An Optimization Model for Energy Generation and Distribution in a Dynamic Facility

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An analytical model is described using linear programming for the optimum generation and distribution of energy demands among competing energy resources and different economic criteria. The model, which will be used as a general engineering tool in the analysis of the Deep Space Network ground facility, considers several essential decisions for better design and operation. The decisions sought for the particular energy application include: the optimum time to build an assembly of elements, inclusion of a storage medium of some type, and the size or capacity of the elements that will minimize the total life-cycle cost over a given number of years. The model, which is structured in multiple time divisions, will employ the Decomposition Principle for large-size matrices, the Branch-and-Bound Method in mixed-integer programming, and the Revised Simplex Technique for efficient and economic computer use.

I. Introduction

The problem of allocating limited resources among competing activities in the "best" possible way has been always the prime concern to any organization. To make decisions after comparing the performance characteristics and life-cycle costs of existing versus new designs or between two new alternate designs is a common engineering practice. However, seeking better decisions with "optimum" designs rather than "working" designs is a superior engineering practice that has progressed only in the last few decades as a result of advances in computer technology. Solving large optimization problems with variables in the order of ten thousands is now feasible (Ref. 1).

In light of decreasing fossil energy resources, erosion of purchasing power due to inflation, rising operation, mainte-

nance and utility costs, and the inevitable need to replace equipment because of obsolescence or wear, increased attention has been given to optimum solutions for energy systems. Optimum configurations of energy generation and distribution systems are sought from the supply (or material) end to the demand (or consumer) end with multiple competing processes and links in between.

Fortunately, all optimization problems are similar in structure (Ref. 2) whereby a measure of the system effectiveness, hereinafter called the "objective function", which involves many decision variables, is to be maximized (or minimized), subject to some limitations or constraints. The aim of any optimization problem is to seek the optimum decision variables that determine the "best" competing activities among limited resources, and determine the maximum (or minimum) objective function.

For a dynamic facility, the design of an efficient and economical energy network is essential, so that the available energy forms are efficiently converted and distributed to the consumer in a cost-effective manner. The development of an optimization model for such a network provides an excellent morphology for the optimal mixing of several different types of resources, multiple components, links, and energy consumers. The optimization model also provides for the optimal operation of such a network when installed.

Currently available simulation models (Refs. 3 to 6) for thermal systems are custom made to solve a preselected design configuration. A few of these models include some component optimizations, yet they have limited applications. An optimization problem with large complex constraints, decision variables, and many possible combinations of energy resources, power plants, and consumers render the model suitable for use with a computer. While the mathematical tools for solving these large optimization problems are available (Refs. 1, 2, 3, and 7), formulating the problem in the "standard" form represents the majority of the effort. As the dynamic facility changes its configuration with time, analytical modeling becomes necessary to operate the facility in its "best" economical condition. Variations in costs, energy prices, weather patterns, load profiles, the energy supply, and reliability constraints could be treated as deterministic, probabilistic, or a combination of both as determined by the analyst.

The Deep Space Network (DSN) facilities that are part of the NASA-owned facilities have identified under several projects the consequences of energy supply shortages and the important need for energy conservation and self-sufficiency. The prime concern has been to reduce operational costs and to improve the facilities reliability and maintainability for successfully supporting deep-space tracking missions. Under the DSN Energy Conservation Project, several feasibility studies are being carried out for the installation of system(s) that will provide reliable energy to the Deep Space Communication facilities in sufficient amount and at a competitive cost.

In Figs. 1 through 4, a few examples are given for the energy systems under study. Figure 1 shows a typical DSN facility in Spain where only diesel engines are used as prime movers. Possible ties to a nearby utility network and further coupling with high-performance fuel cells are being investigated. Decisions to "build" or "no-build," when to start construction, and what capacities should be built are part of the answers sought by the study.

