End-to-End Quality Measure for Transmission of Compressed Imagery Over a Noisy Coded Channel

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For the transmission of imagery at high data rates over large distances with limited power and system gain, it is usually necessary to compress the data before transmitting it over a noisy channel that uses channel coding to reduce the effect of noise-introduced errors. Both compression and channel noise introduce distortion into the imagery. In order to design a communication link that provides adequate quality of received images, it is necessary first to define some suitable distortion measure that accounts for both these kinds of distortion and then to perform various tradeoffs to arrive at system parameter values that will provide a sufficiently low level of received image distortion. This article uses the overall mean-square error as the distortion measure and describes how to perform these tradeoffs.

I. Introduction

For deep space missions to distant planets, the space loss is so high that it is usually necessary to compress the imaging data before transmission in order to meet the data rate requirements with the necessarily limited power levels and antenna gains normally available. Data compression or source coding introduces distortion into the received imagery, and so do errors on the noisy communication channel, even though their effect is reduced by channel coding.

In order to design the communication link, i.e., to choose the appropriate values of the several parameters that can be varied (within limits), we first need a reasonable measure of the distortion introduced into the imagery. This article defines one such measure—the overall mean-square error (MSE)—and describes how to perform the required parameter tradeoffs. The techniques used here apply to any concatenated channel coding scheme (Ref. 1) which uses an interleaver to interleave the 8-bit symbols of the outer code. As a specific example, the coding scheme is assumed to be a Reed-Solomon outer code with 223 information symbols and 32 parity symbols, and a convolutional inner code with rate 1/2. Two interleaving schemes, denoted A and B in Ref. 2, are considered. The source coding scheme is assumed to be Rice's RM2 (Ref. 3).

Traditionally, the channel bit-error-probability (BEP) has been used at JPL as a measure of acceptability of the overall image communication system. For PCM-coded (uncompressed) images, the JPL rule-of-thumb is that a BEP of 10^{-3} to 5×10^{-3} provides acceptable image quality. However, since there are very few images, simulated or otherwise, that include the effects of both channel errors and data compression (source coding) by some algorithm like the Rice algorithms (Refs. 3 and 4), it is not clear what value of BEP can be equated to "acceptable image quality" for compressed imagery. Some simulations of this kind have been done, but only for a compression ratio r (= uncompressed data rate/compressed data rate) of 2 and the BARC algorithm (Ref. 4), whereas

from link analysis it appears that higher values of r and the RM2 algorithm will be needed for missions to the outer planets under certain conditions.

Although it is true that the concatenated channel error rate drops off as the bit-energy-to-noise ratio increases, this increase requires an increased transmitting antenna diameter D_t or higher compression ratio. Beyond a certain point (see Ref. 5 or Eq. A-1 in the Appendix) increasing D_t does not help because pointing losses increase rapidly with D_t. Thus D_t is usually no more than 5 m (unless electronic pointing techniques can be used). The alternative of increasing r causes increased image distortion and there is a theoretical bound on how much r can be increased while keeping the distortion below a certain level, no matter how the distortion is defined and no matter what channel coding or data compression scheme is used. Therefore, it is necessary to perform an analysis that takes both channel and source coding errors into account.

A very large number of simulations of pictures compressed by the RM2 algorithm in the presence of channel errors would be needed before reliable subjective results can be obtained, and such simulations are not currently available and are obviously time-consuming. So we have obtained, instead, an estimate of overall mean-square error (MSE) in RM2-compressed images transmitted over the concatenated code channel. Much as the MSE and other objective measures of picture quality are looked down upon, the alternative is too time-consuming, and the authors of most papers describing a new compression scheme calculate the MSE or something related, in addition to performing a few simulations. See, for example, papers describing new compression schemes such as Blosser, et al. (Ref. 6), or statements about the popularity of MSE in articles dealing with image quality (Refs. 7 and 8); similarly, in the chapters describing various coding schemes, Pratt (Ref. 9) discusses MSE to a great extent. Besides, the correlation between subjective measures and MSE is of the order of 0.7 to 0.8 (Ref. 7), which is quite high, although not high enough to make MSE, in general, a very reliable measure.

