Telemetry Degradation due to a CW RFI Induced Carrier Tracking Error for the Block IV Receiving System With Maximum Likelihood Convolutional Decoding

M. K. Sue
Telecommunications Systems Section

This article is a part of a continuing effort to develop models to characterize the behavior of the Deep-Space Network (DSN) Receiving System in the presence of a radio frequency inteference (RFI), and presents a simple method to evaluate the telemetry degradation due to the presence of a CW RFI near the carrier frequency for the DSN Block IV Receiving System using the Maximum Likelihood Convolutional Decoding Assembly (MCD). Analytical and experimental results are presented.

I. Introduction

This study is part of a continuing effort to study the adverse effects that a radio frequency interference may have on the Deep-Space Network Receiving System. Depending on the frequency and power level, an RFI can have various effects on a receiver, such as saturating receiver components, generating harmonics, and degrading the tracking loop performance. For the Block IV receiver, saturation is the dominating effect when the interference frequency is far away from the carrier frequency and degradation on the carrier tracking loop becomes a significant when the interference frequency is close. The receiver saturation and carrier loop degradation due to CW RFI have been studied and documented (Refs. 1 and 2). The purpose of this study is to evaluate the degradation on the Telemetry System due to a noisy carrier reference caused by a CW RFI with frequency close to the desired carrier frequency, say within 1 kHz.

It is well known that the Communication System employed for deep-space communication is a phase coherent system that requires phase synchronization at the receiver. Synchronization at the receiver is achieved by tracking the carrier phase with a phase-locked loop. The presence of a CW RFI will produce a phase error in the carrier tracking loop that will in turn cause an apparent reduction of the strength of the received signal and consequently an increase in bit error rate. It is the purpose of this article to evaluate this effect using an analytical model previously developed for the prediction of the carrier loop performance in the presence of a CW RFI (Ref. 2). There may be other effects such as Subcarrier Demodulation Assembly (SDA) and Symbol Synchronization Assembly (SSA) degradation, which may further degrade telemetry performance; these effects will not be discussed.

II. Analysis

It is well known that the probability of bit error of a maximum likelihood convolutional decoder with perfect carrier reference is a function of the ratio of signal energy per bit to noise spectral density (Ref. 3), i.e.,

$$P_e = f(\rho) \qquad (1)$$

where ρ is the ratio of signal energy per bit to noise spectral density.

Exact analytical expression for P_e is not attainable for phase coherent demodulation and Maximum Likelihood Convolution Decoding (MCD). However, for the MCD implemented in the DSN, the probability of bit error has been approximated by the following equation (Ref. 4):

$$P_e \approx A \exp{(B\rho)} \qquad (2)$$

where $A = 85.7501$ and $B = -5.7230$ for $K = 7$, rate ½ convolutional code with $Q = 3$. It is noted that Eq. (2) assumes perfect carrier demodulation.

When a CW RFI is present, it creates an imperfect carrier reference that will degrade the telemetry system performance. If we let $\phi(t)$ denote the phase error of the carrier tracking loop and assume that noise contribution to $\phi(t)$ is negligible, then, in the presence of a CW RFI, we have from Ref. 2:

$$\phi(t) = \lambda + m \sin{(\Delta\omega t + \nu)} \qquad (3)$$

where λ is the static phase error (SPE), m is the modulation index, $\Delta\omega$ is the RFI offset frequency in radians per second and ν is the phase angle. Both λ and m are functions of $\Delta\omega$ and the interference-to-signal ratio (ISR) and they are related through the following set of equations (Refs. 5, 6, and 7):

$$\sin{\lambda} = \frac{m^2 \delta \cos{\psi}}{2J_0(m)} \qquad (4)$$

$$\sin{(\lambda - \nu)} = \frac{-m^2 \delta \cos{\psi}}{2R_e J_1(m)} \qquad (5)$$

$$R_e^2 = \left[\frac{m\delta \sin{\psi} + 2J_1(m)\cos{\lambda}}{J_0(m) - J_2(m)}\right]^2$$

$$+ \left[\frac{m^2 \delta \cos{\psi}}{2J_1(m)}\right]^2 \qquad (6)$$

$$\delta = \frac{\Delta\omega}{\alpha_L K_0 |F(j\Delta\omega)|} \qquad (7)$$

where K_0 is the open loop gain, α_L is the limiter suppression factor, $F(j\omega)$ is the loop filter transfer function, ψ is the phase angle of $F(j\Delta\omega)$, $|F(j\Delta\omega)|$ is the amplitude of $F(j\Delta\omega)$, $J_i(\cdot)$ is the Bessel Function of ith order and R_e is the effective interference to signal amplitude ratio at the limiter output.

