Atmospheric Noise Temperature Measurements

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Radiometric microwave noise temperature measurements are used to estimate atmospheric transmission loss. It is common practice to use the following lumped element model expression for the noise temperature contribution,

$$T'' = T_p (1 - 1/L).$$

This relationship is used to estimate the transmission loss L in terms of T'' and the atmosphere effective physical temperature T_p. This report evaluates T_p in terms of assumed distributed loss and temperature models. Simplified expressions are presented for low loss applications. For these applications L is determined directly and accurately without integration or iteration.

I. Summary

Radiometric microwave noise temperature measurements are used to estimate atmospheric transmission loss. It is common practice to use the lumped element model expression for the noise temperature contribution

$$T'' = T_p (1 - 1/L)$$

This relationship is used to estimate the transmission loss L in terms of T'' and the atmospheric effective physical temperature T_p. This report evaluates T_p in terms of assumed distributed loss and temperature models. For exponential loss and linear temperature distributions

$$T_p = T_1 + k (T_2 - T_1)$$

where

$$k = \frac{1 - I}{1 - 1/L}$$

$$I = \frac{1}{(\ln \alpha_2/\alpha_1) e^{\alpha_2/a}} \int_{\alpha_1/a}^{\alpha_2/a} \frac{e^z}{z} dz$$

These equations are evaluated and T_p is presented for a range of values for L and α_2/α_1. For low loss ($L \approx 1$)

$$k \simeq \frac{\alpha_2/\alpha_1}{(\alpha_2/\alpha_1) - 1} - \frac{1}{\ln (\alpha_2/\alpha_1)}$$

As an example, if $\alpha_2/\alpha_1 = 10$, we have $k = 0.6768$. Then for this model

$$T_p \simeq T_1 + 0.6768\,(T_2 - T_1)\,.$$

This provides a useful estimate of T_p (in some applications T_1 is estimated and T_2 measured). Then from the lumped element model

$$L = \frac{1}{1 - T''/T_p}$$

This indicates that for low loss, L can be directly and accurately determined from these models without integration or iteration.

II. Introduction

Radiometer microwave noise temperature measurements are used to estimate atmospheric transmission loss (Ref. 1). It is common to use the lumped element model expression for a lumped element model for the noise temperature contribution

$$T'' = T_p\,(1 - 1/L) \tag{1}$$

where

L = propagation path loss, ratio

T_p = atmosphere effective physical temperature, kelvins

to estimate the transmission loss

$$L = \frac{1}{1 - T''/T_p} \tag{2}$$

This article investigates methods to evaluate T_p required for the appropriate transmission path model.

III. Theory

We have (Ref. 1, Eq. 3, using the same symbols and definitions)

$$T'' = \int_0^l \alpha(x)\,T(x)\,e^{-\int_x^l \alpha(x')\,dx'}\,dx \tag{3}$$

Substitute, $x = yl$

$$T'' = \int_0^1 \alpha(y)\,l\,T(y)\,e^{-\int_{yl}^1 \alpha(y')l\,dy'}\,dy \tag{4}$$

Using

$$Z = -\int_{yl}^1 \alpha(y')\,l\,dy' \tag{5}$$

results in

$$T'' = -\int_0^{Z_1} T(Z)\,e^z\,dZ \tag{6}$$

where

$$Z_1 = -\int_0^1 \alpha(y')\,l\,dy'$$

IV. Application

Consider the variable parameter model (Ref. 1, model 2) with exponential propagation constant and linear temperature distributions,

$$\begin{aligned} \alpha(y) &= \alpha_1\,e^{aly} \\ T(y) &= T_1 + (T_2 - T_1)\,y \end{aligned} \tag{7}$$