Combining MSE calculations with link analysis, one can select the overall communication system parameter values.

II. The System

Figure 1 shows the system block diagram.

The output of the source encoder is a stream of bits consisting of a continuously variable number of bits per pixel encoded. This stream is divided into 8-bit 'symbols' and a number k of these (which we will take to be 223) are encoded

into an n-symbol 'codeword' by the outer encoder (n is taken to be 255). A block of I (taken to be 16) codewords, called a 'code block' or CB is interleaved in the interleaver, and the output, again regarded as a stream of bits, is encoded by the rate 1/2 inner encoder. This is then modulated and transmitted to the receiving end, where the procedure is reversed.

For convenience, we assume that the imaging camera produces pictures consisting of 800 lines per picture, 800 pixels per line and 8 bits per pixel. The camera rate can then be defined as R_c = the number of pictures per hour, or R_d = bits/sec. The compression ratio r is defined as

$$r = \frac{8 \text{ bits/pixel}}{\text{average number of bits/pixel at source encoder output}}$$

We consider cases where $r \leq 16$. The quantities R_s, R_b, R_o, R_d are data rates (bits/sec) at the channel output, the inner decoder output, the outer decoder output, and the camera output respectively. They are related by:

$$R_d = r \times R_o, R_o = \frac{223}{255} R_b, R_b = \frac{1}{2} R_s$$

E_s, E_b, E_o are the received energy per bit measured at the channel output, the inner decoder output, and the outer decoder output. They are related by:

$$E_o = \frac{255}{223} E_b, E_b = 2E_s$$

P_b is the probability of a bit error on the inner channel, while P_o is the probability of a codeword error in the overall channel.

III. Picture Quality

The picture quality measure defined here will be calculated in several steps. In Section III.A.1, below, the effects of a channel error, i.e., the basic equations describing the effect of an error in a codeword output by the outer decoder, will be considered. The procedure followed by Rice (Ref. 2) will be used, but slightly more refined calculations will be given. In Section III.A.2, these results are used to obtain a mean-square error, $MSE^{(c)}$, due to channel errors. Section III.B deals with the $MSE^{(s)}$ due to source coding distortion. In Section III.C, the $MSE^{(s)}$ due to source coding distortion is combined with the results of Section III.A to get an end-to-end picture quality measure, namely, the overall (normalized) $MSE^{(t)}$.

A. Effect of Channel Errors on Picture Quality

1. Basic equations. The only property of the RM2 algorithm that we will use in this and the next subsection, is that RM2 divides each picture into 64×64 pixel sub-pictures called "source blocks" (SB), encodes each of these independently, and serially outputs the pixels of each encoded SB, scanned row-wise. Some sort of synchronizing information is also assumed, which enables the start and finish of each encoded source block to be clearly recognized.

RM2 generates an encoded version of each SB by transforming the SB as a whole, so that a single bit error in the SB can conceivably spoil the whole SB. This may be a somewhat conservative assumption. We also assume that the beginning of a channel codeword need not be synchronized with the beginning of an SB, so that there is always a possibility that a single codeword error affects more than one SB, whatever the relative lengths of the codeword and SB. We assume that each SB spanned by an erroneous codeword is "lost" or unusable. This is reasonable if the erroneous bits in an erroneous codeword are distributed uniformly through the codeword, so that each SB spanned, with high probability gets at least one bad bit.

We now calculate the average number of SBs lost per codeword error.