Another example of using the optimization tool is in the search for the optimum-size solar concentrator, as shown in Fig. 2, for gas-fired heat pumps presently near commercializa-

tion. A dual utilization of solar energy for direct power via photovoltaic cells and for thermal energy via solar collectors is shown in Fig. 3 coupled to conventional gas and electric power in a building. A larger combination of links and processes is sketched in Fig. 4, where different forms of energy resources share loads with the conventional utilities in meeting the facility demands.

Conservation measures applied to equipment increase the efficiency of energy utilization, thus reducing the consumption of other conventional energy sources. Also, conservation measures applied to a building envelope (such as adding insulation, lighting reductions, etc.) reduce the consumption levels of original resources. A conservation measure treated as an "energy resource" will have a cost per unit of energy that depends on the measure's implementation cost and its potential reduction of demand.

The sample energy systems shown in Figs. 1 to 4 (or any combination of these systems) require an optimization model that can handle a large number of configurations yet be expressed in simple terms for low computational cost to enable wide user acceptance.

Chapman (Ref. 8) has outlined the first energy distribution model, which provided a starting point for this study. His objective was to determine which of several possible power plants and connecting links (decision variables) need to be constructed while satisfying the needs (constraints) of the energy consumers at a minimum total life-cycle cost (objective function). Each component was assumed to have a known cost function of its capacity, reflecting the initial capital cost plus the maintenance cost over the expected lifetime. Chapman's model was limited in scope and was not tested numerically. As a result of on-going energy conservation activities and changes in economic parameters, the facilities configuration is changing and an update of the initial Chapman model seemed necessary. The following section summarizes the present study objectives

II. Study Objectives

The objectives for developing the energy generation-distribution model are listed below; they are the result of a careful review of the relevant literature.

(1) To provide a working engineering tool for the optimal design of mechanical systems in general, and, in particular, the optimal design of an energy network comprised of a mixture of several different types of energy sources, and serving different consumers. The tool should provide also the optimal construction time and operation of such a network when installed. The

model should include both a "concept optimization" and a "design optimization" within a given concept.

(2) To determine the size or capacity of each add-on element and the distribution pattern for each energy resource, without affecting the network availability, reliability, or continuity of service.

(3) To achieve the above two objectives by an optimum generation-distribution system that either must have a minimum life-cycle cost, or must be built under a limited budget while satisfying other essential constraints such as the energy demands of each consuming building, the capacity constraints of the distribution links, the constraints of equipment reliability and availability during critical tracking periods, the convenience in startup and shutdown, and the periods of restart after a period of shutdown.

(4) The energy model should consider future decreases of building loads as a result of the on-going efforts for energy consumption reduction. Conservation measures should be treated, in a broader view, as an "energy resource" to compete with other conventional and nonconventional resources.

(5) To perform a sensitivity analysis of the model's optimum solution(s) to variations in economic and noneconomic parameters. While near-term cost could be estimated with a reasonable degree of certainty, long-term costs and escalation rates will be probabilistic. This objective is desirable especially when considering the inflation trends experienced during the last decade. Although the study may seek a single optimal solution, sensitivity trends of cost escalations and operation and maintenance changes compared to those of existing systems have to be considered as well.

III. Approach

A broad variety of energy generation-distribution systems and combinations utilizing several energy resources could be proposed, as shown in Figs. 1 through 4. The major operation is how one should approach the optimal system design. Optimization (sometimes called programming), whether it is in linear or nonlinear form, has been applied to many technical and management decisions. Linear optimization is used when both the constraints and the objective function are expressed by linear or piecewise linear functions of the decision variables. Decomposition techniques are employed if the problem is large and can be partitioned in special form (Ref. 1). The optimum feasible solution will be the particular selection of the decision variables that satisfy all the constraints while minimizing the total life-cycle cost.

