Performance of Concatenated Codes for Deep Space Missions

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Computer simulation results are presented on the performance of convolutional codes of constraint lengths 7 and 10 concatenated with the (255, 223) Reed-Solomon code (a proposed NASA standard). These results indicate that as much as 0.8 dB can be gained by concatenating this Reed-Solomon code with a (10, 1/3) convolutional code, instead of the (7, 1/2) code currently used by the DSN.

I. Introduction

The purpose of this article is to present new results on the combined performance of short constraint length Viterbi-decoded convolutional codes and Reed-Solomon codes. When one coding scheme is superimposed upon another the resulting combination is called a concatenated code. Those interested in learning about these coding schemes can find elementary presentations in Ref. 1. Our interest is in their performance.

The DSN currently has both (7, 1/2) and (7, 1/3) Viterbi decoders. The performance of several convolutional codes of rates 1/2 and 1/3 with constraint lengths between 7 and 10 have been known for some time (Refs. 2, 3). At the time that the DSN Viterbi decoders were built, hardware speeds were not fast enough to build Viterbi decoders of constraint lengths beyond 7 that were sufficiently reliable and inexpensive. However, with current and expected technological advancements in mind, we have given another look at the possible performance of Viterbi decoders of constraint length 10 and rates 1/2 and 1/3.

This article not only extends previous Viterbi performance results, but also contains new performance results for convolutional codes concatenated with a (255, 223) Reed-Solomon code. (The performance of the DSN (7, 1/2) code concatenated with this Reed-Solomon code appears in Ref. 4.) The Reed-Solomon bit-error probability depends not only on the (average) Viterbi bit error rate, but also on the lengths of the Viterbi error bursts and the density of the errors within the bursts. Consequently, additional simulations are required to gather these statistics.

The Galileo Project and the International Solar Polar Mission are planning to employ a concatenated Reed-Solomon/Viterbi coding scheme for telemetering science and engineering data over the space communications channel. Even the Voyager mission has this capability on board. The reason for using a concatenated coding scheme over convolutional coding alone is due to its more efficient use of signal power to achieve bit error probabilities in the 10^{-5} range. Such low error rates are necessary to make data compression schemes workable. Data compression algorithms, while promising to remove substantial information redundancy, are very sensitive to transmission errors.

The quest for good codes is more than just an academic pursuit. A 1 dB or 26 percent improvement from coding is equivalent to enhancing the utilization of the current DSN by arraying a 34-m and a 64-m antenna. With current DSN antenna costs estimated near $100 million, such a gain represents $26 million. Similar tradeoffs can be made on board a spacecraft.

A block diagram of a concatenated coding system is shown in Fig. 1. Binary data generated on board the spacecraft are first encoded by the Reed-Solomon encoder. This encoder also interleaves the Reed-Solomon symbols so as to minimize the effect of error bursts on individual Reed-Solomon codewords. After this first level of coding the data pass to the convolutional encoder. The modulator converts these binary data to a phase modulated radio frequency signal which is amplified and sent out towards the Earth. Two modulation stages are actually performed in the transmitter. The binary data are first multiplied by a square wave subcarrier, and then the resulting waveform is used to phase-modulate a high frequency sinusoidal carrier.

On the ground, the analog signal is detected and tracked by the receiver. A carrier reference is derived and is used to heterodyne the signal to subcarrier frequency. The subcarrier demodulator assembly (SDA) removes the square wave subcarrier, and the symbol synchronizer assembly (SSA) attempts to recover the original coded bit stream. Due to channel noise (and other degradations caused within the receiver system) the SSA does not output the original binary sequence. Instead, it outputs a stream of quantized estimates of these bits. The Viterbi decoder takes these estimates as inputs and decodes the convolutional level of the coding. The Reed-Solomon decoder then deinterleaves the symbols and does the final decoding.

The simulations discussed in this article assumed that there are no losses from carrier and subcarrier tracking and demodulation, and that the Viterbi decoder retains node synchronization at all times. Studies of these degradations are being undertaken, and the results will appear in future publications. For the purpose of this article, only signal degradation caused by the Gaussian noise of the space channel is assumed. The comparisons made in this article should remain valid when degradations are added.

