Temperature Control Simulation for a Microwave Transmitter Cooling System

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*This article, the first **of** two, describes and analyzes the thermal performance **of** a temperature control system **for** the antenna microwave transmitter (klystron tube). In this article, only the mathematical model is presented along with the details **of a computer** program which is written **for** the system simulation and the performance parameterization. Analytical expressions are presented in Appendixes.*

I. Introduction

The microwave transmitter is one of the vital subsystems in the Deep Space Network Antenna Tracking System. The transmitter (klystron tube) converts the electrical energy to a microwave signal with a conversion efficiency in the order of 40-60 percent. Due to the inherently large heat generation in the transmitter and the sensitive temperature control requirements, a well-designed cooling system is required. The transmitter cooling design is commonly made, based on past engineering experience taken from the working transmitters. A detailed thermal analysis of an optimized system was not necessary. However, due to the increasing complexity of add-on components in the electrical circuit, for better controls and continuous upgrade of the existing system performance, the need for a new cooling system with an optimum design is found inevitable. In order to support this new cooling system design, this work is initiated to give a detailed thermal analysis and to evaluate the thermal performance of the system. A short computer program was written to analyze the transient cooling process. This article, the first of two, includes the system governing equations needed for a detailed transient thermal analysis and a detailed description of the simulation program. The second phase analysis is intended to evaluate the thermal behavior of the new system under varying operating conditions, with different component selection, location and size on the temperature control requirements.

II. System Description

Figure 1 illustrates one proposed design of the new microwave transmitter cooling system. The numbers assigned to the various points in Fig. 1 represent the fluid stations. The system is comprised of (1) a cross flow air-to-liquid heat exchanger (A), (2) a temperature control valve (B) which controls the amount of fluid flowing out from the heat exchanger (station 1) and the bypass of the returning flow (station 3) to deliver the desired fluid temperature (station 4), (3) a filter (C) which removes contaminants resulted from the corrosion in the system; and (4) a pressurized storage tank (D) which performs as a thermal flywheel or damper for all possible thermal fluctuations in the circuit. The storage tank is charged with nitrogen gas for maintaining the desired pressure. An electric resistance heater is provided inside the storage tank which operates only when the control valve (B) fails to supply a fluid temperature above the temperature setpoint of the tank. A pump (E) pumps the fluid from state 9 to 10 and maintains the fluid in the loop at the design pressure. A flow meter (F) monitors the system flow rate at station 11. A purity loop (G) purifies the working fluid of the cooling system from state 31 to state 32. A pressure control valve (H) controls the working pressure at the microwave transmitter station 17. A flow bypass valve (I) regulates the proper flow to the microwave transmitter. The microwave transmitter includes two major assemblies, a water load (J) and the klystron assembly itself (K). The changes in fluid states due to the piping pressure drop and heat losses are given different numbers as in the stations 1-2, 4-5, 6-7, 8-9, 10-11, 12-13, 13-14, 13-31, 15-16, 16-17, 16-26, 17-18, 17-21, 19-20, 22-23, 27-28, 32-33, 34-35, 35-3, and 35-36.

III. Thermal Analysis

The following assumptions were made in the mathematical formulation of the system:

(1) The entire system, excluding the storage tank, is assumed at steady state, with the fluid flowing at a constant rate.

(2) The system is located in an environment with uniform ambient temperature and solar radiation.

(3) Axial conduction heat transfer is neglected from one end of a component to another.

(4) Sky and ambient temperatures are assumed approximately the same for simplicity.

(5) The fluid inside the storage tank is assumed to be thoroughly mixed to give a uniform temperature. The fluid temperature leaving the tank is assumed to be the same as the fluid temperature inside the storage tank.

(6) The walls of the tank and the various pipes are assumed isothermal.

(7) The temperature of nitrogen gas inside the storage tank is assumed to be the same as the fluid temperature, with negligible heat transfer effect.

(8) The storage tank is assumed in the form of a cylinder exposed partially to solar radiation. For simulation purposes, the surface area of the storage tank which is exposed to solar radiation is assumed to be a ratio λ of the total tank surface area.