In the present study, we will try to set up and formulate the problem for using the well-developed linear programming (LP) methods. Linear programming is known to be very efficient for solving large optimization problems (Ref. 1). In mathematical terms, we will seek to minimize the total life-cycle cost (TLCC) of the system (objective function), which is written in linear terms of N_u unknowns (or decision variables), and set up the constraint equations bounding the solution. Hence, we seek to minimize

$$TLCC = \sum_s C_s X_s \tag{1a}$$

Subject to N_g constraint equations of the equality type:

$$\sum_s a_{rs} X_s = b_r \tag{1b}$$

and

$$X_s \geqslant 0 \tag{1c}$$

where $s = 1, 2, \cdots N_u$ and $r = 1, 2, \cdots N_g$. Equation (1c) constrains all the unknown variables to be nonnegative. Inequality constraints with greater than or equal to ($\geqslant$) or less than or equal to ($\leqslant$) can be converted to equality-constraints by using additional "slack" variables. The format of Eq. (1) is a "standard" format for optimization problems. Several methods are available in the literature (Ref. 1, 2) for solving Eq. (1), of which the Simplex Algorithm is extremely efficient, especially when the number of variables is large. Several versions of the Simplex Algorithm, such as the Revised-Simplex, the Dual-Simplex, and the Revised-Dual Simplex, can be used to reduce the computer memory use and cost. The Dantzig's Decomposition Principle (Ref. 1) is particularly useful for large problems if the elements of the constraint matrix, a_{rs}, are diagonalized. In addition, if some of the unknown variables must be integers, the problem is called Mixed-Integer Programming and the Branch-and-Bound Technique or equivalent is further used. For more details on the various methods to solve Eq. (1), see Refs. 1, 2, 3, and 7.

Although the DSN ground stations are typically distributed facilities, selecting a single facility or grouping the facilities into a facility complex will be made for simplicity. Hence, the energy generation and distribution model will be restricted to only *one* consumer to represent one building, a group of buildings, or a complex facility. Existing as well as newly built power plants dealing with various energy types will be

examined for operation in optimum conditions to satisfy the building demands while simultaneously minimizing the total life-cycle cost of the energy network over a certain planning period. The computerized model should be modular and open-ended to accommodate a distributed network consisting of a number of energy sources, conversion processes, storage nodes, consumers, and the internal transmission network that links supply and demand. Requirements for this model will be to provide characteristics of the facility elements as a basis for load analysis, design improvement, and "build" or "no-build" decisions. The format of the model should: (1) be amenable to checking and future modification, (2) require user information that is normally available and clearly definable to avoid ambiguity, and (3) be capable of such efficient execution that undue demands are not placed upon the computer facilities and excessive run-time costs are not entailed.

The type of elements considered in the model are conversion processes, and storage distribution nodes and links. Identical mathematical representations are used to describe any element type within a given time step. However, the presence or absence of particular features for individual elements are accounted for by a set of dichotomous (binary) indices assigned to the individual assemblies. Further, the distribution model allows: (1) the transfer of electrical energy between power plants, (2) intermediate storage-distribution nodes at various sites between plants, and (3) the substitution of electrical energy for direct heating and cooling for any consumer.

Interactions between nodes, processes, and assemblies will include:

(1) The economical competition of a mixture of energy sources and conversion processes to supply the loads at particular consumer sites.

(2) The logistics of the energy transmission network and storage media considerations for load leveling or "shedding."

(3) Selection of component location to be either centralized or distributed with respect to the location of energy resources and consumer sites.

(4) Synthesis of a network to meet time-varying energy usage at the consumption locations and a reliable installed capability to meet peak consumption requirements.

(5) The effects of daily, monthly, and seasonal variations on power generation capability, on environmental influences upon demand, and on the total operation cost including the time of use.

IV. Assumptions

Mathematically, the model is comprised of a collection of equations that describe the essential properties and controls of the system being modeled. Derivation of energy (or material) relationships is essential. The total system cost (objective function) will be the sum of the total life-cycle cost of individual subsystems, which in turn will include the initial capital cost plus the operation, maintenance, and replacement costs over the study period. A block diagram for any energy generation-distribution system is shown in Fig. 5., where conversion processes are symbolically denoted by circles and storage-distribution nodes are denoted by rectangles. The following assumptions are made:

(1) Type of energy sources: These are end nodes, represented by rectangles in Fig. 5, which include but are not limited to:

(a) Electromagnetic waves in the solar band (solar radiation).