Whenever a codeword error occurs, it affects only 223 consecutive information symbols. In interleaver scheme A (Ref. 2), these information symbols also correspond to contiguous portions of the picture encoded. However, interleaver B (Ref. 2) forms each codeword by choosing every 16th symbol output by the source coder as successive information symbols in the codeword. Because of this, a single codeword error can span a whole information code block (CB). Thus, the two cases (A and B) need to be treated separately.

a. Interleaver A. Suppose codeword No. 2 is in error, then the symbols labelled 224, 225, ... 446 may be in error. Hence only one or two SBs are lost by a single codeword error for $r \leqslant 16$. The number of lost SBs per codeword error depends on the relative location of the SBs and the codeword. Figure 2(A) illustrates the situation for $r = 4$. One SB is lost in case A-1 and two in case A-2.

The number of bits per source block, b_r, after source coding with compression ratio r, is

$$b_r = \frac{8}{r} \times 64 \times 64 \quad \text{bits per compressed SB}$$

Let S_A be the number of bits affected by a codeword error, and let δ_r be the probability that one compressed source block

with compression ratio r is lost by a codeword error. Then

$$S_A = 223 \times 8 = 1784 = \text{(bits affected by a codeword error)}$$

$$\delta_r = \frac{b_r + 1 - S_A}{b_r}$$

Hence the average number of lost source blocks per codeword error when using interleaver type A and compression ratio r, A_r is

$$A_r = 1 \times \delta_r + 2 \times (1 - \delta_r) = 2 - \frac{b_r + 1 - S_A}{b_r}$$

b. Interleaver B. Suppose codeword No. 2 is in error. Then the symbols labelled 2,18,34, ... 3554, (in which consecutive symbols 1,2, ... represent the successive elements in an SB scanned row-wise) may be in error. Let S_B be the range in number of bits affected by a codeword error when using interleaver type B. Also let C_r be the smallest number of compressed source blocks which are affected by a codeword error, and γ_r be the probability that C_r source blocks are lost by a codeword error.

$$S_B = 3553 \times 8 = 28424 \text{ (bits affected by a codeword error)}$$

$$C_r = \lceil S_B / b_r \rceil$$

$$\gamma_r = \frac{C_r \times b_r + 1 - S_B}{b_r}$$

where $\lceil x \rceil$ is the smallest integer greater than or equal to x. Figure 2(B) shows these situations ($b_4 = 8192$ bits, $C_4 = 4$). The case B-1, where four source blocks are lost, happens with probability γ_4. The only alternative case (B-2), where five source blocks are lost, happens with probability $(1 - \gamma_4)$. Hence, in general, the average number of lost source blocks per codeword error when using interleaver type B and compression ratio r, B_r is

$$B_r = C_r \times \gamma_r + (1 + C_r) \times (1 - \gamma_r) = 1 + C_r$$

$$- \frac{C_r \times b_r + 1 - S_B}{b_r}$$

The values of A_r and B_r are shown in Table 1 for values of r between 2 and 16. These numbers are refined versions of those calculated by Rice (Ref. 2). For $r = 1$, i.e., the no compression case,

$$A_1 = B_1 = \frac{223 \times 8}{64 \times 64 \times 8}$$

since no source block structure is used.

2. Average normalized mean-square error $\text{MSE}^{(c)}/\sigma^2$ due to channel errors.

We now use the basic results obtained in the previous section to calculate the MSE contribution of channel errors.

a. Average fraction of unusable part of a picture. Let N_p be the average number of pictures transmitted between codeword errors. Then

N_p = Avg. # of pictures per codeword error

= (# of pictures per SB) $\times$ (# of SBs per codeword)

$\times$ (# of codewords per codeword error)

$$= \frac{1}{157} \times \frac{223}{\frac{1}{r} \times 64 \times 64} \times \frac{1}{P_o} = \frac{1}{P_o} \times r \times \frac{223}{64 \times 64 \times 157}$$

where

# of source blocks per picture

$$= \left\lceil \frac{800 \times 800 \text{ pixels per picture}}{64 \times 64 \text{ pixels per SB}} \right\rceil = 157$$

and

# of compressed source blocks per codeword

$$= \frac{223 \times 8 \text{ bits per codeword}}{64 \times 64 \times \frac{8}{r} \text{ bits per SB}} = \frac{223}{\frac{1}{r} \times 64 \times 64}$$