Using the above assumptions, two systems, an insulated system and a noninsulated system, have been analyzed. Appendixes A and B give the details of the heat balance for the insulated and noninsulated systems. The temperature distribution of any piece of piping, whether it is insulated or not, is characterized by the expression in Eq. (A-25).

$$T_f(x) = T_{f,e} + \left[\frac{E_1 - B_1\,(T_{f,e} - T_a)}{B_1}\right](1 - e^{-C_1 X}) \qquad (1)$$

where B_1 is the overall heat transfer coefficient between the fluid and the ambient air, E_1 is the solar energy absorbed by the pipe, $T_{f,e}$ is the fluid temperature to the pipe entrance, C_1 is the pipe characteristic constant, x is the distance measured from the pipe entrance, and $T_f(x)$ is the fluid temperature at that location.

The general solution for the time history of the storage tank outlet temperature can be expressed as in Appendix B by an equation similar to Eq. (1) as,

$$T_f(t) = T_f(0) + \left[\frac{D_2}{D_1} - T_f(0)\right](1 - e^{-D_1 t}) \qquad (2)$$

where $T_f(t)$ is the fluid temperature leaving the tank after a time internal t, and D_1 and D_2 are the storage tank characteristic constants that depend on the mass flow rate, thermal conductances, and the tank inlet fluid temperature as shown in Appendix B. The value D_2/D_1 represents the final equilibrium temperature of the tank with a constant inlet temperature.

In order to determine the time increment for a stable and convergent solution of the transient analysis, the time constant $1/D_1$ is calculated. When time t equals D_1, the temperature difference $(D_2/D_1) - T_f(0)$ would drop by 36.8 percent from its initial value. A new time increment is calculated for each time interval.

The temperature rise due to the pump dissipating work can be determined by using the thermodynamics relationship for the incompressible flow as

$$\Delta T_f = \frac{W_p}{\dot{m}_f C_p} \qquad (3)$$

where W_p is the actual pump work in the system and $\dot{m}_f$ and C_p are the fluid mass flow rate and the fluid specific heat, respectively. The heat dissipation in the system can be correlated with the temperature rise of the cooling fluid. The general expression for the fluid outlet temperature $T_{f,ex}$ is

$$T_{f,ex} = \frac{Q + \dot{m}_f C_p T_{f,e}}{\dot{m}_f C_p} \qquad (4)$$

where Q is the heat generated by the klystron systems and $T_{f,e}$ is the fluid temperature at the klystron inlet.

Stations 4, 24, 29, and 34 represent the fluid mixing locations. The general expression for the mixed fluid temperature is

$$T_{f,m} = \beta T_{f,1} + (1 - \beta) T_{f,2} \tag{5}$$

where β is the mass ratio of the flow rate in the first stream, whose temperature is $T_{f,1}$, to the total mass flow. $T_{f,2}$ is the fluid temperature of the second stream. The fluid temperature at station 35 represents the return fluid temperature just before the heat exchanger bypass loop. The bypass ratio γ is determined by the temperature control valve and is calculated by iteration at each time step. The final γ value is determined when the system reaches the steady state operating conditions. Due to the insufficient physical information of the heat exchanger, it is assumed that the heat exchanger is of the counter flow type. The fluid temperature leaving the exchanger, $T_{f,ex}$, can be expressed in terms of the entering fluid temperature $T_{f,e}$ as (Ref. 1)

$$T_{f,ex} = T_{f,e} - \frac{(T_{f,e} - T_a)\,(1 - e^p)}{\left(\dfrac{\dot{m}_f C_p}{\dot{m}_a C_a}\right) - e^p} \tag{6}$$

where

$$p = UA\left(\frac{1}{\dot{m}_f C_p} - \frac{1}{\dot{m}_a C_a}\right)$$

UA = the product of the overall heat transfer coefficient and the total surface area given in Ref. 2.