(b) Kinetic energy of wind currents (wind energy).

(c) Solid fossil fuel such as coal and shale.

(d) Gaseous fossil fuel such as natural gas, methane, or any gaseous coal product.

(e) Liquid fossil fuel such as diesel oil, gasoline, or any liquid coal product.

(f) Direct electrical energy supplied by a utility company through an electrical network.

(g) "Energy conservation," which is treated broadly as an "energy resource." Special care should be taken, however, when it is included in the analysis.

(h) Water or process steam as a material or energy resource, respectively.

(i) Air as a material resource for combustion processes.

(2) Energy (or material) conversion processes: These are designated by circles in Fig. 5, and include but are not limited to any of the following:

(a) Low- and high-concentration-ratio solar collectors for converting solar radiation into thermal energy.

(b) Photovoltaic cells for direct conversion of solar radiation into electrical energy.

(c) Wind turbines for direct conversion of wind kinetic energy into electrical energy.

(d) Solar ponds for heating, electrical power applications, or both.

(e) Internal or external combustion engines, using fossil fuel for electrical power generation, with or without waste-heat recovery.

(f) Boilers for hot water or steam generation using fossil fuel combustion.

(g) Fuel cells for direct conversion to electricity with and without waste-heat recovery.

(h) Mechanically or electrically powered heat pumps for both heating and cooling.

(i) Electrical resistance heaters.

(j) Heat-powered absorption or jet (ejector) refrigeration units for cooling.

(k) Mechanically or electrically driven vapor compression chillers.

(l) Power cycles for electrical power generation driven by such high-temperature heat engines as organic or steam Rankine, Stirling, and Brayton.

(m) Waste-heat recuperators, regenerators, or heat exchangers for heat recovery.

(n) Empty (or dummy) processes to by-pass a stage of nodes to another stage and simplify the mathematics of the problem.

(3) Storage-distribution nodes: Each is represented by a rectangle in Fig. 5, and assumed to consist of one type of energy form or storage media such as (but not limited to):

(a) Electrical energy.

(b) Fossil fuel.

(c) Low-temperature or high-temperature thermal energy, such as cold water or hot water tanks.

(d) Chemical energy (or material) as in oxygen gas, hydrogen gas, and water.

(e) An empty (or a dummy) node for the mathematical simplification of the problem.

Storage capability (such as a flywheel, a battery, or water tanks) could be added to each node. Storage-distribution nodes are controlled by conservation laws relating the influx and outflux of energy (or material). The first law of thermodynamics, the continuity equation, or Kirckhoff's law for electrical nodes could be applied, for example. However, by selecting each node to handle only one type of energy (or material), only one conservation equation is needed for each node. No lateral or cross exchange between nodes within the same stage of nodes is assumed. If lateral nodal linkage

is desired, the system network should be organized by utilizing an additional number of empty (or dummy) elements.

(4) Staging: A set of conversion processes and storage-distribution nodes need to be constructed between the available energy sources (supply) on one end and the facility loads (demand), on the other. A large number of different possibilities arise as a result of different types of supply and demand. Grouping of conversion processes and storage-distribution nodes into "stages" is done for convenient sequence identification.

A stage of conversion processes will consist of a number of processes that have the same "level." Hence, all processes in the first stage have to be undergone before the processes in the second stage can begin, and so on. The same is applicable for storage-distribution nodes. Theoretically, there is no limit to the number of processes, nodes, or stages in any system configuration. The only limit is the memory of the available computer to solve the problem.

An assembly is defined here as an integral collection of conversion processes, storage-distribution nodes, and their associated links that could have a "stand alone" function. For example, an add-on photovoltaic array, with its electrical linkage to an existing electrical node is treated as an assembly. In the model, "build" or "no-build" decisions are associated with assemblies rather than elements due to the above integral function.

(5) Conservation equations: Energy (or material) balance of each storage-distribution node is mandatory. The choice to "build" or "no-build" a storage capacity for a given node could be determined by the model. Storage capacities, if a "build" decision is made, are either known in advance or considered part of the decision variables.

