The Effect of Noisy Carrier Reference on Telemetry with Baseband Arraying

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Antenna arraying is coherently adding individually received signals from different receiving stations to improve telemetry performance. The array configuration of the current DSN Network Consolidation Project (NCP) will consist of a 64-meter station and three 34-meter stations.

This article examines the effect of noisy carrier reference on telemetry link performance in this NCP configuration when individually received signals are combined at baseband. This imperfect carrier reference causes a degradation in detection performance in coherent communication systems. A measure of this degradation is the radio loss, which is the amount of increase in data signal-to-noise ratio (SNR) per bit required to achieve the same bit error rate when carrier reference is perfect. Performance analysis and numerical results are obtained for the Voyager high rate telemetry link with maximum likelihood convolutional decoding.

The arraying of antennas provides not only improved performance due to an increase in effective antenna aperture, but also a decrease in radio loss with respect to a single antenna. This telemetry link performance improvement is a function of the carrier loop SNR and data bit error rate. When the carrier loop SNR's are low, it provides a significant improvement in the telemetry link performance since the decrease in radio loss with respect to a single station is substantial.

I. Introduction

Antenna arraying is a technique for coherently adding the received signals from different stations in order to achieve improved downlink telemetry performance. Previous articles in the DSN Progress Report (Refs. 1 and 2) discuss the use of arraying. Wilck talked about the baseband combiner used to array the signals from DSS's 12, 13 and 14 for Mariner 10. Arraying provided a 0.8-dB gain in telemetry signal-to-noise ratio. Brockman discussed carrier arraying for improved tracking capability.

In this report we consider baseband arraying of N stations. Results are applied to an array consisting of a 64-meter station and three 34-meter stations, which is the current DSN NCP configuration. Radio loss for a high rate telemetry link is analyzed when baseband arraying is used to add the signals. In this analysis we have assumed that the subcarrier tracking and symbol synchronization are perfect. It is also assumed that there is no carrier arraying. Numerical results are obtained for the Voyager high rate telemetry link with maximum likelihood decoding. Performance analysis shows that the bit error rate is less sensitive to the noisy carrier phase reference when baseband arraying is employed than when a single station is used.

II. System Model and Performance

A system for baseband arraying of N stations is depicted in Fig. 1. Consider the case where the telemetry signal is an RF carrier that is phase-modulated by a squarewave subcarrier ($\text{Sin}\ \omega_{sc}t$) at a peak modulation index θ. The subcarrier is bi-phase modulated with a binary data stream $D(t)$. The telemetry signal can be expressed (without loss of generality, assume station 1) as

$$S_1(t) =$$

$$\sqrt{2P_1}\ \sin(\omega_c t + \phi_{1c} + D(t)\,\theta\ \ \text{Sin}(\omega_{sc}t + \phi_{1sc})) + n_1(t) \tag{1}$$

where P_1 is the total received power, ω_c is the carrier radian frequency, ϕ_{1c} is the carrier phase, ω_{sc} is the subcarrier radian frequency, ϕ_{1sc} is the subcarrier phase, and $n_1(t)$ is the additive white Gaussian noise with two-sided spectral density $N_{01}/2$.

The signal $S_1(t)$ is coherently demodulated to the subcarrier frequency by a reference signal generated by the carrier tracking loop,

$$r_1(t) = \sqrt{2}\cos(\omega_c t + \hat{\phi}_{1c}) \tag{2}$$

where $\hat{\phi}_{1c}$ is the PLL estimate of the carrier phase.

The resulting data signal for station 1 is

$$\begin{aligned} Y_1(t) &= S_1(t)\, r_1(t) \\ &= \sqrt{P_1}\ \sin\theta\ D(t)\ \ \text{Sin}(\omega_{sc}t + \phi_{1sc})\cos\phi_1 + \tilde{n}_1(t) \end{aligned} \tag{3}$$

where $\phi_1 = \phi_{1c} - \hat{\phi}_{1c}$ and $\tilde{n}_1(t)$ is white Gaussian noise with two-sided spectral density $N_{01}/2$ (Refs. 3 and 4).