C_a = the specific heat of the air

C_p = the specific heat of the fluid

$\dot{m}_a$ = the mass flow rate of the air

$\dot{m}_f$ = the mass flow rate of the fluid

IV. Computer Program Methodology

A computer program entitled Kystron Cooling Project (KCP) is written using the mathematical model developed in Section III and Appendixes A and B. The program includes a main program and five subroutines. The program starts its computation at the heat exchanger outlet (station 1). Initially, the fluid temperature is assumed 10°F above the ambient condition and all fluid is flowing through the heat exchanger (γ equals to zero). The program then proceeds its computation in the sequence as described in Fig. 1. For each of the pipe loss calculations, the subroutine PIPE is called to compute the fluid exit temperature. The storage tank calculation is handled in the TANK subroutine, and the time constant $(1/D_1)$ will be used for the next time increment. In each of the pipe loss calculations, HTCOE and AHTOE subroutines are called to calculate the heat transfer coefficient of the fluid inside the pipe and the heat transfer coefficient to the ambient air, respectively, for different pipe sizes and lengths. The WRTEMP subroutine prints out the fluid condition at each of the stations indicated in Fig. 1. A temperature sensor is built into the program which compares the calculated temperature at station 18 and the design setpoint of the klystron assembly. The difference of the two temperatures will be adjusted by the temperature control valve at station 2. The valve determines the amount of the fluid flow passing through the heat exchanger which mixes with the returning flow (3) to deliver the new adjusted temperature by iterations. When the design condition of the klystron assembly is met for three consecutive times, the system has reached steady state operating condition, and the time required for the first occurrence is considered to be the system response time.

V. Summary

The computer program has not only analyzed the performance of a cooling system, but it also has provided a helpful tool for the designer in setting design requirements. However, in order to evaluate the performance of a system, detailed information of the system is needed. When this study was initiated, several cooling design concepts were considered. Physical dimensions and system control requirements were not available. The second phase of this study will report the performance and design requirements of a finally selected cooling system.

Definition of Terms

A	area
$B_1 - B_6$	thermal conduction
C_a	air specific heat
C_p	fluid specific heat
$C_1 - C_2$	constants
D	diameter
$D_1 - D_2$	constants
$E_1 - E_4$	energy flux
G	heat capacity
$H_1 - H_7$	heat rate
h	convection heat transfer coefficient
I	solar flux
J	constant
K	thermal conductivity
L	pipe length
M	total fluid mass inside the storage tank
$\dot{m}$	mass flow rate
Q	heat rate
T	temperature
t	time
U	effective heat transfer coefficient
V	velocity
x	distance
Y	thickness
α	absorptivity
ρ	density
μ	viscosity
δ	parameter
ϵ	emissivity
β	mass flow ratio
γ	heat exchanger bypass ratio
λ	ratio of the tank surface area exposed to insulation
γ_1	surface area ratio of tank wall and tank insulation

Subscripts

a	ambient air
e	entrance
ex	exit
f	fluid
I	insulation
	inside
m	mixing fluid
o	outside
ti	tank insulation
tw	tank wall
w	pipe wall
x	location