Formula to compute α_L (in the presence of RFI) and R_e can be found in Ref. 2.

It is noted that $\phi(t)$ is sinusoidal with a frequency equal to the RFI offset frequency. If we assume that the RFI offset frequency is much lower than the telemetry bit rate, then $\phi(t)$ is essentially constant over the length of most decoder error. Under this condition, the conditional probability of bit error for a given phase error can be expressed as (Refs. 3 and 4):

$$P_e(\phi) = A \exp{(B\rho \cos^2{\phi(t)})} \qquad (8)$$

Since we have assumed that $\phi(t)$ is deterministic, we can obtain an average probability of bit error by taking a time average of $P_e(\phi)$ over one period of $\phi(t)$. If we let P_R denote the average probability of bit error in the presence of RFI, then we have from Eq. (8) the following:

$$P_R = \langle P_e(\phi)\rangle = \frac{1}{T}\int_0^T A \exp{(B\rho \cos^2{\phi(t)})}\, dt \qquad (9)$$

where $T = 2\pi/\Delta\omega$ and $\langle\cdot\rangle$ denotes the time average.

It is noted that the approach taken here to derive the average probability of bit error is based on the assumption that the phase error varies slowly compared to the data rate. This approach is often referred to as the "high-rate model."

Usually, a figure of merit used to determine telemetry performance degradation due to tracking error is the equivalent loss of signal energy-to-noise ratio, which is often referred to as "radio loss." Alternatively, we can define a parameter, γ, as the ratio of the probability of bit error under the influence of a CW RFI to that with no RFI and use this ratio as a measure of the RFI effects on the Telemetry System. Recalling that the probability of bit error with no RFI is given by Eq. (2), we can obtain γ by dividing Eq. (9) by Eq. (2). After simplification, γ becomes:

$$\gamma = \frac{1}{T}\exp{\left(\frac{-B\rho}{2}\right)}\int_0^T \exp{\left[\frac{B\rho}{2}\cos{(2\lambda + 2m\sin{(\Delta\omega t)})}\right]}\, dt$$

$$(10)$$

The parameter γ is the bit error multiplication factor. For a given bit error rate in the absence of RFI, the bit error rate under the influence of a RFI is simply equal to the product of the bit error rate with no RFI and the multiplication factor.

It is noted that the foregoing analysis is based primarily on two assumptions: (1) the data rate is much higher than the RFI offset frequency, and (2) other effects such as SDA, SSA and noise contribution to the carrier tracking error are negligible. These assumptions, particularly the high-rate assumption, will probably place some restrictions on the applications of the model. It is therefore necessary to examine possible restrictions that may result from any of these assumptions. The assumption that the noise contribution to the phase error is negligible has been proven to be a satisfactory assumption with reasonably accurate results even at the recommended minimum signal level (10-dB carrier margin), as indicated in Ref. 2. Hence no restriction results from this assumption. The assumptions that the data rate is much higher than the offset frequency and that the SDA and SSA effects are negligible require that the offset frequency be much smaller than the data rate and the subcarrier frequency. This restricts the range of the offset frequency for which the model in Eq. (10) is valid. This restriction fortunately does not limit the usefulness of this model because the power level required for an RFI to produce significant degradation on the carrier loop at large offset frequency will, in general, be so strong that receiver saturation may become dominant.

It is further noted that the expression for γ given by Eq. (10) does not account for signal inversion effects caused by the phase error, $\phi(t)$. When $|\phi(t)| \leqslant \pi/2$, the effect is a reduction of the received signal strength. When $|\phi(t)|$ is larger than $\pi/2$ and less than or equal to π, the effect is a reduction in signal strength and inversion of the received signal. For the type of RFI considered, $|\phi(t)|$ does not exceed $\pi/2$ until the interference power is approximately within 1 dB of the power level required for an interference to cause the receiver to drop out of lock. For the cases considered with γ ranging from 1 to 10, $|\phi(t)|$ is always less than $\pi/2$; hence signal inversion effects can be ignored. For those who are interested in signal inversion effects, expressions are provided in Appendix A for the calculation of γ and the probability of bit error.