Similarly, the demodulated signal for the ith station, $i = 2, 3, \ldots, N$ is

$$\begin{aligned} Y_i(t) = \sqrt{P_i}\ \sin\theta\ D(t - \tau_i)\ \ \text{Sin}(\omega_{sc}(t - \tau_i) \\ + \phi_{isc})\cos\phi_i + \tilde{n}_i(t) \end{aligned} \tag{4}$$

where

$$\phi_i = \phi_{ic} - \hat{\phi}_{ic}$$

and $\tilde{n}_i(t)$ is the Gaussian noise process resulting from $n_i(t)$ after carrier demodulation. ϕ_{isc} is ϕ_{1sc} delayed by τ_i, and ϕ_i, ϕ_j are independent phase processes for any $i \neq j$. Without loss of generality, let station 1 be the reference station, i.e., $\tau_1 = 0$, and let $\tau_N \geqslant \tau_i$, $i = 1, 2, \ldots, N$.

The corresponding loop signal to noise ratios are

$$\rho_{li} = \frac{P_i \cos^2\theta}{N_{0i}\, B_{Li}\Gamma_i}\ ; \qquad i = 1, 2, \ldots, N \tag{5}$$

where N_{0i} is the one-sided noise spectral density for $n_i(t)$ in station i, and Γ_i is the limiter performance factor (Refs. 3 and 4).

The received signals should be synchronized. To do so, the relative time delay τ_i for each station should be known. Here it is assumed that τ_i, $i = 1, 2, \ldots, N$, is perfectly estimated; i.e.,

$$\hat{\tau}_i = \tau_i\ ; i = 1, 2, 3, \ldots, N \tag{6}$$

Now, we delay the signal from station i by $\hat{\tau}_N - \hat{\tau}_i$, weight the signal from station i by the constant β_i (let $\beta_1 = 1$) and add coherently. Clearly the independent additive noises $n_i(t)$ from each receiver add incoherently, or in a mean square sense.

The resultant signal can be written as

$$\begin{aligned} Z(t) &= \sum_{i=1}^{N} \beta_i\, Y_i(t - \hat{\tau}_N + \hat{\tau}_i) \\ &= \left[\sin\theta\, D(t - \hat{\tau}_N) \sum_{i=1}^{N} \beta_i \sqrt{P_i}\cos\phi_i\right] \\ &\quad \times \text{Sin}(\omega_{sc}t - \omega_{sc}\tau_N + \phi_{Nsc}) \\ &\quad + \sum_{i=1}^{N} \beta_i\, \tilde{n}_i(t + \hat{\tau}_i - \hat{\tau}_N) \end{aligned} \tag{7}$$

After subcarrier tracking (assuming perfect tracking) and demodulation we have

$$\begin{aligned} X(t) &= \sin\theta\, D(t - \hat{\tau}_N) \sum_{i=1}^{N} \beta_i \sqrt{P_i}\cos\phi_i \\ &\quad + \sum_{i=1}^{N} \beta_i\, \hat{n}_i(t + \hat{\tau}_i - \hat{\tau}_N) \end{aligned} \tag{8}$$

where $\hat{n}_i(t)$, $i = 1, 2, \ldots, N$ are independent low-pass Gaussian noise processes derived from $\tilde{n}_i(t)$ after the subcarrier demodulation process.

The sampled signal at the output of the integrate and dump circuit is

$$x_k = a_k \sin\theta \sum_{i=1}^{N} \beta_i \sqrt{P_i} \cos\phi_i + \sum_{i=1}^{N} \beta_i n_{ik} \qquad (9)$$

where $n_{ik}, i = 1, 2, \ldots, N$ are Gaussian noise samples and a_k is the data symbol. At the input of the Viterbi decoder the sample x_k is 3-bit quantized. Given ϕ_i and a_k, the signal-to-noise ratio of sample x_k is

$$SNR = \frac{(\overline{x}_k)^2}{\sigma^2_{x_k}} \qquad (10)$$

Here

$$\overline{x}_k = \left(\sum_{i=1}^{N} \beta_i \sqrt{P_i} \cos\phi_i\right) a_k \sin\theta \qquad (11)$$

and

$$\sigma^2_{x_k} = \overline{(x_k - \overline{x}_k)^2} = \frac{1}{2T_s} \sum_{i=1}^{N} \beta_i^2 N_{0i} \qquad (12)$$

where T_s = symbol time.