Then, with Eq. (5),

$$\begin{aligned} Z &= (\alpha_1/a)\,(e^{aly} - e^{al}) \\ &= (\alpha_1/a)\,(e^{aly}) - (\alpha_2/a) \\ Z_1 &= \frac{(\alpha_1 - \alpha_2)}{a} \end{aligned}$$

and

$$T'' = -\int_0^{(\alpha_1-\alpha_2)/a} \left[T_1 + (T_2 - T_1)\frac{\ln\left(\frac{aZ}{\alpha_1} + \frac{\alpha_1}{\alpha_2}\right)}{al} \right] e^z dz \tag{8}$$

Expanding

$$T'' = T_1\,(1 - 1/L) + (T_2 - T_1)\,(1 - I) \tag{9}$$

where $(al = \ln \alpha_2/\alpha_1, \alpha_1/a = \ln L/(\alpha_2/\alpha_1 - 1), (\alpha_2 - \alpha_1)/a = \ln L)$

$$I = \frac{1}{(\ln \alpha_2/\alpha_1)\, e^{\alpha_2/a}} \int_{\alpha_1/a}^{\alpha_2/a} \frac{e^z}{Z}\, dZ$$

I is solely a function of α_2/α_1 and L. Equating Eqs. (1) and (9)

$$T_p = (1 - k)\, T_1 + kT_2 \tag{10}$$

where

$$k = \frac{1 - I}{1 - 1/L}$$

The solution for k is obtained from numerical integration of I. The results are shown in Fig. 1. Over this range of parameters,

$$k \simeq 0.5 + 0.01768\, L\,(\mathrm{dB}) + 0.01768\,(\alpha_2/\alpha_1)\,(\mathrm{dB})$$
$$- \, 0.000368\, L\,(\mathrm{dB})\,(\alpha_2/\alpha_1)\,(\mathrm{dB}) \tag{11}$$

For example, if (α_2/α_1) (dB) = L (dB) = 10, we have $k \simeq 0.8168$ so that for $T_1 = 250\,K$ and $T_2 = 290\,K$, $T_p \simeq 282.7\,K$, which agrees with previous calculations (Ref. 1, Table 2, case 3).

For $L \approx 1$, (from expansion of Eq. 8)

$$I \approx 1 - (\alpha_2/\alpha_1)\frac{L - 1}{(\alpha_2/\alpha_1) - 1} + \frac{L - 1}{\ln (\alpha_2/\alpha_1)}$$

so that

$$k \cong L\left[\frac{\alpha_2/\alpha_1}{(\alpha_2/\alpha_1) - 1} - \frac{1}{\ln (\alpha_2/\alpha_1)}\right]$$

$$\approx \left[\frac{\alpha_2/\alpha_1}{(\alpha_2/\alpha_1) - 1} - \frac{1}{\ln (\alpha_2/\alpha_1)}\right] \tag{12}$$

and similarly for $(\alpha_2/\alpha_1) \approx 1$ (also from Ref. 1, model 3)

$$I \approx \left(\frac{1}{\ln L} - \frac{1}{L \ln L}\right)$$

so that

$$k = \frac{1 - \dfrac{1}{\ln L} + \dfrac{1}{L \ln L}}{1 - 1/L} \tag{13}$$

for both $L \approx 1$ and $\alpha_2/\alpha_1 \approx 1$ this reduces to $k \simeq 0.5$

In Figs. 2, 3 and 4 T_p is shown plotted using model 2 for a range of values of α_2/α_1, L, T_1, and T_2. Precise transmission loss estimates can be made from radiometer noise temperature measurements for this model without integral evaluations using Eqs. (2) and (10) [Eqs. 12 or 13 if appropriate]. The procedure is to calculate the loss using Eq. (2) iteratively with T_p evaluated from Eqs. (10) and (11). The required initial estimate of T_p is given by 280 K, $(T_1 + T_2)/2$ or other choice as appropriate.