Let's assume that the probability of more than one codeword error for each bad picture is very small, and hence ignored. Then N_p is equivalent to the average number of

pictures transmitted per bad picture, and A_r (or B_r) is equivalent to the average number of lost source blocks in one bad picture when using compression ratio r and interleaver type A (or B). Here a bad picture means the picture is corrupted by channel errors, i.e., contains lost SBs.

The quality factor due to channel errors, Q_c, is defined as the average usable fraction of a picture. Then in terms of the user's choice parameter β (which we will soon explain) and with the terms already defined, Q_c can be expressed as:

$$Q_c = \left(\frac{\text{\# of good pictures}}{\text{total \# of pictures}} \right)$$

$$+ \beta \times \left(\frac{\text{\# of bad pictures}}{\text{total \# of pictures}} \right)$$

$$\times \left(\frac{\text{\# of good SB's in a bad picture}}{\text{total \# of SB's in a bad picture}} \right)$$

where

$$\frac{\text{\# of bad pictures}}{\text{total \# of pictures}} = \frac{1}{N_p} = 1 - \frac{\text{\# of good pictures}}{\text{total \# of pictures}}$$

and

$$\frac{\text{\# of good SB's in a bad picture}}{\text{total \# of SB's in a bad picture}} = \frac{157 - A_r \text{ (or } B_r)}{157}$$

Therefore,

$$Q_c = \left(1 - \frac{1}{N_p} \right) + \beta \times \frac{1}{N_p} \times \left(1 - \frac{A_r \text{ (or } B_r)}{157} \right)$$

$$= 1 - P_o \times \frac{1}{r} \times \frac{64 \times 64}{223} \left\{ (1 - \beta) \times 157 + \beta \times A_r \text{ (or } B_r) \right\}$$

The β is a user's choice parameter taking on values between 0 and 1. Here $\beta = 0$ means the user throws away a bad picture however good some portions of it may be. This may not be reasonable since the user always wants to maximize his information return; $\beta = 1$ means the user utilizes all of the good portion of a bad picture. But this may also be unreasonable because of edge effects of the lost source blocks and because when $r = 1$ (no compression case), it is usually hard to tell which part of a bad picture is really bad. We will choose $\beta = 0.99$ for later calculations.

We then define $Q_{\bar{c}} = 1 - Q_c$, which is the average unusable fraction of a picture. In Fig. 3, $1/Q_{\bar{c}}$ in dB (at $P_o = 10^{-4}$) is plotted for various values of r and β. It is noticeable that for $r > 1$, regardless of the value of β, the quality improves ($Q_{\bar{c}}$ decreases) when we compress more (r increases). This is due to the fact that the total number of pictures increases with r although the total number of lost SBs per codeword error remains almost the same or very slowly increases. The case of $r = 1$ is quite interesting. For $\beta = 1$, the quality with $r = 1$ is higher than with $r \geqslant 2$. But for $\beta < 0.9$, the quality with $r = 1$ is worse than with $r \geqslant 2$; and for $\beta = 0.99$, the quality with $r = 1$ is almost the same as with $r \geqslant 2$.

b. Calculation of $MSE^{(c)}/\sigma^2$. The relationship between the average unusable fraction of a picture, $Q_{\bar{c}}$, and the average normalized MSE due to channel error, $MSE^{(c)}/\sigma^2$, is now calculated. The subscript or superscript "c" is for channel error. The assumptions used in this calculation are that the gray level distribution of a pixel is uniform from 0 to 255, and that when a pixel is corrupted by channel errors, the gray level of the reproduced pixel is arbitrarily changed to one of these 256 levels. Then the average MSE (mean-square error) for that pixel is:

$$MSE = \frac{1}{256 \times 256} \times \sum_{i=0}^{255} \sum_{j=0}^{255} (i-j)^2 = 10922.5$$