(6) Capacity Bounds: If a particular link, conversion process or storage node is to be constructed, its capacity must: (a) exceed the prescribed lower bound for design feasibility, (b) lie below the prescribed upper bound, which may not be necessary in some cases, and (c) provide enough margin above the peak power experienced during operation. These constraints are useful in forcing the mathematics of the optimum solution to yield a practical design for each element size and to meet peak demands without overloading.

(7) Reliability: The reliability of an add-on or new system should be higher than or at least equal to the reliability of the existing system. To satisfy this reliability condition, two approaches could be followed. The first

is to calculate the optimum element capacity, followed by a slight overdesign. The optimum capacity is divided then into a large number of small modules rather than a small number of large modules. The final "practical" system selected by this approach may not be the least expensive, since an overdesign is later incorporated after the analytical solution is obtained. The second approach is to modify the original problem statement by additional size and cost constraints to keep the system's reliability at least the same. The optimum solution in the second approach does not need a readjustment for reliability after it is obtained. The selection of one of the two approaches is optional. However, the first approach is assumed throughout due to its simplicity.

(8) Dummy Elements: The generation-distribution system should allow loading and unloading various links producing the same kind of output energy (or material) when all are connected at a distribution-storage node. Empty (or dummy) elements are permitted to form an empty set with a null effect on the flux passing through. Also, dummy elements should be assigned numbers and be treated the same way as active elements in forming the matrix of equation coefficients.

(9) Centralization: Centralized, distributed, and semicentralized semidistributed systems could be parameterized by the linkage distances, which are either known in advance or assumed to be part of the decision variables.

The present model identifies each element in the system simply by its location and number, and allows changes to be made in the original system configuration without much effort. The cost of installing any element in an assembly is itemized into several linearized parts, which are addressed below.

V. Cost Analysis

Life-cycle cost (LCC) is an evaluation method that takes into account relevant cost over a selected time period of a system of elements, materials, and operation. It incorporates initial investment costs, future replacement costs, operation and maintenance cost, and salvage values, adjusting them to a consistent time basis and combining them in a single measure that makes it easy to compare alternative options (Refs. 9-13).

The discount rate i' % for a certain time period is the rate of interest that reflects the investor's "opportunity cost" of money, excluding general inflation. The effect of discounting is to obtain the present value equivalent of future cash flow.

Whether a discount rate of 7% is selected, as recommended for federal buildings (Ref. 9), or 2% as in DSN practice (Ref. 11) is immaterial at this point. Different discount values will change the present worth equivalent of future savings for projects displacing the use of a conventional fuel. The discount expression is given by:

$$PW \text{ equivalent} = \frac{\text{future cash flow occurring at period } m}{(1 + i')^m}$$

Several criteria for LCC analysis are explained in detail in the literature (Ref. 9). Examples are the total life-cycle cost (TLCC), net savings, savings-to-investment ratio, simple payback period, discounted payback period, and annual levelized cost. In this study, the total life-cycle cost (TLCC) is used as the objective function. This sums all significant costs of a system over a certain period, discounted to present value at a selected base year. Detailed life-cycle cost elements could be classified into direct costs for work force and indirect costs for such items as services, travel, and computer programming. A broader classification is made according to the execution sequence (Ref. 12) as follows:

(1) Future research and development cost.

(2) Future planning cost including cost of feasibility studies.

(3) Implementation cost including cost of design reviews, testing, quality assurance, and installation.

(4) Maintenance and operations cost including cost of utility and preventive maintenance.

(5) Sustaining cost including the cost of replacement and modifications.

(6) Salvage cost.

Each of the above cost elements should be multiplied by an inflation (or discount) factor and an overhead factor. Cost estimates are provided in constant dollars computed at a base year, which is usually the year in which the study is performed.