For example, if $T'' = 254.4$ K, $(\alpha_2/\alpha_1) = 10$, $T_1 = 250$ K, $T_2 = 290$ K, we have (using for initial estimate, $T_p = (T_1 + T_2)/2 = 270$ K) from Eq. (2)

$$L = 12.4 \text{ dB}$$

First iteration (Eqs. 11, 10 and 2)

$$k = 0.8504$$

$$T_p = 284.0$$

$$L = 9.8 \text{ dB}$$

Second iteration

$$k = 0.814$$

$$T_p = 282.6$$

$$L = 10.0 \text{ dB}$$

which indicates rapid convergence. For most applications, the second iteration is not required. For example, starting with $T_p = 270$ K as above and $T'' = 210.6$ K, 139.5 K and 57.1 K results in (with 1 iteration) $L = 6.02$ dB, 3.01 dB, and 1.00 dB respectively, (in agreement with previous calculations, Ref. 1, Table 2). This technique is useful for other models with application of Eqs. (4) and (6). The solution for k is tabulated in Table 1 for various models. Further, these methods apply not only to atmospheric measurements, but to any determination of loss from radiometer calibrations such as transmission line loss (required for thermal load standards, Ref. 2) or radome loss calibration (Ref. 3).

For an application example at small loss, assume $(\alpha_2/\alpha_1) \simeq 10$, $L \approx 1$. Then

$$T_p \simeq T_1 + 0.6768\,(T_2 - T_1) \tag{14}$$

In some applications, T_1 would be estimated and T_2 measured. Using Eqs. (14) and (2):

Case 1

$$\left.\begin{aligned} T_1 &= 250\text{ K} \\ T_2 &= 290\text{ K} \\ T'' &= 10\text{ K} \end{aligned}\right] \Rightarrow \begin{aligned} T_p &\simeq 277.1\text{ K} \\ L &\simeq 0.159\text{ dB} \end{aligned}$$

Case 2

$$\left.\begin{aligned} T_1 &= 250\text{ K} \\ T_2 &= 310\text{ K} \\ T'' &= 10\text{ K} \end{aligned}\right] \Rightarrow \begin{aligned} T_p &\simeq 290.6\text{ K} \\ L &\simeq 0.152\text{ dB} \end{aligned}$$

The small difference between L calculated for cases 1 and 2 illustrates that for small losses L is insensitive to errors in T_p. In these examples, the error in T_p using Eq. (14) is approximately 1.3 K, resulting in an error in L of approximately 0.01 dB. This example demonstrates that for low loss applications, determine or estimate, α_2/α_1, calculate k from Eq. (12), T_p from Eq. (10) and L from Eq. (2). This is accomplished directly without integration or iteration.

For comparison, a linear/linear variable parameter model (Ref. 1, model 6) is analyzed in Appendix A and summarized in Fig. A-1 and Table 1. This model results in (using parameters of Case 1 above)

$$k \simeq 0.6364$$

$$T_p \simeq 275.5\text{ K}$$

$$L = 0.161\text{ dB}$$

This indicates that for low loss, the solution for L is insensitive to the model choice.

V. Conclusion

Various expressions are derived for the effective physical temperature T_p of the atmosphere for use in the lumped element model expression for the noise temperature contribution

$$T'' = T_p\,(1 - 1/L)$$

We have

$$T_p = T_1 + k\,(T_2 - T_1)$$

where

$$k = \frac{1 - I}{1 - 1/L}$$

I is derived and evaluated for a range of values for L and (α_2/α_1) for 2 models. Low loss, useful and accurate approximations are given by

$$k \simeq \frac{\alpha_2/\alpha_1}{\alpha_2/\alpha_1 - 1} - \frac{1}{\ln \alpha_2/\alpha_1} \qquad \text{model 2 (exponential/linear)}$$

$$\simeq 0.6768 \text{ (for } \alpha_2/\alpha_1 = 10)$$

$$k \simeq 1 - \frac{1 + 1/3\,[(\alpha_2/\alpha_1) - 1]}{(\alpha_2/\alpha_1) + 1} \qquad \text{model 6 (linear/linear)}$$