III. Numerical Results

The parameter γ has been evaluated using results of Eq. (2) for the DSN Block IV receiver with maximum likelihood convolutional decoding for three RFI offset frequencies, i.e., 10, 100, and 1000 Hz. The receiver is assumed to be in the typical operational configuration, i.e., 2-kHz predetection noise bandwidth, wide mode with $2BLO$ equal to 10 Hz for S-band and 30 Hz for X-band, where $2BLO$ is the threshold loop noise bandwidth. Three operating points corresponding to a bit error rate of 10^{-3}, 10^{-4}, and 10^{-5} have been chosen. The resulting bit error rate multiplication factor has been plotted as a function of interference-to-signal power ratio with RFI offset frequency as a parameter (Figs. 1 through 6). Based

on these curves, it is observed that γ is relatively insensitive to the operating points.

It is noted that the phase jitter used to generate the curves for the 10-Hz case is twice the theoretical jitter obtained from Ref. 2. The factor of 2 is necessary to account for the inaccuracy of the phase-locked loop model. As it was pointed out, the predicted phase jitter is about half of the measured phase jitter for small offset frequency and small interference-to-signal ratio (Ref. 2).

IV. Experimental Verification

Experimental verification was performed in TDL using the Block IV receiver, Block III SDA, and a Viterbi decoder with $K = 7$, $R = 1/2$ and $Q = 3$. Comparison of experimental and theoretical results are shown in Figs. 4 and 5. For the cases compared, good agreement between the measured and predicted values is observed for the 10-Hz offset and the 100-Hz offset. For the 1000-Hz offset, experimental results indicate that the TDL system is 4 to 6 dB more sensitive to RFI than predicted. Judging from the simplifying assumptions made in the analysis and measurement accuracy, this discrepancy is not unexpected.

V. Conclusion

A method to evaluate the telemetry degradation due to the presence of a CW RFI near the carrier frequency has been presented for the DSN Block IV Receiving System using Maximum Likelihood Decoding. This method is based on two assumptions: (1) the data rate is much higher than the RFI offset frequency, and (2) other effects such as SSA, SDA and noise effects on the phase locked loop are negligible. These assumptions are valid for most deep-space applications with small offset frequency. As the RFI offset frequency becomes large, the accuracy of this model becomes poor. Experimental results obtained from TDL indicate that reasonably accurate prediction is obtainable even at an offset frequency of 1000 Hz for a system with a data rate of 40 kbits/s and a subcarrier frequency of 370 kHz.

Based on experimental and analytical results, it can be concluded that degradation on the Telemetry System is not as severe as that on the carrier loop for the type of RFI considered. The protection criteria proposed for the carrier loop will provide sufficient protection for the Telemetry System with a maximum increase in BER by a factor less than 2. Therefore, a separate protection criterion is not needed to protect the Telemetry System for the type of RFI considered. A curve of protection criteria proposed for the carrier loop is shown in Fig. 9 for reference.