II. Summary of Simulation Results

Figure 2 indicates the performances of several decoding schemes as a function of bit-energy to noise ratio. In particular it shows the relative performances of several Viterbi decoded convolutional codes including the (7, 1/2) code, which is the present standard for deep space applications. Also shown in Fig. 2 are Shannon's theoretical performance limits for rate 1/2 and rate 1/3 binary codes and the performance of uncoded transmission. The Shannon limits represent the best possible error performance for binary codes of these rates (Ref. 5). It is easily seen that the (7, 1/2) code is 2.3 dB away from the theoretical limit at an error probability of 5×10^{-3}. Also, the (10, 1/3) code is less than 2 dB from Shannon's limit for rate 1/3 binary codes.

Also shown in Fig. 2 are the results of concatenating these convolutional codes with an outer Reed-Solomon (255, 223) code. Ideal interleaving is assumed as well as no system losses other than Gaussian channel noise. The performance of the concatenated scheme is very sensitive to SNR; a 1 dB change can result in a bit-error probability jump of several orders magnitude. Consequently, the use of such a concatenated scheme should be accompanied by tight control of the signal to noise ratio of the communications link. Otherwise, the additional operating margin may negate the advantages derived from coding.

III. Simulated Performance of Several Coding Schemes

The key to computing the performance of the concatenated coding system is determining the Reed-Solomon symbol-error statistics. This information cannot be deduced from Viterbi bit-error performance curves. Consequently, extensive simulations were performed on the Xerox Data Systems Sigma 5 computer to calculate both the Viterbi bit-error and Reed-Solomon symbol-error statistics. Each data point was generated by processing 900,000 bits through a modification of the software Viterbi decoder developed by J. W. Layland. The simulations assumed that there were no system losses due to receiver noise or lack of synchronization. The only degradation present in the simulation was that of the random number generator simulating additive white Gaussian noise to reflect the channel SNR. Also, sufficient Reed-Solomon symbol interleaving was assumed so that the symbol error events were independent. This is referred to as ideal interleaving. It is worth noting that interleaving to a depth of 5 is nearly ideal for the DSN (7, 1/2) inner convolutional code at SNRs above 2.0 dB.

Figure 3 shows the results of these simulations. In addition to the plots of Viterbi bit-error probability, p, as a function of channel SNR (E_b/N_o), each graph displays the Reed-Solomon symbol-error probability, π. The Reed-Solomon bit- and word-error probabilities are calculated from π and other burst statistic information derived from these simulations. These Reed-Solomon performance curves are plotted against concatenated channel SNR which is 0.58 dB greater than that of the Viterbi channel due to the overhead of the Reed-Solomon parity symbols.

IV. Conclusion and Discussion

The Viterbi decoders currently used by the DSN suffer loss of node synchronization at low SNRs. This means that if the signal is too weak, the decoder cannot decide which of the two code symbols associated with each data bit should be first. The concatenated coding system allows transmission of data at SNRs lower than those required for a convolutional-only scheme. This means that node synchronization losses will be higher in the concatenated scheme.

There are also synchronization problems associated with the Reed-Solomon code. A method for determining Reed-Solomon symbol and word boundaries is needed. If a packet telemetry system such as the one proposed by the EEIS (Ref. 6) is to be implemented, then a frame synchronization device is also required.

For this article, only the error correcting capability of the Reed-Solomon code was considered. However, this code is also capable of correcting a number of erasures, i.e., Reed-Solomon symbols that are previously known to be in error. The (255, 233) code can correct E errors and e erasures in each codeword as long as $2E + e < 33$. If erasures can be detected, then the performance of the Reed-Solomon decoder may improve by as much as 0.3 dB.

It should be noted that the loss of node synchronization and subsequent recovery by the Viterbi decoder may cause a deletion or insertion of a bit into the data stream entering the Reed-Solomon decoder. When this occurs, Reed-Solomon symbol and word synch will be lost. In the proposed EEIS packet telemetry scheme a node synch failure could result in a loss of over 8000 information bits. Consequently, the sensitivity of the concatenated coding to node synchronization losses is potentially greater than that of convolutional coding alone.