$$\simeq 0.6364 \text{ (for } \alpha_2/\alpha_1 = 10)$$

For both $L \approx 1$ and $\alpha_2/\alpha_1 \approx 1$, these reduce to $k \simeq 0.5$. With these low loss approximations, T_p can be evaluated in terms of α_2/α_1, T_1 and T_2. Then L can be determined accurately and directly without integration or iteration, using

$$L \simeq \frac{1}{1 - (T''/T_p)}$$

References

1. Stelzried, C., and Slobin, S., "Calculation of Atmospheric Loss From Microwave Radiometer Noise Temperature Measurements," in *TDA Progress Report 42-62*, Jet Propulsion Laboratory, Pasadena, Calif., April 15, 1981.

2. Stelzried, C. T., "Microwave Thermal Noise Standards," *IEEE Trans. Microwave Theory and Techniques*, Vol. MTT-16, No. 9, Sept. 1968, p. 646.

3. Seidel, B. L., and Stelzried, C. T., "A Radiometric Method for Measuring the Insertion Loss of Radome Materials," *IEEE Trans. Microwave Theory and Techniques*, Vol. MTT-16, No. 9, Sept. 1968, p. 625.

Table 1. Summary of various approximations for k required to calculate T_p

Model	Parameter			
	$\alpha(x)$	$T(x)$	L	k
2	$\alpha_1 e^{(x/l)\ln(\alpha_2/\alpha_1)}$	$T_1 + (T_2 - T_1)\frac{x}{l}$	$e^{\frac{(\alpha_2-\alpha_1)l}{\ln(\alpha_2/\alpha_1)}}$	$\simeq 0.5 + 0.01768\,L\,(\mathrm{dB}) + 0.01768\,\frac{\alpha_2}{\alpha_1}\,(\mathrm{dB}) - 0.000368\,L\,(\mathrm{dB})\,\frac{\alpha_2}{\alpha_1}\,(\mathrm{dB})$ $\simeq L\left[\frac{\alpha_2/\alpha_1}{(\alpha_2/\alpha_1)-1} - \frac{1}{\ln(\alpha_2/\alpha_1)}\right](L \approx 1)$ or $\approx \left[\frac{\alpha_2/\alpha_1}{(\alpha_2/\alpha_1)-1} - \frac{1}{\ln(\alpha_2/\alpha_1)}\right](L \approx 1)$ $\approx \frac{1+\frac{1}{L\ln L}-\frac{1}{\ln L}}{1-1/L}\Bigg\}\ \frac{\alpha_2}{\alpha_1} = 1$ (Ref. 1, Model 3) $\simeq 0.5\Big\}\ L = 1, \frac{\alpha_2}{\alpha_1} = 1$
6	$\alpha_1 + (\alpha_2 - \alpha_1)\frac{x}{l}$	$T_1 + (T_2 - T_1)\frac{x}{l}$	$e^{\frac{(\alpha_1+\alpha_2)l}{2}}$	$\simeq 0.5 + 0.01768\,L\,(\mathrm{dB}) + 0.01364\,\frac{\alpha_2}{\alpha_1}\,(\mathrm{dB}) - 0.000309\,L\,(\mathrm{dB})\,\frac{\alpha_2}{\alpha_1}\,(\mathrm{dB})$ $\simeq 1 - \frac{1+\frac{1}{3}\left(\frac{\alpha_2}{\alpha_1}-1\right)}{\frac{\alpha_2}{\alpha_1}+1}\ (L \approx 1)$ $\approx \frac{1+\frac{1}{L\ln L}-\frac{1}{\ln L}}{1-1/L}\Bigg\}\ \frac{\alpha_2}{\alpha_1} = 1$ (Ref. 1, Model 3) $\simeq 0.5\Big\}\ L = 1, \frac{\alpha_2}{\alpha} = 1$