Acknowledgment

References

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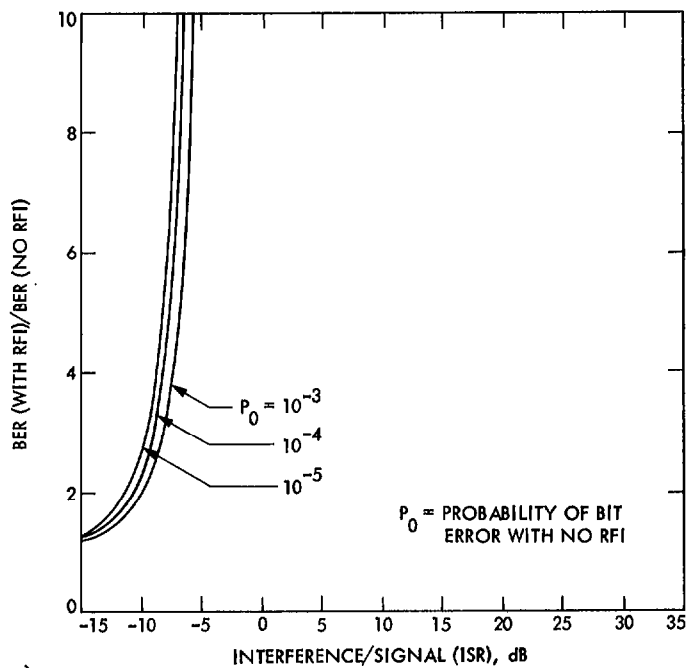
Fig. 1. TLM degradation vs ISR for 10-Hz frequency
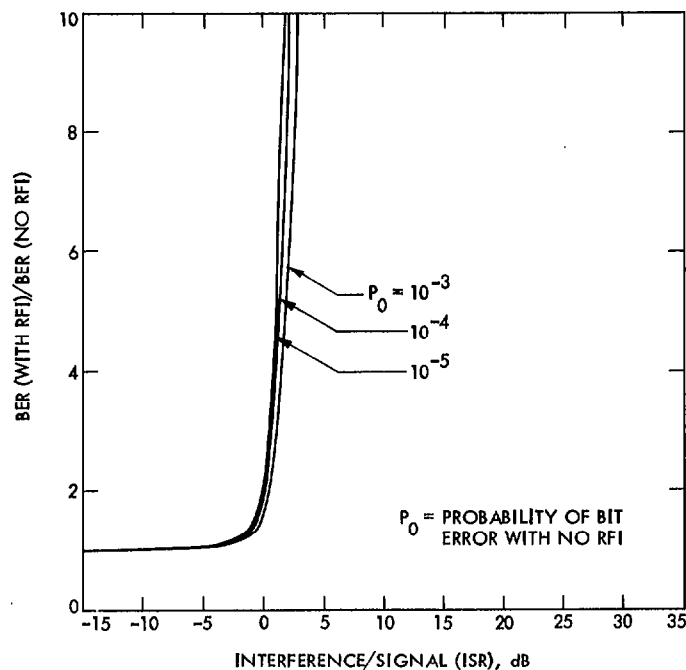offset (wide/2 kHz/10Hz)

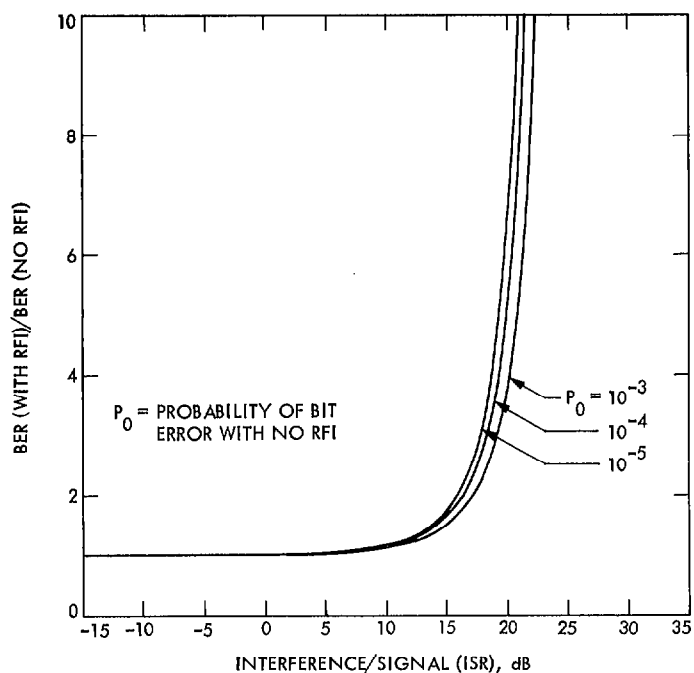Fig. 2. TLM degradation vs ISR for 100-Hz frequency
offset (wide/2 kHz/10Hz)

Fig. 3. TLM degradation vs ISR for 1000-Hz frequency
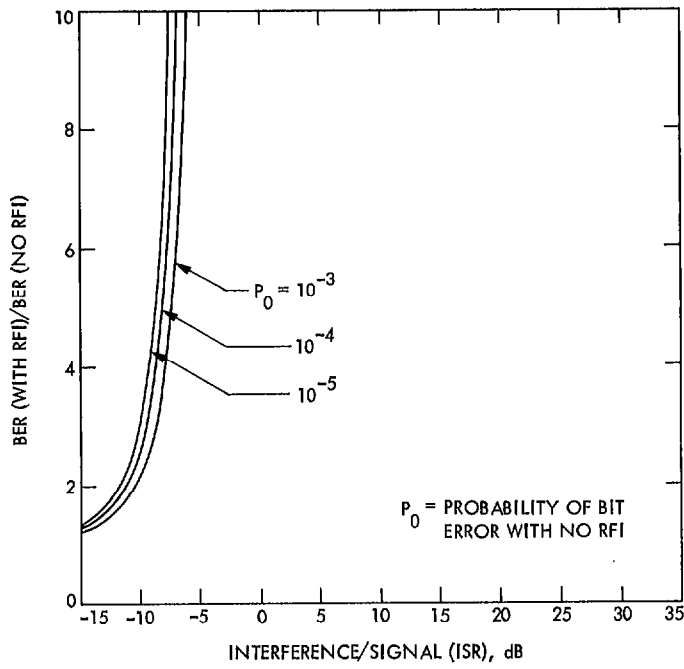offset (wide/2 kHz/10 Hz)