The σ^2 for the uniform 256 gray level distribution is:

$$\sigma^2 = \int_{-128}^{128} x^2 \times \frac{1}{256} \times dx = 5461.33$$

Now suppose we have one bad pixel among 1000 pixels. This is equivalent to $Q_{\bar{c}} = 10^{-3}$. On the other hand, the average normalized MSE for those 1000 pixels with one bad pixel caused by channel errors is:

$$\frac{MSE^{(c)}}{\sigma^2} = \frac{1}{1000} \times \frac{10922.5}{5461.33} = 2.00 \times 10^{-3}$$

Generalizing from the above observation, we have the following simple relationship between $MSE^{(c)}/\sigma^2$ and $Q_{\bar{c}}$:

$$\frac{MSE^{(c)}}{\sigma^2} = 2 Q_{\bar{c}}$$

B. Average Normalized Mean-Square Error Due to Source Coding, $MSE^{(s)}/\sigma^2$

Even when there is no channel error, there usually exists a degradation due to source coding. Rice (Ref. 10) measured root mean-square error (RMSE) for a particular picture, where the value of σ was specified, for values of r between 4 to 16, using RM2. Hence the average normalized MSE due to source coding, $MSE^{(s)}/\sigma^2$, can be obtained from his results for those values of r. For $r = 2$, a rough value of RMSE was obtained by extending the graph of Rice's Fig. 1 (Ref. 10). For $r = 1$, the value of $MSE^{(s)}/\sigma^2$ was calculated with the assumption of the source having a Gaussian distribution.

C. End-to-End Picture Quality Measure $MSE^{(t)}/\sigma^2$

The total end-to-end normalized MSE, $MSE^{(t)}/\sigma^2$ can be defined as the sum of average normalized MSEs for the usable portion of a picture and for the unusable portion of a picture. In the unusable portion, we assume the loss due to channel error dominates over the degradation due to source coding, and ignore the latter. On the other hand, in the usable portion of a picture, since there is no channel effect, only the degradation due to source coding is considered. Recall that $Q_c = 1 - Q_{\bar{c}}$ is the average usable fraction of a picture. Hence

$$\frac{MSE^{(t)}}{\sigma^2} = \frac{MSE}{\sigma^2} \text{ for the usable parts of a picture}$$

$$+ \frac{MSE}{\sigma^2} \text{ for the unusable parts of a picture}$$

$$= \frac{MSE^{(s)}}{\sigma^2} \times (1 - Q_{\bar{c}}) + \frac{MSE^{(c)}}{\sigma^2}$$

$$= \frac{MSE^{(s)}}{\sigma^2} \times (1 - Q_{\bar{c}}) + 2 \times Q_{\bar{c}}$$

Its inverse, i.e., signal-to-noise ratio (SNR), is a better representation of the quality measure, since quality improves with increasing SNR. Hence, we have the picture quality Q from the above results:

$$Q = SNR_{dB} = -10 \log_{10} \frac{MSE^{(t)}}{\sigma^2}$$

In Fig. 4, curves of the picture quality Q in SNR_{dB} versus codeword error probability P_o are shown, with compression

ratio r and interleaver-types as parameters. For very low error cases ($P_o \leqslant 10^{-6}$), the channel error effects diminish, and $MSE^{(t)} \approx MSE^{(s)}$. Also, for these values of P_o, the use of interleaving scheme B is almost the same as that of scheme A. For high error rates ($P_o \geqslant 10^{-3}$), the channel error effect term in the total end-to-end MSE dominates over the term of degradation due to source coding. Also, the order of acceptability in quality is the same as that of Fig. 3, since only channel error terms are considered in this region of P_o.