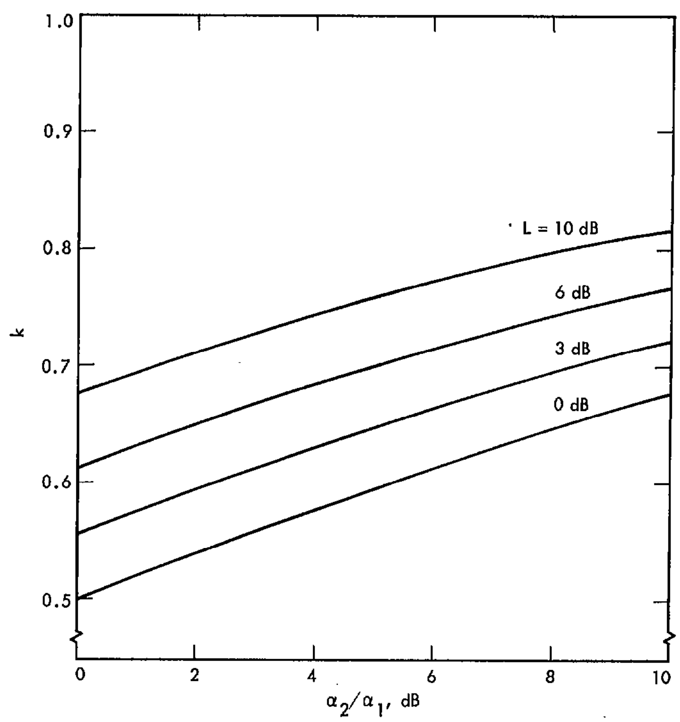

Fig. 1. Plot of k vs α_1/α_2 for various values of L for model 2 (T_1 = 250 K, T_2 = 290 K)

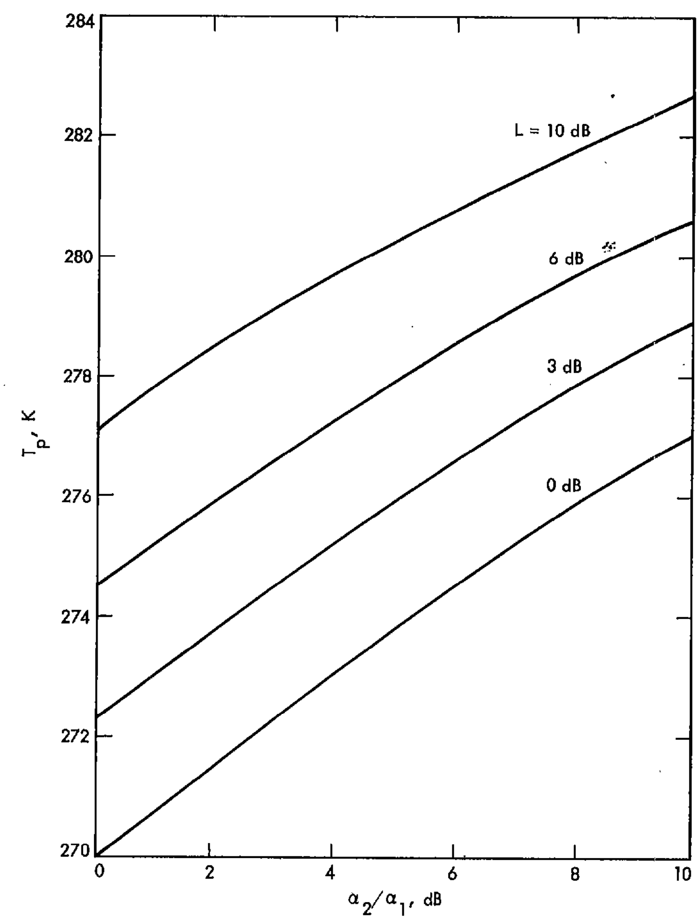

Fig. 2. Plot of T_p vs α_2/α_1 for various values of L for model 2 (T_1 = 250 K, T_2 = 290 K)