References

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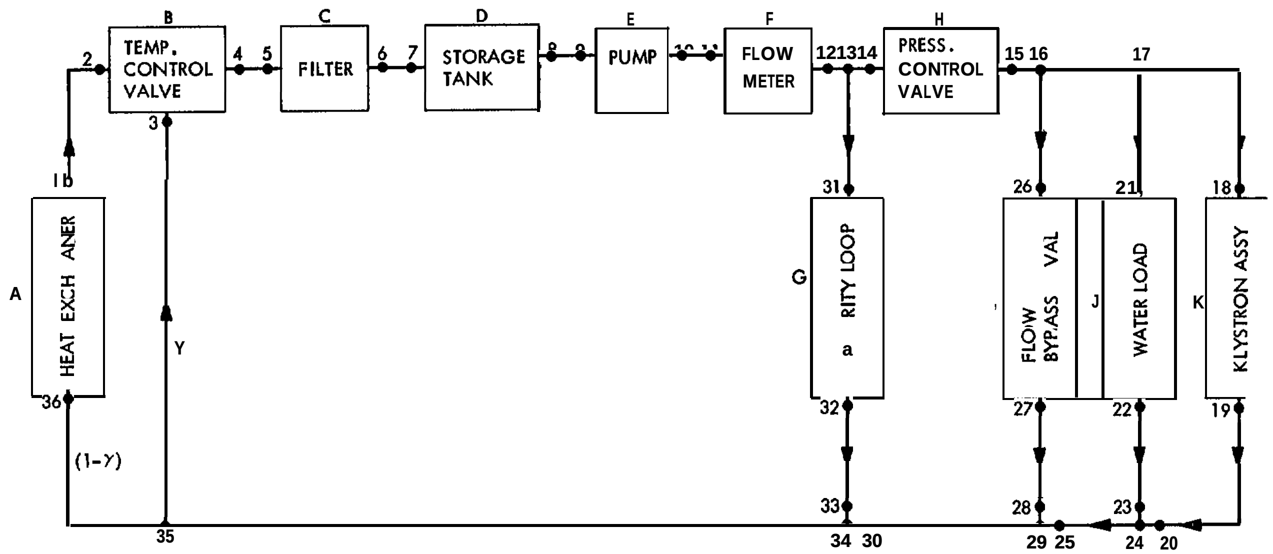

Fig. 1. A candidate cooling loop for the microwave transmitter

Appendix A

Derivation of Temperature Distribution for a Pipe

In this appendix, the derivations of the heat transfer equa-, tions for both an insulated and a noninsulated pipe are made. Following the assumptions made in Section III, Fig. A-l illustrates a segment of an insulated pipe whose length is dx located at a distance x from the inlet fluid section. The differential rates of heat flux are divided as follows:

dQ_1 = total absorbed solar insolation (direct and diffuse) on the outer insulation surface

$$dQ_1 = \alpha_I I D_{I,o}\, dx \quad \text{(A-1)}$$

where α_I is the absorptivity of the pipe insulation

dQ_2 = effective heat transfer between the insulation outer surface and the ambient air which combines the convection and radiation parts

$$dQ_2 = U_I (T_{I,o} - T_a)\, \pi D_{I,o}\, dx \quad \text{(A-2)}$$

where U_I is the effective heat transfer coefficient

dQ_3 = conduction heat transfer from the insulation layer to the pipe wall

$$dQ_3 = \frac{(T_{I,o} - T_{w,i})\, dx}{\left[\dfrac{\ell n\left(\dfrac{D_{I,o}}{D_{w,o}}\right)}{2\pi K_I}\right] + \left[\dfrac{\ell n\left(\dfrac{D_{w,o}}{D_{w,i}}\right)}{2\pi K_w}\right]} \quad \text{(A-3)}$$

dQ_4 = convection heat transfer between the interior pipe wall and the fluid flowing through with a heat transfer coefficient h_f

$$dQ_4 = h_f (T_{w,i} - T_f)\, \pi D_{w,i}\, dx \quad \text{(A-4)}$$

dQ_5 = sensible heat carried by the fluid from section x to section $(x+dx)$

$$dQ_5 = \dot{m}_f C_p \left(\frac{dT_f}{dx}\right) dx \quad \text{(A-5).}$$

Writing the energy balance equations for each of the elementary pipe components at steady state will yield the following equations:

For the insulation surface:

$$dQ_1 - dQ_2 - dQ_3 = 0 \quad \text{(A-6)}$$

For the pipe wall:

$$dQ_3 - dQ_4 = 0 \quad \text{(A-7)}$$

For the fluid:

$$dQ_4 - dQ_5 = 0 \quad \text{(A-8)}$$

The coefficient U_I given in Eq. (A-2), defined as the effective heat transfer coefficient between the insulation outer surface and the ambient air, can be expressed as the summation of a convection heat transfer coefficient h_a and a "linearized" radiation heat transfer one:

$$U_I = h_a + 6\epsilon (T^2_{I,o} + T_a)(T_{I,o} + T_a) \quad \text{(A-9)}$$

The coefficient h_a can be determined by the expression (Ref. 3)

$$h_a = \left(\frac{J K_a}{D_{I,o}}\right) \left(\frac{\rho V_a D_{I,o}}{\mu_a}\right)^n \quad \text{(A-10)}$$

where J and n are constants depending upon the Reynolds number as given in Ref. 3 and V_a is the air velocity.