Therefore, the signal-to-noise ratio is

$$SNR = \frac{\left(\sin\theta \sum_{i=1}^{N} \sqrt{2P_i T_s} \cos\phi_i\right)^2}{\sum_{i=1}^{N} \beta_i^2 N_{0i}} \qquad (13)$$

Note that for a rate 1/2 convolutional code, bit energy is

$$E_{bi} = 2P_i T_s \sin^2\theta \qquad i = 1, 2, \ldots, N \qquad (14)$$

Let $f(E_b/N_0)$ represent the bit error rate for a given bit SNR E_b/N_0. Then the conditional bit error rate is

$$P_b(\phi_1, \phi_2, \ldots, \phi_N) = f\left(\frac{\left(\sum_{i=1}^{N} \beta_i \sqrt{E_{bi}} \cos\phi_i\right)^2}{\sum_{i=1}^{N} \beta_i^2 N_{0i}}\right) \qquad (15)$$

$$\text{where } f(x) = \begin{cases} \exp[-(\alpha_0 + \alpha_1 x)], & x \geqslant \dfrac{\ln(2) - \alpha_0}{\alpha_1} \\ \dfrac{1}{2}, & |x| \leqslant \dfrac{\ln(2) - \alpha_0}{\alpha_1} \\ 1 - \exp[-(\alpha_0 + \alpha_1 x)], & x \leqslant -\dfrac{\ln(2) - \alpha_0}{\alpha_1} \end{cases} \qquad (16)$$

where $\alpha_0 = -4.4514$ and $\alpha_1 = 5.7230$, for standard constraint length 7, code rate 1/2 convolutional code.

Note that $\phi_i, i = 1, 2, \ldots, N$ are independent, having probability density functions

$$p(\phi_i) = \frac{e^{\rho_{li} \cos\phi_i}}{2\pi I_0(\rho_{li})}; \quad i = 1, 2, \ldots, N \qquad (17)$$

where ρ_{li} is the previously defined carrier loop SNR for station i (Eq. 5). The average bit error rate is

$$P_b = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \cdots \int_{-\pi}^{\pi} P_b(\phi_1, \phi_2, \ldots, \phi_N)$$

$$p(\phi_1)p(\phi_2), \ldots, p(\phi_N) d\phi_1 d\phi_2, \ldots d\phi_N \qquad (18)$$

Clearly P_b depends on the weighting parameters $\beta_2, \beta_3, \ldots, \beta_N$. For a given set of $\rho_{li}, i = 1, 2, \ldots, N$ we can find the β_i's which minimize P_b. Analytically this is a very complex task. However, it is straightforward to optimize the β_i's when there are no tracking phase errors. We will use these values of β_i here. These choices of β_i will be suboptimum when the carrier loop SNR is below 15 dB, but the difference in the resulting bit error rate, P_b, is very small.

We want to minimize P_b when there are no phase errors. Note that $f(\cdot)$ (Eq. 16) is a convex, monotonically decreasing function of effective bit SNR. Therefore, minimizing P_b is equivalent to maximizing the effective bit SNR with respect to the β_i's, $i = 1, 2, \ldots, N$.

With no phase errors, the effective bit SNR is

$$SNR = \frac{\left(\sum_{i=1}^{N} \beta_i \sqrt{E_{bi}}\right)^2}{\sum_{i=1}^{N} \beta_i^2 N_{0i}} \qquad (19)$$

Note that this function is convex. Therefore, taking the derivative with respect to β_i, $i = 1, 2, \ldots, N$ and setting it equal to zero, we get

$$\beta_i = \sqrt{\frac{E_{bi}}{E_{b1}} \frac{N_{01}}{N_{0i}}} \tag{20}$$

These choices of β_i minimize the bit error rate. Using (20) in (15), we obtain

$$P_b(\phi_1, \phi_2, \ldots, \phi_N) = f\left[\frac{\left(\sum_{i=1}^{N} \frac{E_{bi}}{N_{0i}} \cos \phi_i\right)^2}{\sum_{i=1}^{N} \frac{E_{bi}}{N_{0i}}}\right] \tag{21}$$