The key elements of the LCC methodology used in this study follow the NBS rules (Ref. 9), which are utilized in Appendix A for the input data. The obligations that occur at different times should be adjusted to a common time basis, which might be: (1) the present, whereby all expenses are converted to an equivalent value occurring now, (2) annually (or levelized), whereby all expenses are converted to an equivalent value occurring in a uniform amount each year over the study period, and (3) the future, whereby all expenses are converted to an equivalent value occurring at some common

time in the future. For convenience, the present worth basis will be used.

The "economic efficiency" of a given system configuration can be improved by the optimal timing of investment decisions. To have a "wait option" means timing the investment so as to capture the largest possible long-run benefits in the face of technological advances and other future changes.

The monthly recurring fuel or nonfuel operation and maintenance costs are assumed to begin to accrue at the beginning of the base year. These are evaluated as lump sum amounts at the end of each *month* over the study period, starting with the end of the first month of a base year. Treating investment costs as a lump sum occurring at the beginning of the construction time is a simplified approach somewhat less accurate than a detailed analysis employing actual scheduling and accounting of costs. The difference in the two approaches is generally not large. Nonannually recurring replacement costs and salvage values are temporarily neglected in the analysis.

The time step-by-step method of calculating the present worth energy cost is suitable since both the quantity and type of energy in each element are expected to change periodically, for each hour, day and month. Due to the time changing of installation costs, the development of new technologies, the declining of a component efficiency with time, the increase of maintenance cost by aging, and the escalation of energy costs and maintenance costs as a result of inflation, it becomes necessary to have the model divided into multiple time steps. The structure details of the model are explained in the next section.

VI. Equation Formulation

The relationships between the decision variables and the constraint equations are grouped by the headings that follow.

A. Time Steps

In the model, a distinction should be made between an energy (or material) flux time step, m (where $m = 1, 2, \cdots N_t$) and a construction time step n, (where $n = 1, 2, \cdots, N_{ct}$) as illustrated in Fig. 6. The first type time steps are assumed to be four for each representative day for each month to account for four different time-of-day uses for N_y years under study. This means a total of $48 N_y$ time steps. Construction time steps, on the other hand, indicate the time-frequency at which management could make the start-of-construction decisions. For instance, due to known budgetary cycles or management directions, the beginning of construction within the user's organization is assumed to take place once a year, once every 6 months, once every 3 months, or once a month. Hence, each

year of the study period will be divided into 1, 2, 4 or 12 construction-time steps where decisions to "build" or "no-build" are made and construction monies are spent. The construction time steps are N_y, $2 N_y$, $4 N_y$, or $12 N_y$, respectively. The selection of small periods for energy (or material) flux computations and large periods for construction time decisions forces the solution and the feasible region[1] to eliminate undesirable answers. Without the construction-time steps, a new assembly built when $m = 59$, for instance, means that construction must start in the second year of study, the month of March and off-peak period, which is not a practical time. Although the selection of n is arbitrary, it is restricted in this model such that the start-of-construction decisions are not within a month. Since four consecutive energy (or material) flux time steps represent a one-month period, the number of decisions to "build" or "no-build" during N_y years becomes:

$$N_{ct} = N_{t/4} \quad = 12 N_y \text{ for monthly construction decisions}$$

$$N_{ct} = N_{t/12} = 4 N_y \text{ for quarterly construction decisions}$$

$$N_{ct} = N_{t/24} = 2 N_y \text{ for semiannual construction decisions} \qquad (2)$$

$$N_{ct} = N_{t/48} = N_y \text{ for annual construction decisions}$$

where $N_t = 48 N_y$.

B. Losses in Transmission Links

The energy (or material) flux travelling through the transmission links between a conversion process and a storage-distribution node are generally expressed as:

$$\overline{E}(k) = E(k) - \alpha(k) L(k) E^\gamma(k)$$

where $\overline{E}(k)$ is the destination energy (or material) flux at the receiving end of the link k, $E(k)$ is the origin energy (or material) flux at the sending end of the link, $L(k)$ is the link length, $\alpha(k)$ is a proportionality loss coefficient, and γ is an exponent between 1 and 2. The coefficient α is a characteristic link constant. To suit a linear programming format, we assume

$$\overline{E}(k) = T(k) E(k) \qquad (3)$$

where k is the number assigned to the link joining the process i and the node j, and $T(k)$ is the transmission efficiency, taken

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[1] This is the mathematical region which contains the feasible solutions of the problem that satisfy all the constraint equations.

as a constant. Note that the indices i, j, and k will take the following values:

$$i = 1, 2, \cdots N_c, j = 1, 2, \cdots N_o, \text{ and } k = 1, 2, \cdots N_Q$$

where N_c, N_o, and N_Q are the number of processes, storage-distribution nodes, and connecting links, respectively.