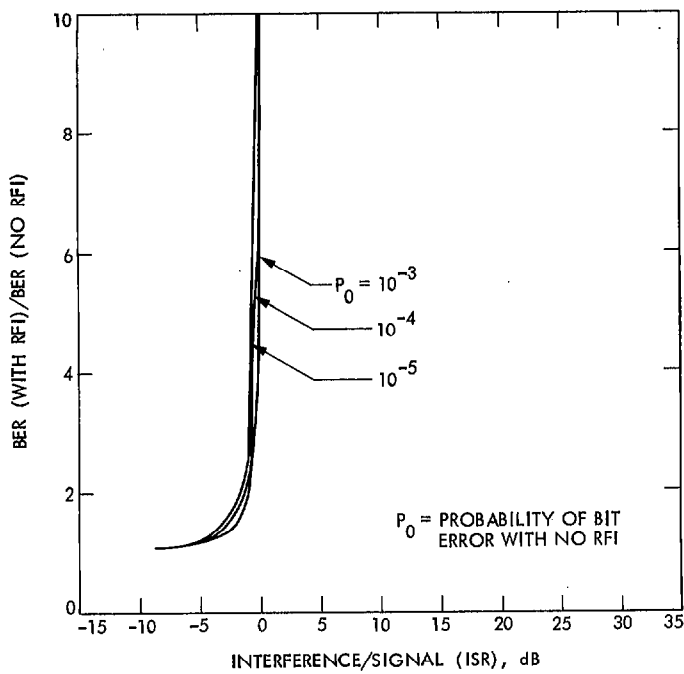**Fig. 4. TLM degradation vs ISR for 10-Hz frequency offset (wide/2 kHz/30 Hz)**

Fig. 6. TLM degradation vs ISR for 1000-Hz frequency offset (wide/2 kHz/30 Hz)
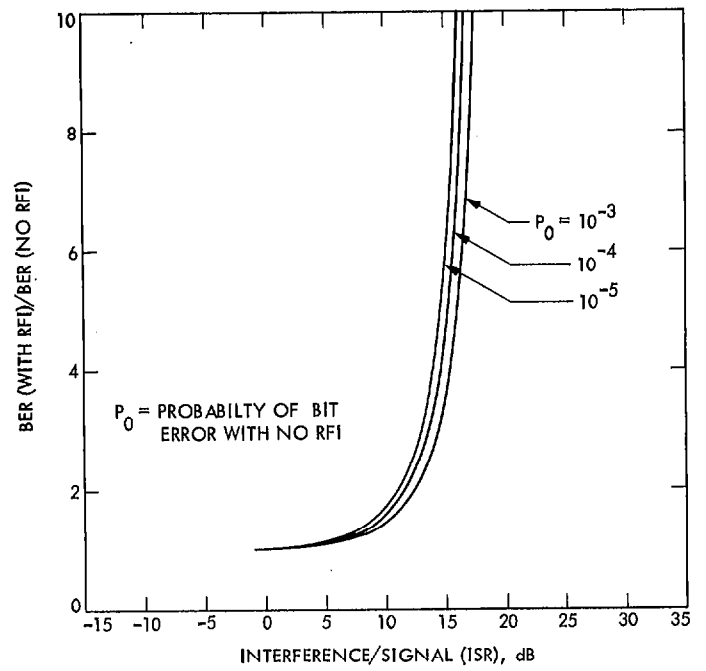