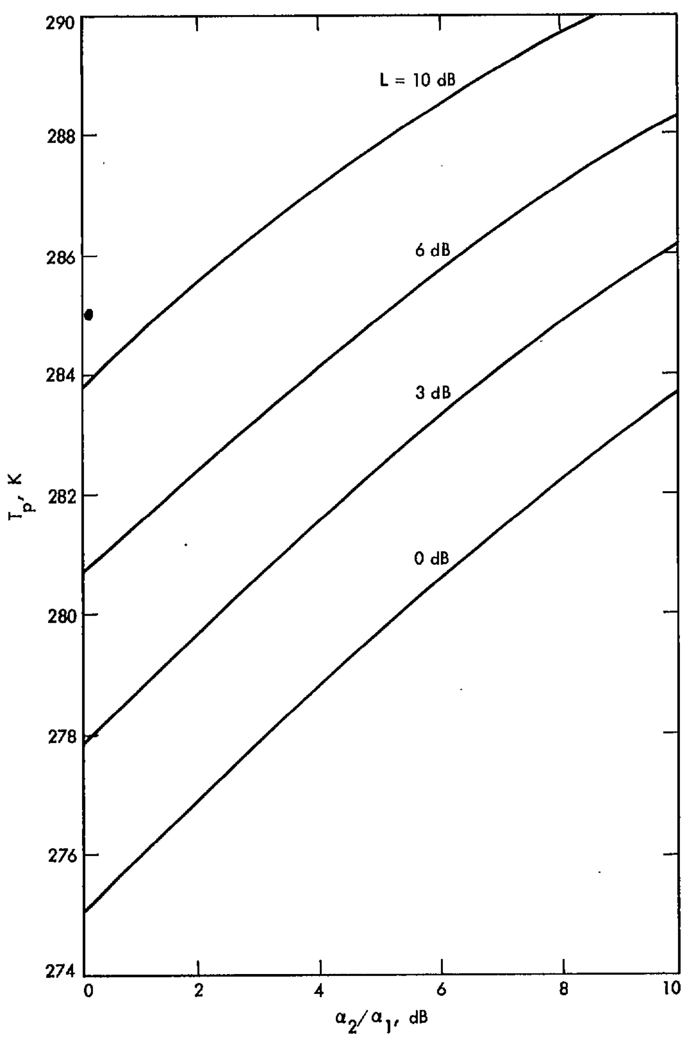

Fig. 3. Plot of T_p vs α_2/α_1 for various values of L for model 2 (T_1 = 250 K, T_2 = 300 K)