The effects of carrier, subcarrier, symbol, and bit tracking in the system are also important to the overall performance of the coded channel since poor tracking increases the number of Viterbi decoder bit errors.

The strict error rate requirements of data compression are a major reason for investigation concatenated coding schemes. These requirements stem from the removal of redundant information, hence compression. As an example, one of the data compression schemes under consideration (Ref. 7) reduces the number of bits per picture by over one half without loss of information. The reconstructed compressed data, however, are more sensitive to transmission errors than the original data. Hence error correcting coding schemes must be used. Notice that the concatenated schemes described in this article more than double the number of bits that are transmitted per information bit. This seems to neutralize the useful effects of data compression. Actually, this is not the case since an SNR of 9.5 dB would be required if no coding were employed to achieve an error rate of 10^{-5} (see Fig. 2), whereas the concatenated scheme with the (7, 1/2) inner code requires only 2.3 dB, and only 1.6 dB is required when the (10, 1/3) inner code is used. It might be beneficial to consolidate data compression and channel coding into a one-step process.

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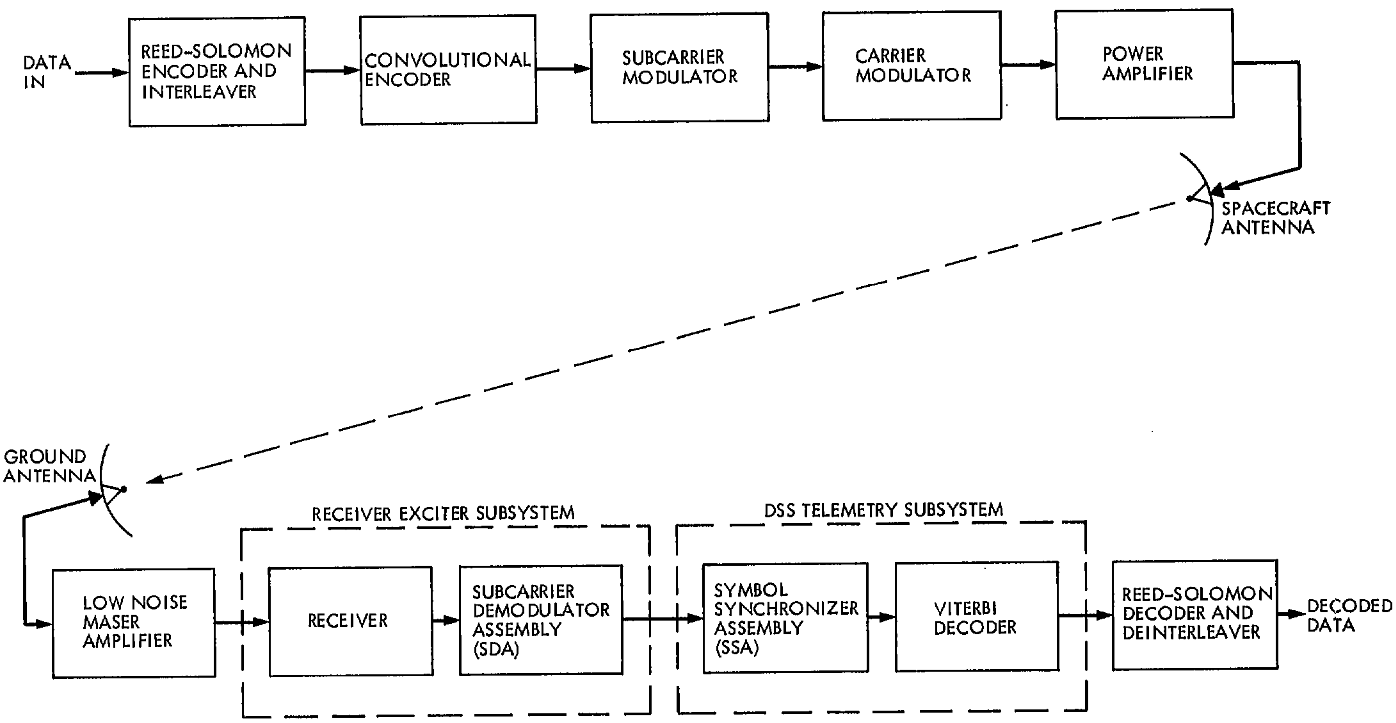