Using (21) in (18) we get the average bit error rate. (Gauss-Chebyshev quadrature formula is used to compute (22).

$$P_b = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \cdots \int_{-\pi}^{\pi} f\left[\frac{\left(\sum_{i=1}^{N} \frac{E_{bi}}{N_{0i}} \cos \phi_i\right)^2}{\sum_{i=1}^{N} \frac{E_{bi}}{N_{0i}}}\right]$$

$$p(\phi_1)p(\phi_2) \ldots p(\phi_N) d\phi_1 d\phi_2 \ldots d\phi_N \tag{22}$$

III. Numerical Results for NCP

The current NCP configuration consists of four antennas: a 64-m antenna and three 34-m antennas. Let station 1 be the one with a 64-m antenna. One of the 34-m stations has both transmit and receive capabilities, i.e., T/R, while the other two can only receive, i.e., listen only or L/O.

We assume the additive noises associated with each received signal are independent with equal noise spectral densities, i.e., $N_{01} = N_{02} = N_{03} = N_{04}$. A Block IV receiver with a 30-Hz threshold loop noise bandwidth is assumed for each station. It is also assumed that the received powers for the 34-m T/R station and the two 34-m L/O stations are less than the received power for the 64-m station by 5.8 dB, and 4.6 dB, respectively.

From (20) we have

$$\beta_1 = 1$$

$$\beta_2 = 1/2\,(-5.8 \text{ dB}) = 0.513$$

$$\beta_3 = \beta_4 = 1/2\,(-4.6 \text{ dB}) = 0.589$$

Using these parameters, numerical results have been obtained. The telemetry bit error rate is evaluated as a function of E_{b1}/N_{01}. This is shown in Fig. 2 for different values of ρ_{l1}, the carrier loop SNR for station 1. Comparisons between the radio loss for a single 64-m station and the array for bit error rates of 5×10^{-3} and 5×10^{-5} are illustrated in Figs. 3 and 4, respectively. When the carrier loop SNR is greater than 15 dB there are essentially no differences between the two curves. When the carrier loop SNR is less than 15 dB, the array has a decrease in radio loss with respect to a single station.

Reference 5 provided the telemetry total power to noise spectral density ratio as a function of time during Uranus encounter for DSS 43 in Australia (a typical case). The carrier margin was derived assuming an 80° modulation index. The carrier margin for DSS 43 during Uranus encounter is plotted as a function of time in Fig. 5. Corresponding to Fig. 5, the radio loss is plotted as a function of time in Fig. 6 for a four-element array in Australia during Uranus encounter.

IV. Conclusion

The arraying of N antennas, in this case a 64-m station and three 34-m stations, provides not only gain due to an increase in effective antenna aperture, but also a decrease in radio loss. The amount of this decrease is a function of the desired bit error rate and carrier loop SNR. When the carrier loop SNR is very high, the radio loss is negligible so the array gain is due only to the increased antenna aperture. For lower loop SNR's the decrease in radio loss is substantial, as shown in the radio loss curves. For the NCP configuration, with the Voyager spacecraft during Uranus encounter, radio loss can be expected to be less than 0.6 dB for the majority of the pass. This assumes a BER of 5×10^{-3}. The radio loss should always be under 1.0 dB.

References

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4. Lindsey, W. C., *Synchronization Systems in Communication and Control*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1972.