The coefficient h_f given in Eq. (A-4), defined as the convection heat transfer coefficient between the pipe inner surface and the fluid, is obtained from the Nusselt number Nu_D and the diameter $D_{w,i}$ as follows:

$$h_f = \frac{Nu_D K_f}{D_{w,i}} \quad \text{(A-l 1)}$$

The Nusselt number can be determined by applying any one of the following expressions depending on the flow condition:

(a) If Reynolds number Re_D is greater than 7000, the flow is fully developed turbulent (Ref. 3) and the Nusselt number is given by

$$Nu_D = .023\, Re_D^{.8}\, Pr^{.3} \tag{A-12}$$

where Pr is the Prandtl number

(b) if Re is less than 2100, the flow is laminar and the Nusselt number is obtained from

$$Nu_D = 3.66 + \frac{.067\left(\frac{D_{w,i}}{L}\right) Re_D\, Pr}{1 + .04\left[\left(\frac{D_{w,i}}{L}\right) Re_D\, Pr\right]^{2/3}} \tag{A-13}$$

where L is the length of the pipe

If Reynolds number ranges $2100 < Re_D < 7000$, the flow is considered as a transition flow. Since very few experimental data are available in this flow regime, a straight line interpolation for the Nusselt numbers between the laminar regime and the turbulent regime is assumed as a first approximation

$$Nu_D = Nu_D\,(2100) + (Re_D - 2100)$$

$$\left[\frac{Nu_D\,(7000) - Nu_D\,(2100)}{7000-2100}\right] \tag{A-14}$$

Equations (A-6) through Eq. (A-8) can be rewritten after substituting all the dQ values and dividing by $(D_{I,o}\, dx)$ as follows:

For the insulation surface

$$E_1 - B_1 (T_{I,o} - T_a) - B_2 (T_{I,o} - T_{w,i}) = 0 \tag{A-15}$$

For the pipe wall:

$$B_2 (T_{I,o} - T_{w,i}) - B_3 (T_{w,i} - T_f) = 0 \tag{A-16}$$

For the fluid:

$$B_3 (T_{w,i} - T_f) - G \frac{dT_f}{dx} = 0 \tag{A-17}$$

where

$$B_1 = U_I\, \pi$$

$$B_2 = \frac{1}{\left[\frac{\ln\left(\frac{D_{I,o}}{D_{w,o}}\right)}{2\pi K_I}\right] + \left[\frac{\ln\left(\frac{D_{w,o}}{D_{w,i}}\right)}{2\pi K_w}\right] (D_{I,o})} \tag{A-18}$$

$$B_3 = h_f \pi \left(\frac{D_{w,i}}{D_{I,o}}\right)$$

$$E_1 = \alpha_I I$$

$$G = \frac{\dot{m}_f\, C_p}{D_{I,o}} \tag{A-19}$$

Expressing the temperature $T_{I,o}$ and $T_{w,i}$ in terms of T_a using Eqs. (A-15) and (A-16) yields

$$T_{I,o} = \frac{E_1 + B_1 T_a + B_2 T_{w,i}}{B_4}$$

$$T_{w,i} = \frac{E_2 + B_3 T_f}{B_5} \tag{A-20}$$

where

$$B_4 = B_1 + B_2$$

$$B_5 = B_3 + B_2 - \left(\frac{B_2^2}{B_4}\right)$$

$$E_2 = E_1\left(\frac{B_2}{B_4}\right) + \left(\frac{B_1 B_2}{B_4}\right) T_a \tag{A-21}$$

Substituting $T_{w,i}$ in Eq. (A-17), the differential equation for the fluid in the pipe at any position x from the entrance will be

$$\frac{dT_f}{dx} + C_1 T_f = C_2 \tag{A-22}$$

where

$$C_1 = \left(\frac{B_3}{G}\right)\left(1 - \frac{B_3}{B_5}\right)$$

$$C_2 = \frac{B_3 E_2}{G B_5} \tag{A-23}$$

Note that C_1 is positive since $(B_5 - B_3)$ can be proved to be equal to $B_1 B_2 / B_4$.