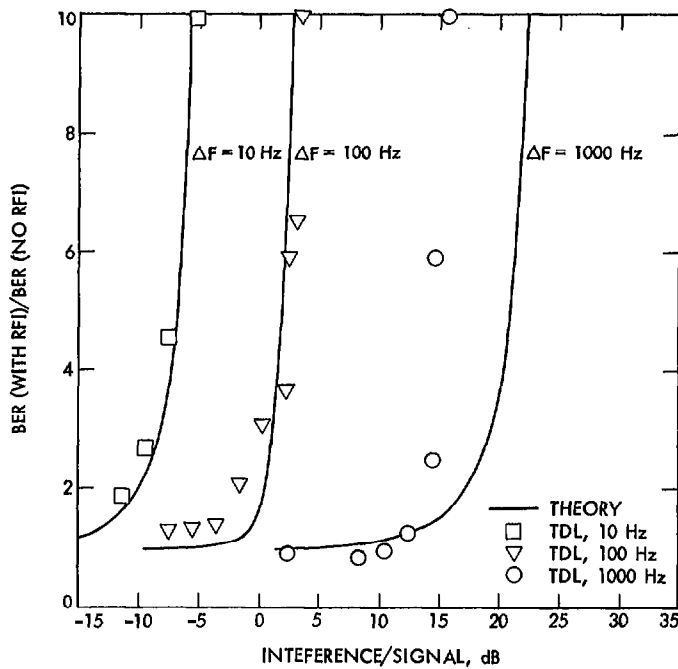**Fig. 5. TLM degradation vs ISR for 100-Hz frequency offset (wide/2 kHz/30 Hz)**

Fig. 7. Comparison of experimental and analytical
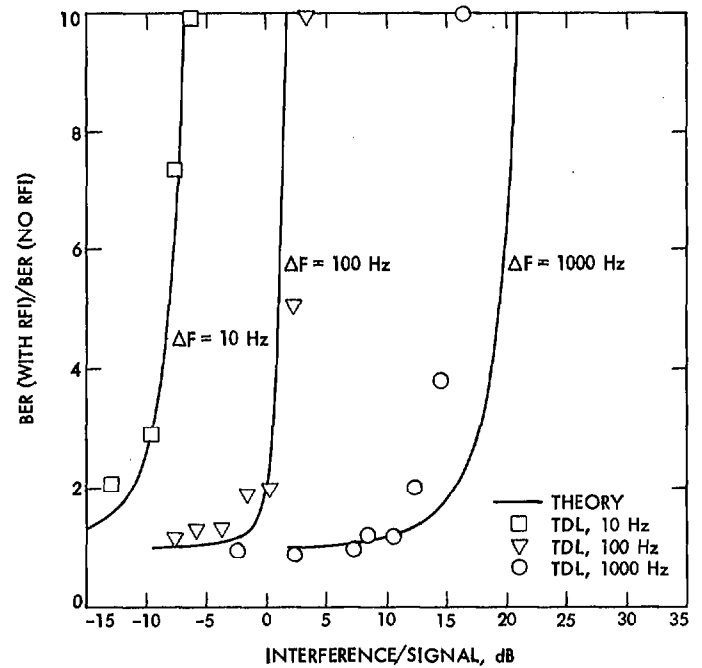results for BER (no RFI) = 10^{-3}

Fig. 8. Comparison of experimental and analytical
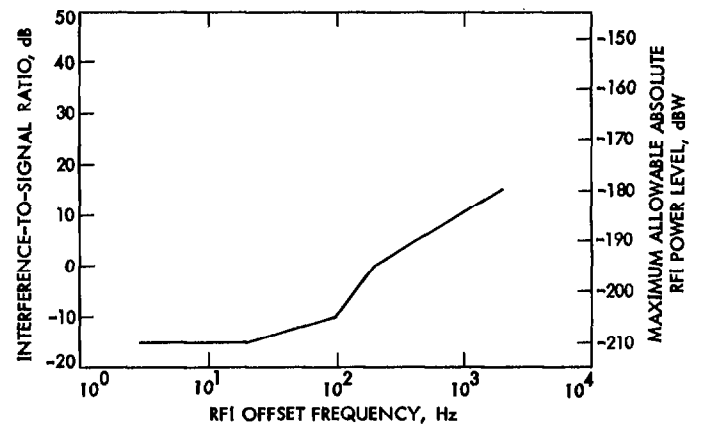results for BER (no RFI) = 10^{-5}

Fig. 9. Recommended protection criteria for CW RFI for Block IV
Receiver (absolute power level based on 10−db carrier
margin and −205 dBW noise power in a 10-Hz bandwidth)
(from Ref. 2)