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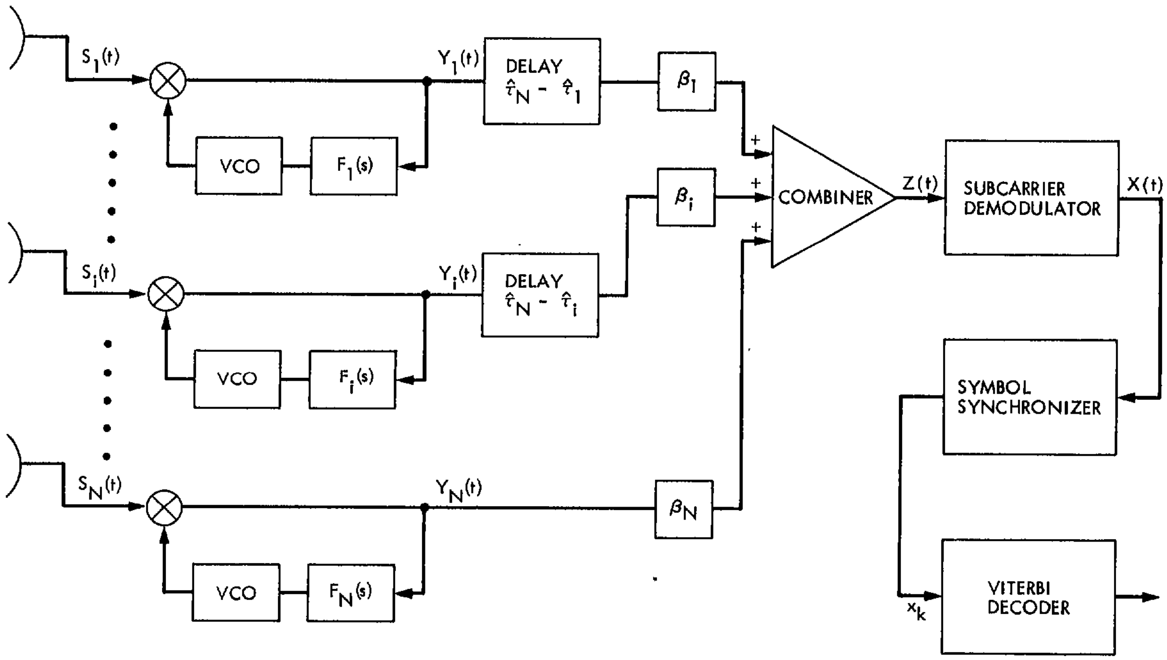