Fig. 1. The concatenated coding system

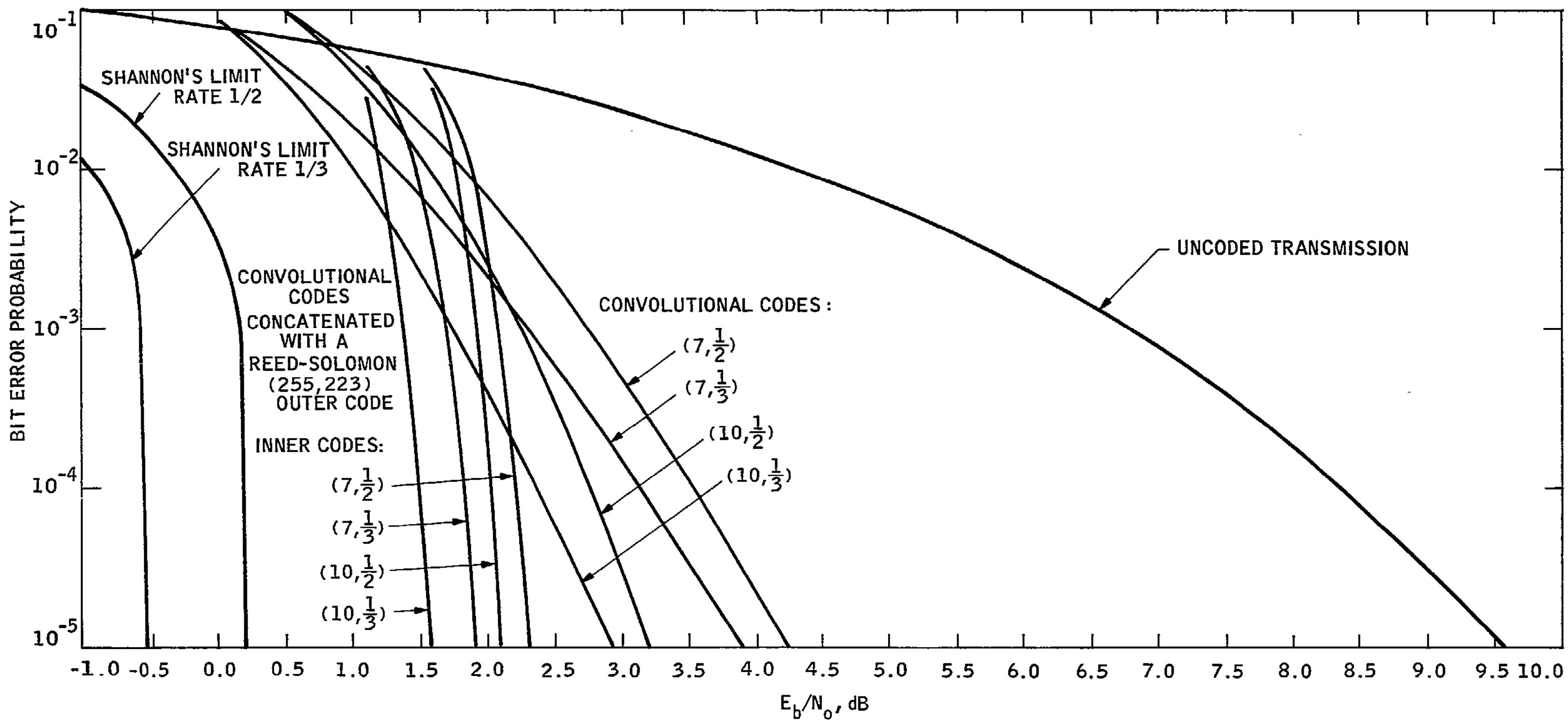

Fig. 2. Comparison of several coding schemes