The general solution of the differential Eq. (A-22) is

$$T_f(x) = \delta e^{C_1 x} + \frac{C_2}{C_1} \tag{A-24}$$

where δ is an arbitrary constant which is determined by the following boundary condition:

At the pipe inlet section ($X = 0$) the fluid temperature $T_{f,e}$ is assumed given. Substituting in Eq. (A-24), the temperature distribution $T_f(x)$ can be reduced to:

$$T_f(x) = T_{f,e} + \left[\frac{E_1 - B_1 (T_{f,e} - T_a)}{B_1}\right] (1 - e^{-C_1 x}) \tag{A-25}$$

The quantity $[E_1 - B_1 (T_{f,e} - T_a)]$ represents the net energy exchange between the fluid and the ambient air. B_1 is, therefore, the overall heat transfer coefficient as given by Eq. (A-18), and E_1 is the solar energy absorbed by the system as given in Eq. (A-19).

The second term of Eq. (A-25) $(1 - e^{-C_1 x})$ is considered a dimensionless flow factor. C_1 as given by Eq. (A-23) is a function of the heat transfer coefficients (B's) and the mass flow rate ($\dot{m}_f$).

Equation (A-25) can be applied to a noninsulated pipe, as well with the following changes:

(1) The absorptivity α_w of the pipe wall should be used to replace α_I of the insulation in Eq. (A-1).

(2) The outside diameter $D_{w,o}$ of the pipe should replace $D_{I,o}$ of the insulation.

(3) The outer surface temperature $T_{w,o}$ of the pipe should replace $T_{I,o}$ of the insulation.

(4) The heat conduction term B_2 in Eq. (A-18) should be modified as follows:

$$B_2 = \frac{1}{\left[\dfrac{\ell n \left(\dfrac{D_{w,o}}{D_{w,i}}\right)}{2\pi K_w}\right] (D_{w,o})}$$

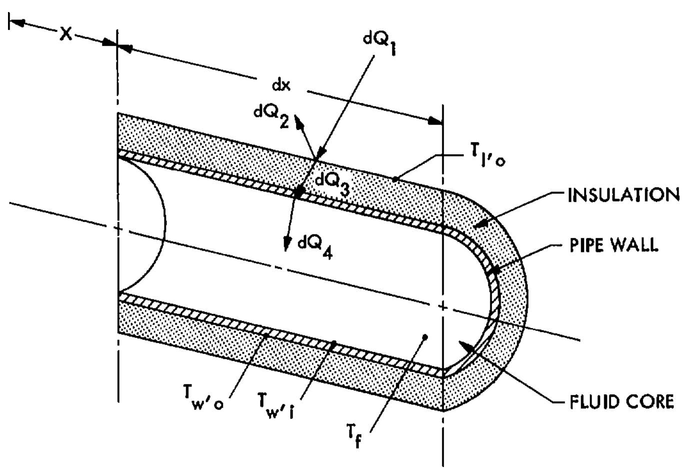

Fig. A-l. Segment of a pipe with thickness dx

Appendix B

Derivation of Temperature Variation for a Storage Tank

In this appendix, the heat transfer equations for an insulated and a noninsulated fluid tank are derived, using the assumptions listed in Section III in the text.

Figure B-l illustrates a cross section of an insulated storage tank with an insulation thickness Y_I. The heat flux is divided as follows:

H_1 = total absorbed solar radiation (direct and diffuse) on an area with a ratio λ of the outer tank surface area.

$$H_1 = \lambda \alpha_{ti} I A_{ti\,o} \tag{B-1}$$

where α_{ti} is the absorptivity of the insulation envelope.