Fig. 1. Configuration for arrayed network with baseband arraying

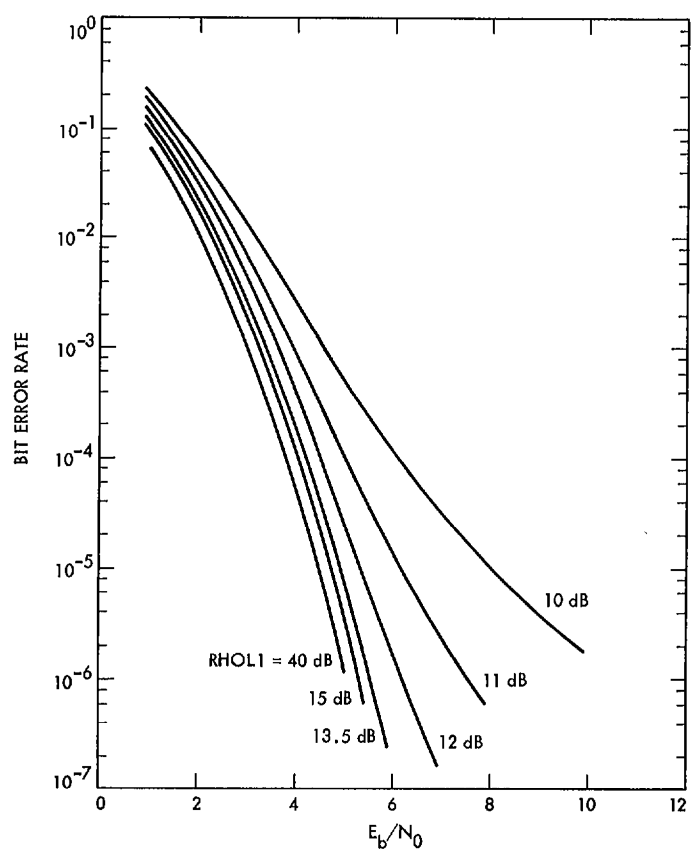

Fig. 2. Performance curves for a four-element array using a baseband combiner

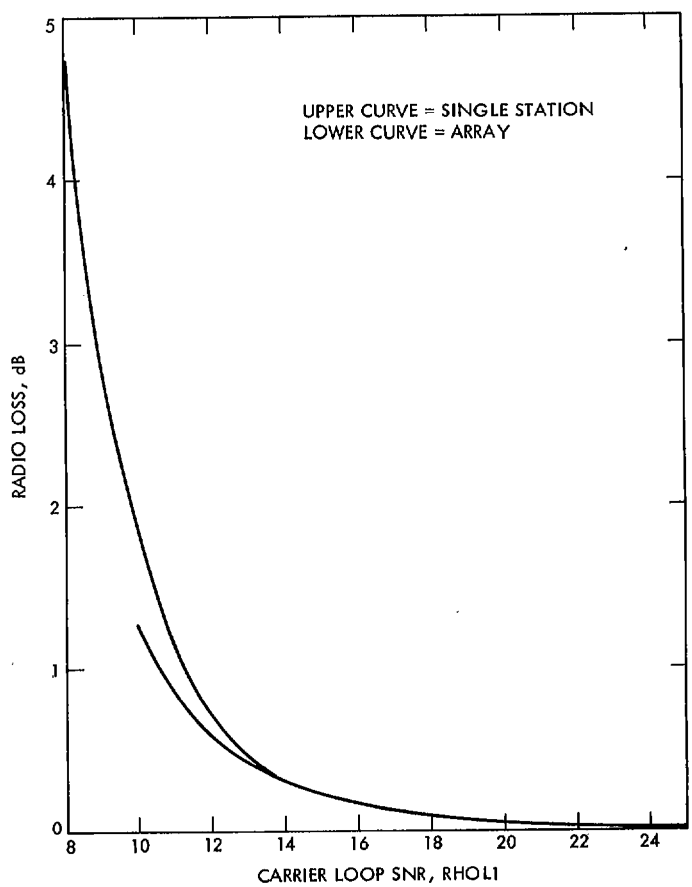

Fig. 3. One-way high rate radio loss for a BER of 5×10^{-3}