The SNRs for the 5-bit and 6-bit uniform Max (Ref. 11) quantizers are 24.6 dB and 29.8 dB respectively. Imagery quantized using the 5-bit uniform quantizer is generally considered to be "usable" (Ref. 9). Hence the desired quality Q in terms of SNR will be in the range of 25 to 30 dB.

IV. Using the Quality Measure in Overall System Design

Figure 4 gives the picture quality as a function of codeword error probability P_o, for the case where an $n = 255$, $k = 223$ outer code having 8 bits/symbol is concatenated with a rate 1/2 inner code. For any specific concatenated code with these parameters, we can relate P_o to the bit energy to noise ratio E_o/N_o. Then, using link analysis (see Appendix), we can relate E_o/N_o to system parameters by an equation such as Eq. (A-2). By this sequence of steps, the overall system parameters like antenna diameter, transmitter power, etc., can be chosen to obtain a desired level of picture quality for a given camera rate R_d (bits/sec) or R_c (pictures/hour).

V. Summary

We have shown how to obtain an end-to-end picture quality measure Q when compressed imagery is transmitted over a concatenated channel. Calculations are given for the specific case of an outer code with 223 information symbols and 32 parity symbols, 8 bits/symbol, and a rate 1/2 inner code, where the compression ratio is between 1 and 16. Similar calculations can be made for any other concatenated channel, to obtain quality Q as a function of codeword error probability P_o for various compression ratios. Link analysis, together with performance curves for the specific code used, enable Q to be incorporated into the overall system parameter design.

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Table 1. Average number of lost source blocks per codeword error

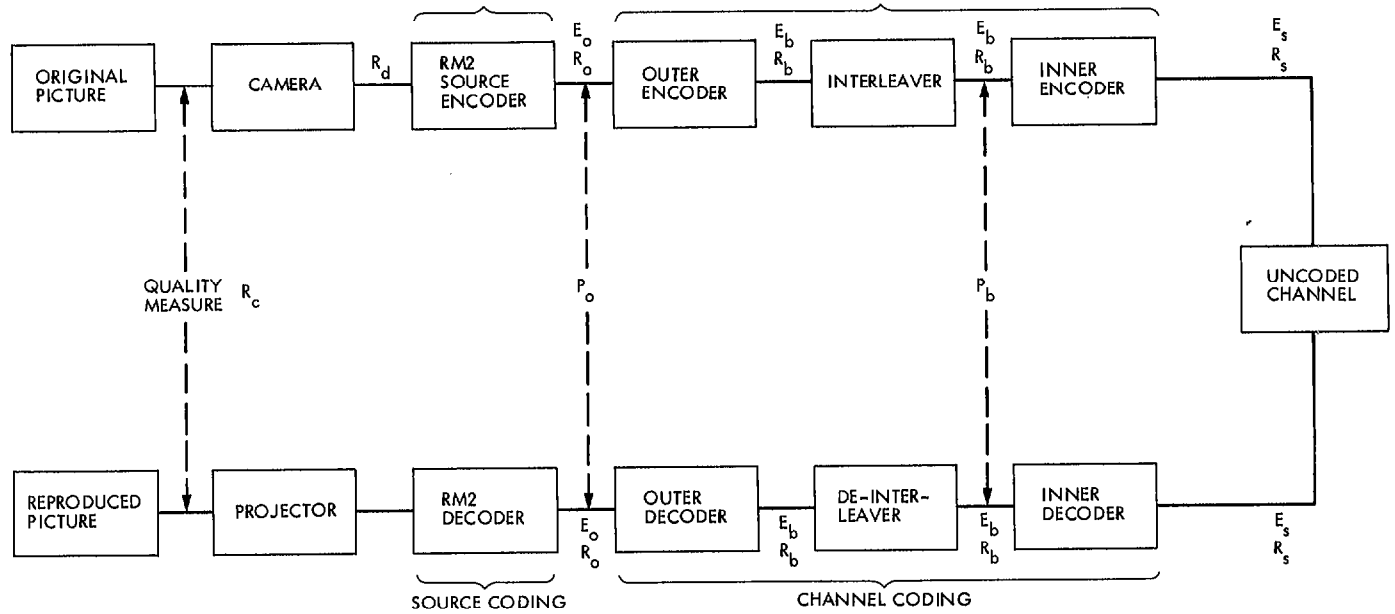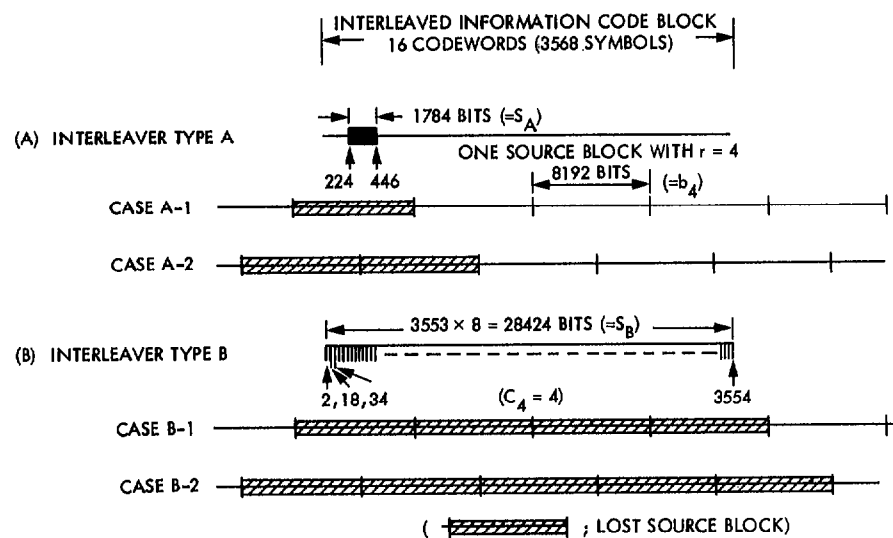| r	2	4	8	16
A_r	1.109	1.218	1.435	1.871
B_r	2.735	4.470	7.939	14.88