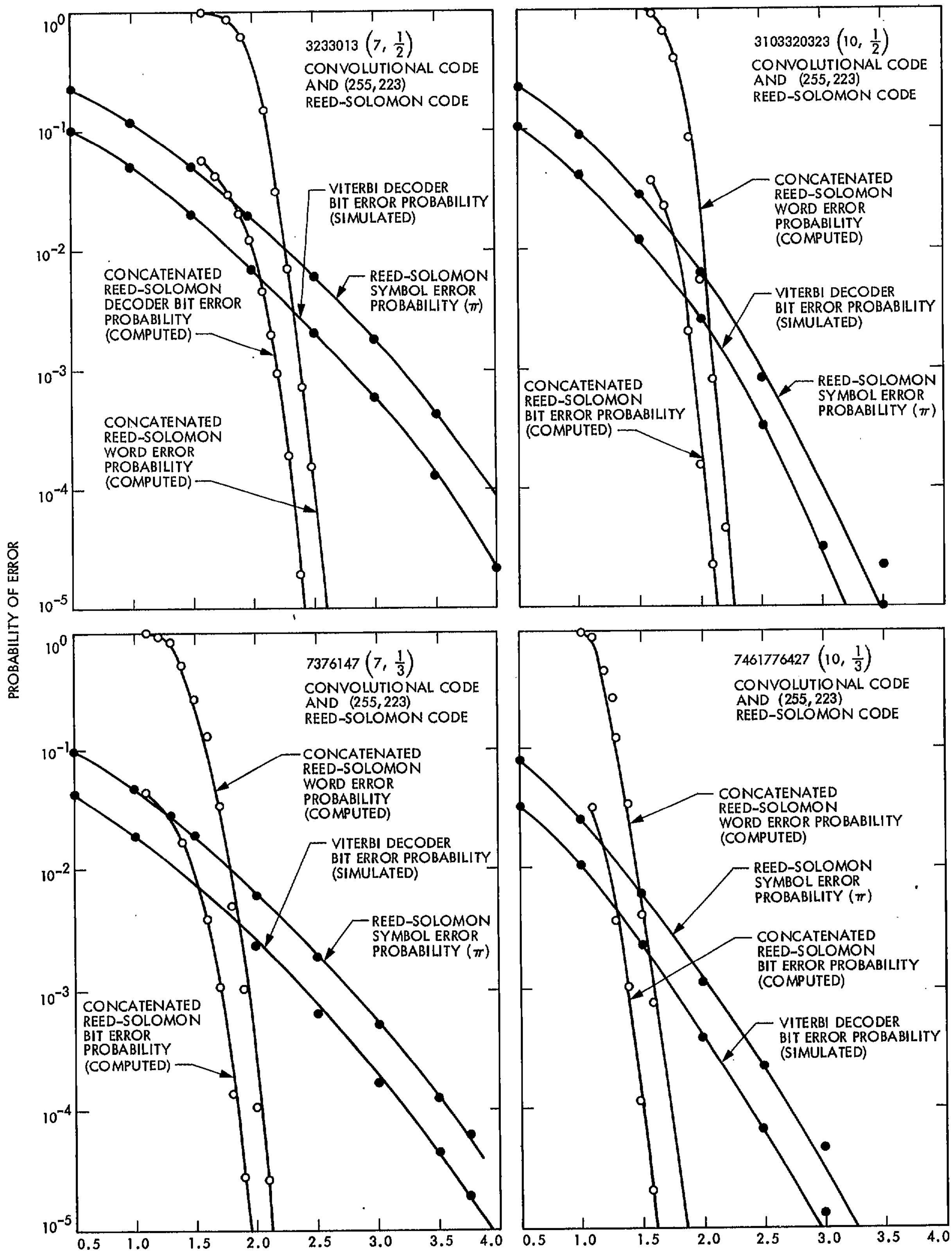

Fig. 3. Performance statistics for ideally interleaved concatenated coding scheme, assuming no system losses