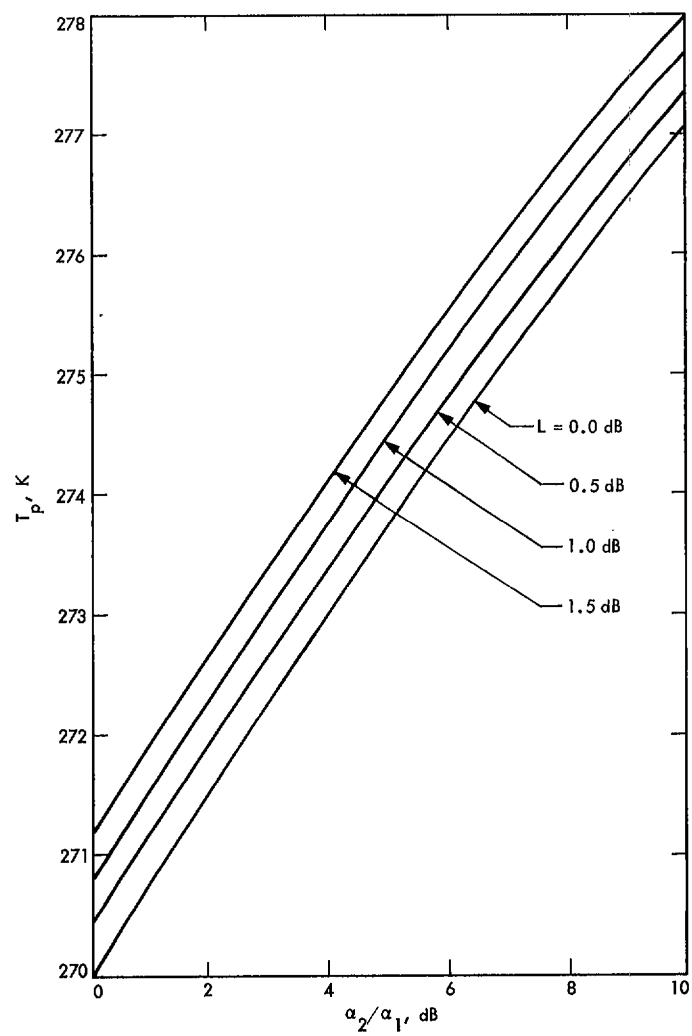

Fig. 4. Plot of T_p vs α_2/α_1 for small increments of L for model 2 (T_1 = 250 K, T_2 = 290 K)

Appendix A
Linear/Linear Variable Parameter Model

Consider the linear/linear variable parameter model (Ref. 1, model 6) with

$$\alpha(y) = \alpha_1 + (\alpha_2 - \alpha_1)y$$
$$T(y) = T_1 + (T_2 - T_1)y \qquad \text{(A-1)}$$

Then

$$T'' = \frac{2 \ln L}{L\left(\dfrac{\alpha_2}{\alpha_1} + 1\right)}$$

$$\int_0^1 \left[1 + \left(\frac{\alpha_2}{\alpha_1} - 1\right) y\right] e^{\frac{\ln L}{\frac{\alpha_2}{\alpha_1}+1}\left[2y + \left(\frac{\alpha_2}{\alpha_1} - 1\right) y^2\right]}$$

$$\cdot\ [T_1 + (T_2 - T_1)y]\, dy \qquad \text{(A-2)}$$

Expanding

$$T'' = T_1\,(1 - 1/L) + (T_2 - T_1)\,(1 - I) \qquad \text{(A-3)}$$

where

$$L = e^{\frac{(\alpha_1 + \alpha_2)\,l}{2}}$$

$$I = \frac{1}{L} \int_0^1 e^{\left[2y + \left(\frac{\alpha_2}{\alpha_1} - 1\right) y^2\right]\left(\frac{\ln L}{\frac{\alpha_2}{\alpha_1}+1}\right)} dy$$

Equating Eqs. (1) and (A-3)

$$T_p = T_1 + k\,(T_2 - T_1) \qquad \text{(A-4)}$$

where

$$k = \frac{1 - I}{1 - 1/L}$$

The solution for k from integration of I is shown plotted in Fig. A-1 over a range of values for (α_2/α_1) and L. This can be approximated with the expression

$$k \simeq 0.5 + 0.01768\, L\ (\text{dB}) + 0.01364\, \alpha_2/\alpha_1\ (\text{dB})$$
$$-\ 0.000309\, L\ (\text{dB})\, \alpha_2/\alpha_1\ (\text{dB}) \qquad \text{(A-5)}$$

for $L \approx 1$

$$k \simeq 1 - \frac{1 + 1/3\ [(\alpha_2/\alpha_1) - 1]}{(\alpha_2/\alpha_1) + 1} \qquad \text{(A-6)}$$

$$k \simeq 0.6364\ (\text{for}\ \alpha_2/\alpha_1 = 10)$$

$$k \simeq 0.5 \qquad (\text{for}\ \alpha_2/\alpha_1 = 1)$$

T_p is shown plotted in Figs. A-2 for T_1 = 250 K and T_2 = 290 K for a range of values for (α_2/α_1) and L.