H_2 = effective heat transfer between the insulation outer surface and the ambient air.

$$H_2 = U_{ti} (T_{ti\,o} - T_a) A_{ti,o} \tag{B-2}$$

where U_{ti} is the effective heat transfer coefficient which is determined by equations similar to Eqs. (A-9) and (A-l 0) and $T_{ti,o}$ is the outside tank insulation surface temperature.

H_3 = conduction heat transfer through the insulation layer with thickness Y_{ti}.

$$H_3 = \frac{(T_{ti,o} - T_{tw,i}) A_{tw,o}}{\left(\frac{Y_{ti}}{K_{ti}}\right) + \left(\frac{Y_{tw}}{K_{tw}}\right)} \tag{B-3}$$

H_4 = convection heat transfer from the storage wall to the fluid inside the storage.

$$H_4 = h_{f,T} (T_{tw,i} - T_f) A_{tw,i} \tag{B-4}$$

where $h_{f\,T}$ is the convection heat transfer coefficient between ‘the tank wall and the fluid inside the tank.

H_5 = sensible heat gain by the fluid inside the storage tank.

$$H_5 = M C_p \frac{dT_f}{dt} \tag{B-5}$$

where M is the total fluid mass inside the tank

H_6 = extracted sensible heat gain by the fluid passing through the storage tank.

$$H_6 = \dot{m}_f C_p (T_f - T_{f,e}) \tag{B-6}$$

where $\dot{m}_f$ is the steady mass flow rate in and out of the tank, and $T_{f,e}$ is the fluid temperature at the storage tank entrance. $T_{f,e}$ is assumed to be constant in each time step calculation.

H_7 = heat generated by the auxiliary heater located inside the tank. Three energy balance equations are found by grouping Eqs. (B-l) to (B-6) and H_5 as follows:

For the insulation surface

$$H_1 - H_2 - H_3 = 0 \tag{B-7}$$

For the tank wall

$$H_3 - H_4 = 0 \tag{B-8}$$

For the fluid inside the tank

$$H_5 = H_4 + H_6 + H_7 \tag{B-9}$$

After substituting all the values into Eqs. (B-7) to (B-9) and dividing by $(A_{ti,o})$, the energy balance equations become

For the insulation surface

$$E_1 - B_1 (T_{ti\,o} - T_a) - B_2 (T_{ti\,o} - T_{tw\,i}) \tag{B-10}$$

For the tank wall

$$B2 (T_{ti,o} - T_{tw,i}) - B_3 (T_{tw,i} - T_f) = 0 \tag{B-l 1}$$

For the fluid inside the tank

$$M C_p \frac{dT_f}{dt} = B_3 (T_{tw,i} - T_f) + G (T_f - T_{f,e}) + \frac{H_7}{A_{ti,o}} \tag{B-12}$$

where

$$\left.\begin{aligned} B_1 &= U_{ti} \\ B_2 &= \frac{\gamma_1}{\left(\frac{Y_{ti}}{K_{ti}}\right) + \left(\frac{Y_{tw}}{K_{tw}}\right)} \\ B_3 &= h_{f,T}\,\gamma_1 \end{aligned}\right\} \tag{B-13}$$

$$\left.\begin{aligned} \gamma_1 &= \frac{A_{tw,i}}{A_{ti,o}} \\ E_1 &= \lambda \alpha_{ti} I \\ G &= \frac{\dot{m}_f C_p}{A_{ti,o}} \end{aligned}\right\} \tag{B-14}$$

Using Eqs. (B-10) and (B-11), temperatures $T_{ti,o}$ and $T_{tw,i}$ can be expressed as

$$\left.\begin{aligned} T_{ti,o} &= \frac{E_1 + B_1 T_a + B_2 T_{tw,i}}{B_4} \\ T_{tw,i} &= \frac{E_2 + B_3 T_f}{B_5} \end{aligned}\right\} \tag{B-15}$$

where $B_4 = B_1 + B_2$

$$\left.\begin{aligned} B_5 &= B_2 \left(1 - \frac{B_2}{B_4}\right) + B_3 \\ E_2 &= \left(\frac{B_2}{B_4}\right) E_1 + \left(\frac{B_1}{B_4}\right) B_2 T_a \end{aligned}\right\} \tag{B-16}$$