Fig. 1. System diagram

Fig. 2. Effect of a codeword error on the compressed source blocks;
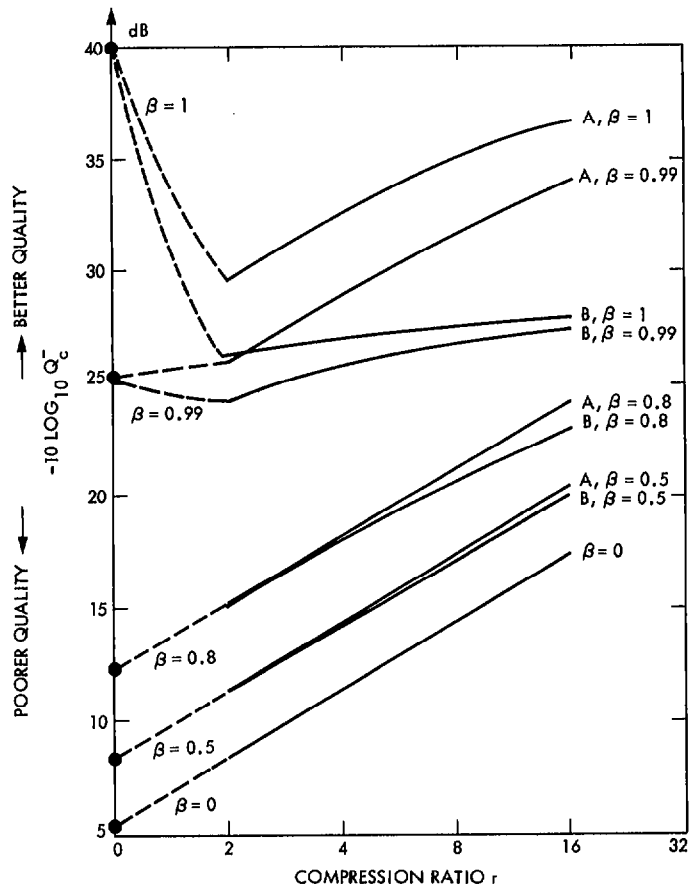$r = 4$, codeword #2 in error
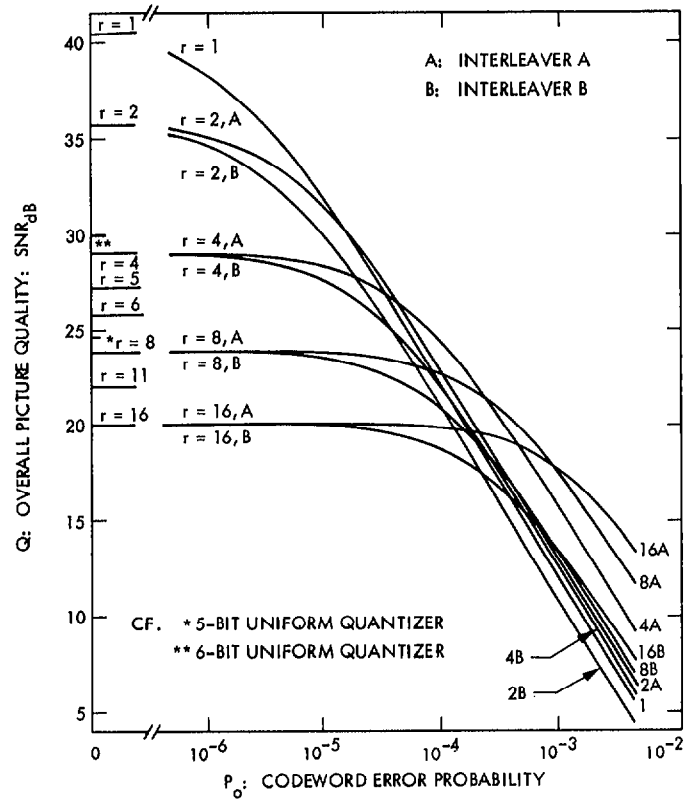
Fig. 3. $1/Q_C^-$ at $P_o = 10^{-4}$

Fig. 4. Picture quality vs P_o with compression ratio r and types of interleaver A and B as parameters

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Appendix

Let

S_t = Transmitter power

f_o = carrier frequency

L = earth-spacecraft separation

η_t = transmitting antenna area efficiency

N_o = one-sided noise spectral density of receiving system

G_r = receiving antenna gain

L_w = attenuation due to weather

M_w = operating margin calculated from parameter tolerances

D_t = transmitting antenna diameter

L_p = transmitting antenna pointing loss which is a function of D_t given by

$$L_p = \frac{\sin^2 (2.78\ \theta_{pt}/\theta_{Bt})}{(2.78\ \theta_{pt}/\theta_{Bt})^2} \qquad \text{(A-1)}$$

where

θ_{pt} = pointing error in degrees

θ_{Bt} = antenna beamwidth in degrees

$$= 177.9259 \times \eta_t^{-1/1.96} \left(\frac{\pi D_t}{\lambda}\right)^{-2/1.96}$$

L_1 = all other system losses

We further assume that the spacecraft camera output rate R_d in bps is fixed, corresponding to 800×800 pixels per picture, 8 bits/pixel, and R_c pictures per hour, so that the data compressor output rate is $R_o = R_d/r$ bps. In terms of these parameters, the link performance equation is

$$\frac{E_o}{N_o} = \left(\frac{S_t}{R_d}\right) r(D_t^2\ L_p) M_w L_w \left(\frac{\eta_t G_r L_1}{16 N_o L^2}\right) \qquad \text{(A-2)}$$