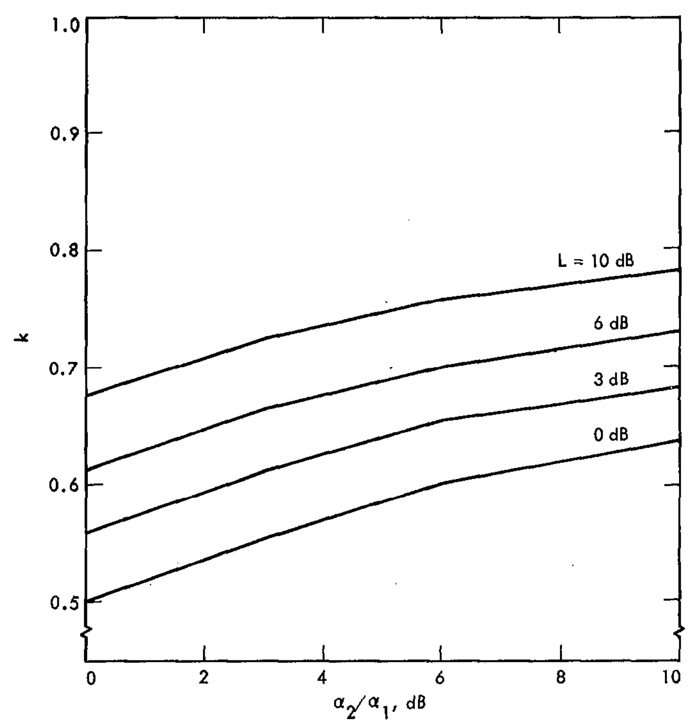

Fig. A-1. Plot of k vs α_2/α_1 for various values of L for model 6 (T_1 = 250 K, T_2 = 290 K)

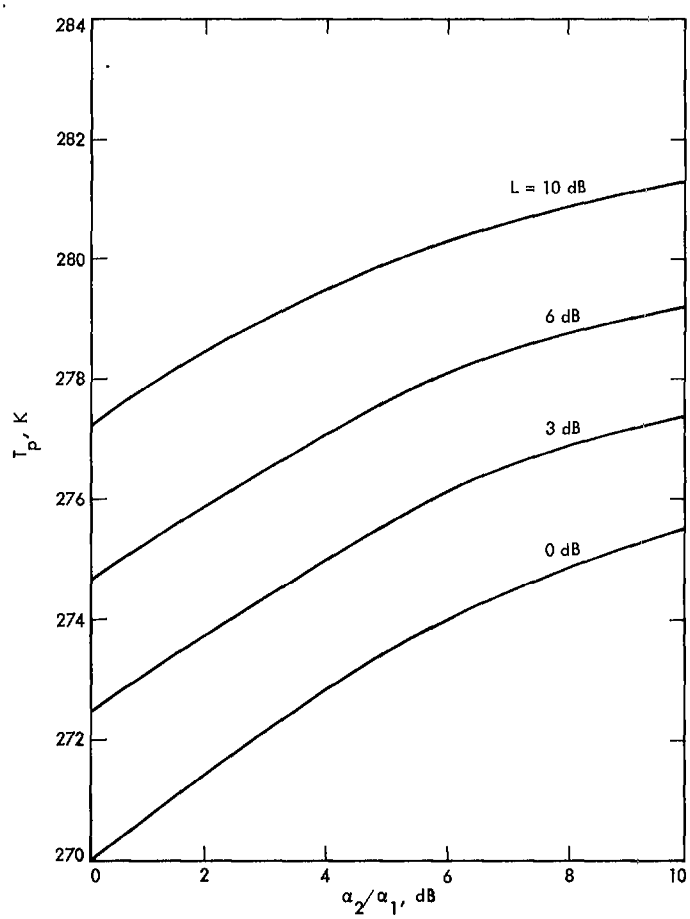

Fig. A-2. Plot of T_p vs α_2/α_1 for various values of L for model 6 (T_1 = 250 K, T_2 = 290 K)