substituting $T_{tw,i}$ into Eq. (B-12) yields

$$\frac{dT_f}{dt} = D_1 T_f + D_2 \tag{B-17}$$

where

$$\left.\begin{aligned} B_6 &= B_3 \left(\frac{B_3}{B_5} - 1\right) + G \\ D_1 &= \frac{B_6}{M C_p} \\ E_3 &= \left(\frac{B_3}{B_5}\right) E_2 \\ E_4 &= \frac{H_7}{A_{ti,o}} \\ D_2 &= \frac{E_5 - G T_{f,e} + E_4}{M C_p} \end{aligned}\right\} \tag{B-18}$$

The final general solution for this differential equation is

$$T_f(t) = \delta e^{-D_1 t} + \frac{D_2}{D_1} \tag{B-19}$$

where δ is an arbitrary constant determined by the initial condition:

At time equals zero, the fluid temperature inside the tank $T_f(0)$ is given. Substituting in Eq. (B-19) and rearranging terms; the temperature variation $T_f(t)$ is given by

$$T_f(t) = T_f(0) + \left[\frac{D_2}{D_1} - T_f(0)\right] (1 - e^{-D_1 t}) \tag{B-20}$$

The quantity $[(D_2/D_1) - T_f(0)]$ can be expressed as the amplitude of disturbance which depends on the thermal conductances (B's), the heat fluxes (E's), the flow capacitance (G) and the storage tank inlet temperature ($T_{f,e}$). If the mass flow ($\dot{m}_f$) is equal to zero, the quantity would reduce to become the static disturbant amplitude in the stagnation condition. The factor $(1 - e^{-D_1 t})$ is defined as the time decay factor which describes the time history of the storage tank.

Equation (B-20) can be applied to a noninsulated tank with the following changes:

(1) The outside tank area $(A_{tw,o})$ should be used to replace the outside tank insulation area $(A_{ti,o})$ in all appropriate equations.

(2) The tank wall absorptivity (α_{tw}) should replace the tank insulations absorptivity (α_{ti}) in Eq. (B-l).

(3) The outer tank wall temperature $(T_{tw,o})$ should replace the outer tank insulation's temperature $(T_{ti,o})$ in Eqs. (B-2) and (B-3).

(4) B_2 in Eq. (B-13) should be modified as follows:

$$B_2 = \frac{\lambda}{\left(\dfrac{Y_{tw}}{K_{tw}}\right)} \tag{B-21}$$

(5) The effective heat transfer coefficient U should be recalculated using the outside diameter of the tank instead of the outside diameter of the insulation.

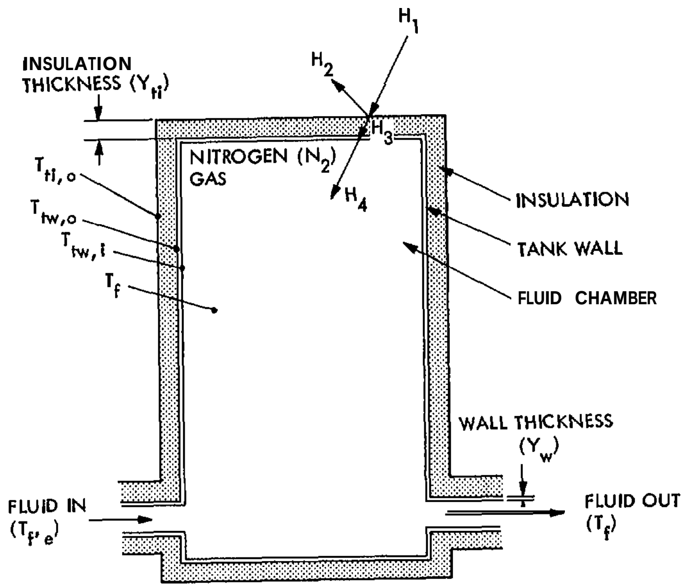

Fig. B-1. Cross section of a storage tank