Appendix A

Effects of Signal Inversion

The probability of bit error for a phase coherent system with Maximum Likelihood Convolutional Decoding is a function of the signal energy per bit to noise spectral density ratio, ρ. In the presence of a phase error, $\phi(t)$, the effect is a reduction of the received signal strength when $|\phi(t)| \leqslant \pi/2$. When $\pi/2 < |\phi(t)| \leqslant \pi$, the effect is a reduction of signal strength and an inversion of the received signal. If we assume that the code is transparent and let P_1 and P_2 denote the probability of bit error for $|\phi(t)| \leqslant \pi/2$ and $\pi/2 < |\phi(t)| \leqslant \pi$ respectively, then from Eq. (2) we have

$$P_1(\phi) = A \exp (B\rho \cos^2 \phi(t)) \quad \text{for } |\phi(t)| \leqslant \pi/2 \tag{A-1}$$

$$P_2(\phi) = 1 - P_1(\phi) = 1 - A \exp (B\rho \cos^2 \phi(t))$$

$$\text{for } \pi/2 < |\phi(t)| \leqslant \pi \tag{A-2}$$

and the conditional probability of bit error, $P_e(\phi)$, is

$$P_e(\phi) = \begin{cases} P_1(\phi) & |\phi(t)| \leqslant \pi/2 \\ P_2(\phi) & \pi/2 < |\phi(t)| \leqslant \pi \end{cases} \tag{A-3}$$

In the presence of a CW RFI, the phase error process under a strong signal condition can be approximated by a deterministic time function as follows:

$$\phi(t) = \lambda + m \sin (\Delta\omega t + \nu) \tag{A-4}$$

where $\Delta\omega$ is the RFI offset frequency in rad/s and ν is a phase constant. For the purpose of this analysis, ν can be assumed to be zero with no loss of generality. Since $\phi(t)$ is periodic, we can obtain the average probability of bit error, P_R, by averaging the conditional probability of bit error, $P_e(\phi)$, over one period of $\phi(t)$, i.e.,

$$P_R = \frac{1}{T} \int_0^T P_e(\phi)\, dt \tag{A-5}$$

where T is the period of $\phi(t)$, i.e., $T = 2\pi/\Delta\omega$.

To evaluate Eq. (A-5), it is necessary to determine the time periods during which $\pi/2 < |\phi(t)| \leqslant \pi$. Noting that the maximum value of $|\lambda|$ is $\pi/2$ and that m cannot be larger than $\pi/2$ in order for a practical receiver to remain in lock, we conclude that there is at most one such time period during which $\pi/2 < |\phi(t)| \leqslant \pi$. If we let t_1 and t_2 denote the end points of this time period, then from Eq. 4 we have:

$$t_1 = \left(\frac{1}{\Delta\omega}\right) \sin^{-1}\left(\frac{\frac{\pi}{2} - \lambda}{m}\right) \tag{A-6}$$

$$t_2 = \frac{\pi}{\Delta\omega} - t_1 \tag{A-7}$$

Having determined t_1 and t_2, we can now proceed to evaluate Eq. (A-5).

$$P_R = \frac{1}{T}\left[\int_0^{t_1} P_1(\phi)\, dt + \int_{t_1}^{t_2} P_2(\phi)\, dt + \int_{t_2}^T P_1(\phi)\, dt\right] \tag{A-8}$$

Substituting Eqs. (A-1) and (A-2) into (A-8) and rearranging terms, we have for P_R the following equation:

$$P_R = \frac{1}{T} \int_0^T A \exp [B\rho \cos^2 \phi(t)]\, dt$$

$$- \frac{2}{T} \int_{t_1}^{t_2} A \exp [B\rho \cos^2 \phi(t)]\, dt \tag{A-9}$$

$$+ \frac{t_2 - t_1}{T}$$

where t_1 and t_2 are given by Eqs. (A-6) and (A-7). The corresponding bit error multiplication factor becomes

$$\gamma = \frac{1}{T} \exp\left[-\frac{B\rho}{2}\right] \left[\int_0^T \exp\left[\frac{B\rho}{2} \cos 2\phi(t)\right] dt \right.$$

$$-2 \int_{t_1}^{t_2} \exp\left(\frac{B\rho}{2} \cos 2\phi(t)\right) dt$$

$$\left. + \frac{t_2 - t_1}{A} \exp\left(-\frac{B\rho}{2}\right)\right] \qquad \text{(A-10)}$$