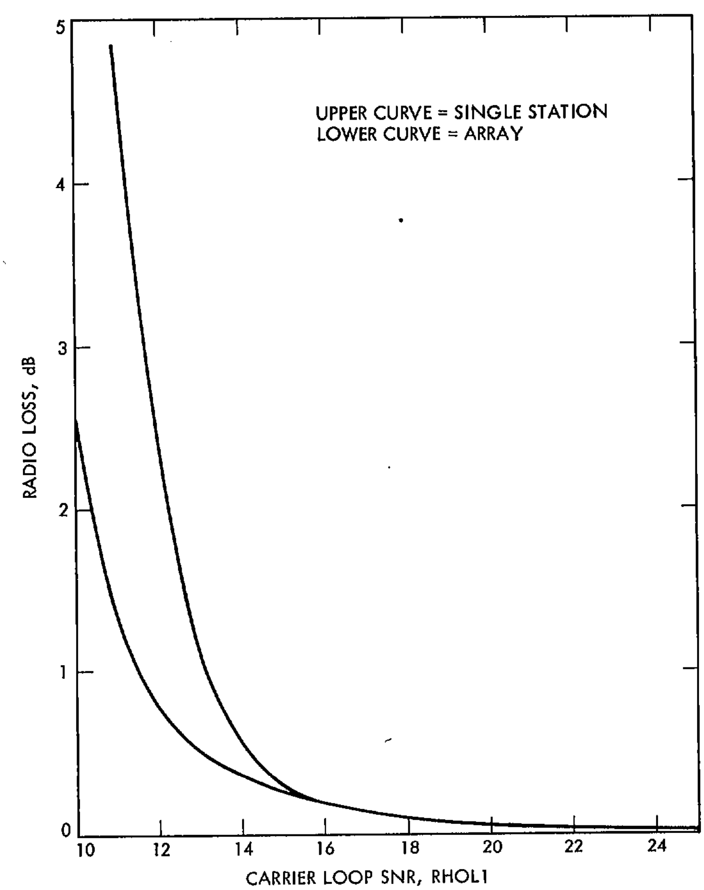

Fig. 4. One-way high rate radio loss for a BER of 5×10^{-5}

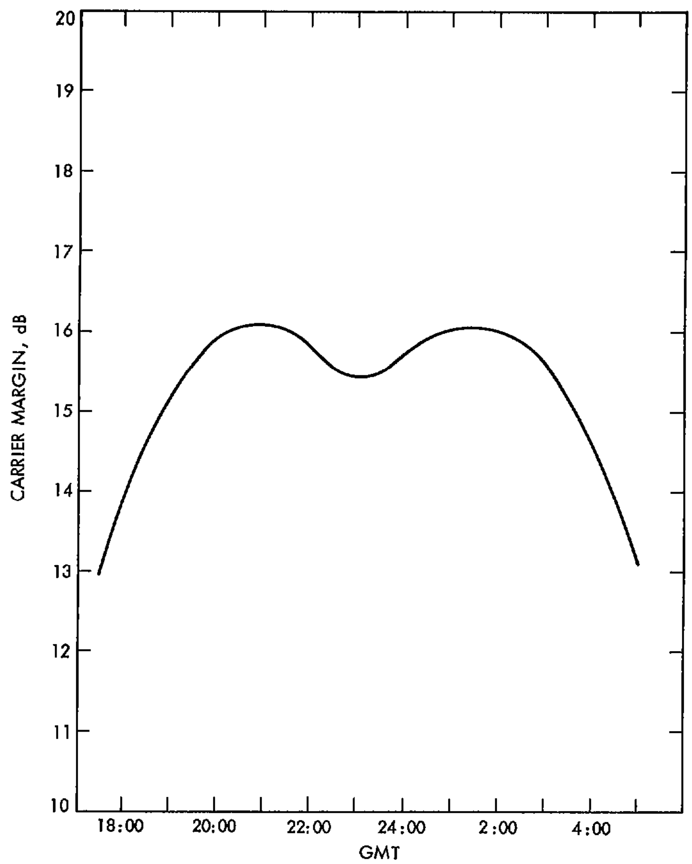

Fig. 5. Carrier margin for DSS 43 at Uranus

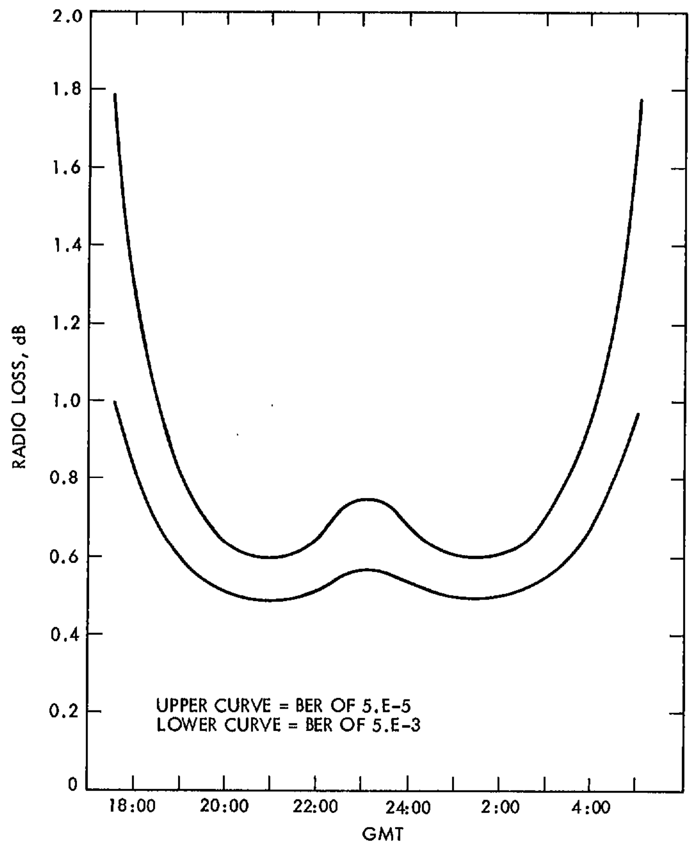

Fig. 6. Radio loss for Australian array at Uranus