included for completeness.

Length gives the length of the code.

$\Sigma(\text{Sidelobes})^2$ gives the lowest sum of the squares of the sidelobes discovered for any code of that length. When a code which has a lower peak sidelobe is found which has a higher than optimal $\Sigma(\text{Sidelobes})^2$, both codes are given.

When the value for the peak sidelobe or sum of squares is in parentheses, the authors feel that a better but as yet undiscovered code probably exists. When the value is underlined, a better value might exist. In all other cases, the value can be proved to be optimal.

The codes are written in hex notation. The first bit is always a +. For example, the code for 29 is given in hex as 14A7C111. In binary, this would be 0001 0100 1010 0111 1100 0001 0001 0001. By removing the leading zeroes, we get the code:

+ - + - - + - + - - + + + + - - - + - - - + - - - +

In some cases a code from Table 2 was found in an earlier work. In these cases, the reference number is given.

References

1. MacMullen, A., *Radar Antennas, Transmitters, and Receivers*, pp. PC-1-56, Technology Service Corporation, April 1977.
2. Boehmer, A., "Binary Pulse Compression Codes," IEEE Transactions on Information Theory, Volume 13, No. 2, p. 156, April 1967.
3. Lindner, J., "Binary Sequences up to Length 40 with Best Possible Autocorrelation Functions," Electronic Letters, Vol. II, No. 21, p. 507, 16 October 1975.
4. Tyler, S., *Optimum Binary Codes of Lengths 2^n* , Technology Service Corporation Memorandum, 12 July 1978.
5. Turyn, R., *Optimum Codes Study*, Sylvania Electronic Systems Final Report AF19(604)-5473, 29 January 1960.
6. Tausworthe, R., *Correlation Properties of Cyclic Sequences*, JPL Technical Report No. 32-388, 1 July 1963.
7. Tyler, S., *Binary Codes of Lengths 2^n with good periodic correlation properties*, Technology Service Corporation Memorandum TSC-PM-A207-26, 7 August 1978.
8. Bailey, J., and Tyler, S., *Periodic Binary Waveforms with Optimum Autocorrelation Functions* (to be published).

Table 1. Optimal codes of length less than 28

| Length | Peak
Sidelobe | Σ (Sidelobes) ² | Code (hex) |
|--------|------------------|-----------------------------------|------------|
| 3 | 1 | 2 | 4 |
| 4 | 0 | 0 | E |
| 5 | 1 | 4 | 1D |
| 6 | 2 | 20 | 25 |
| 7 | 1 | 6 | 4B |
| 8 | 4 | 16 | CB |
| 9 | 3 | 24 | 1F4 |
| 10 | 2 | 36 | 350 |
| 11 | 1 | 10 | 716 |
| 12 | 4 | 16 | 941 |
| 13 | 1 | 12 | 1F35 |
| 14 | 2 | 52 | 36A3 |
| 15 | 1 | 14 | 647A |
| 16 | 4 | 48 | FAC4 |
| 17 | 3 | 64 | 19A3D |
| 18 | 2 | 68 | 31EDD |
| 19 | 1 | 18 | 7A86C |
| 20 | 4 | 64 | F6E8E |
| 21 | 3 | 52 | 117BCE |
| 22 | 2 | 84 | 3D1231 |
| 23 | 1 | 22 | 6650FA |
| 24 | 4 | 32 | DC20D4 |
| 25 | 3 | 72 | 18B082E |
| 26 | 2 | 100 | 2C1AEB1 |
| 27 | 3 | 74 | 5A3C444 |

Table 2. Best results for codes of length 28 to 64

| Length | Peak
Sidelobe | Σ (Sidelobes) ² | Code (hex) | Ref. |
|--------|------------------|-----------------------------------|------------------|------|
| 28 | 4 | <u>80</u> | B30FDD4 | |
| 29 | 3 | <u>92</u> | 14A7C111 | |
| 30 | 2 | 116 | 33927FAB | (6) |
| 31 | 1 | 30 | 4B3E3750 | (6) |
| 32 | 4 | <u>80</u> | 89445BC1 | (7) |
| 33 | 3 | 64 | 18A5C2401 | |
| 34 | 2 | 132 | 24D1F7112 | (8) |
| 35 | 1 | 34 | 71F721592 | (6) |
| 36 | 4 | 64 | C6859AE80 | |
| 37 | 3 | 84 | 1BD623E316 | |
| 38 | 2 | 148 | 3D69144620 | (8) |
| 39 | (5) | (118) | 7C744B905E | |
| 39 | 3 | (134) | 5CC00AD278 | |
| 40 | 4 | <u>80</u> | 918547E90C | |
| 41 | 3 | 104 | 1F0D19DF14A | (8) |
| 42 | 2 | 164 | 33A970D33F4 | (8) |
| 43 | 1 | 42 | 653BE2E00D6 | (6) |
| 44 | 4 | (144) | A042EA0F334 | |
| 45 | 3 | (124) | 17473C9B1AD0 | |
| 46 | 2 | 180 | 3B9BA0712495 | |
| 47 | 1 | 46 | 7BCAE4D82C20 | (6) |
| 48 | 4 | <u>112</u> | CBF089223A51 | |
| 49 | 3 | (192) | 120AF28D1C5E0 | |
| 50 | 2 | 196 | 2E92B0050EE1C | |
| 51 | (5) | (226) | 60B957CC485B0 | |
| 52 | 4 | <u>192</u> | F9AFCD7E6A0CA | |
| 53 | (5) | (228) | 12030BA906D987 | |
| 53 | 3 | (260) | 196EB81901D769 | (6) |
| 54 | (6) | (276) | 30EA0DB237B100 | |
| 55 | (5) | (230) | 74E705812DC456 | (6) |
| 56 | 4 | (272) | DEC4518357C968 | |
| 57 | (5) | (248) | 166EA046116D4F0 | |
| 58 | (6) | (292) | 2C985A631F53A00 | |
| 59 | 1 | 58 | 5D49DE7C1846D44 | (6) |
| 60 | 4 | <u>224</u> | FA32C756BD9E480 | (6) |
| 61 | (5) | (268) | 18F5981E02FBDBA4 | |
| 61 | 3 | (300) | 1B89F34A052CF91D | (6) |
| 62 | 2 | 244 | 225746DC22583D20 | |
| 63 | 1 | 62 | 4314F4725BB357E0 | (6) |
| 64 | (8) | (352) | A804EA630D727C2C | |
| 64 | 4 | (384) | EC10845E8B3CB0AC | |