C. Conversion Efficiency

In handling the conversion efficiency expressions, the following convention is used. A simple process is defined as having one input link and one output link. A compound process involves, in general, more than one input link and more than one output link, and could be decomposed into a number of simple processes as shown in Fig. 7. For any link k, connected to a process inlet, a coupling identifier $p(i, j) = -k$ is assigned. For any outlet link, an identifier $p(i, j) = k$ is assigned. The conversion efficiency (or yield) F is defined for a simple process, as sketched in Fig. 8, by the ratio of output flux to input flux in the output and input links, respectively, or

$$F(k_{in}, k_{out}) = \frac{E(k_{out})}{\overline{E}(k_{in})} \qquad (4)$$

where the first index, k_{in}, refers to the process inlet link number, the second index, k_{out}, refers to the process outlet link number. Combining Eqs. (3) and (4) to eliminate $\overline{E}$, the set of conversion efficiency equations are written in terms of the E unknowns as:

$$E(k_{out}) - F(k_{in}, k_{out}) \, T(k_{in}) \, E(k_{in}) = 0 \qquad (5)$$

Equation (5) represents N_c linear equations for N_c simple processes, applicable for each time step.

D. Storage-Distribution Nodes

Energy (or material) conservation equations must be written for the storage-distribution nodes, including those end-type nodes that represent either the energy supply reservoirs or building demands. Selecting a uniform state, uniform flow (USUF) control volume for the storage-distribution nodes, N_{sd}, will encompass internal flux changes during the time interval $\Delta t(m)$ from $t(m - 10$ to $t(m)$. From Fig. 9, a summation over all process inlet and outlet links would give:

$$\underbrace{\Delta t(m) \sum_{k_{out}} \overline{E}(k_{out}, m)}_{\substack{\text{Sum over inlet links to a node} \\ \text{(process outlet links where } p \\ \text{is positive)}}}$$

$$\underbrace{- \Delta t(m) \sum_{k_{in}} E(k_{in}, m)}_{\substack{\text{Sum over outlet links from} \\ \text{a node (process inlet links} \\ \text{where } p \text{ is negative)}}} \qquad (6)$$

$$\underbrace{- SC(j, m) + SC[j, (m - 1)]}_{\substack{\Delta \text{ Energy stored, within node from} \\ \text{initial time } t(m - 1) \text{ to final time} \\ t(m)}} = 0$$

where E, $\overline{E}$ are rates computed at time $t(m)$, and $SC(j, m)$ is the internal energy (or material) content of storage node j at time $t(m)$. The fluxes E and $\overline{E}$ are assumed constant during the interval $\Delta t(m)$. If no nodal storage exists, the conservation equations for simple distribution nodes N_d will be reduced to the case of a steady-state, steady-flow (SSSF) control volume where all $SC(j, m)$ are eliminated:

$$\sum_{k_{out}} \overline{E}(k_{out}, m) - \sum_{k_{in}} E(k_{in}, m) = 0 \qquad (7)$$

Combining Eqs. (3) and (6) give for the conservation of the jth node out of N_{sd} nodes:

$$\Delta t(m) \sum_{k_{out}} T(k_{out}, m) E(k_{out}, m) - \Delta t(m) \sum_{k_{in}} E(k_{in}, m)$$

$$- SC(j, m) + SC(j, (m - 1)) = 0 \qquad (8)$$

Also, for N_d distribution nodes, Eqs. (3) and (7) Give:

$$\sum_{k_{out}} T(k_{out}, m) E(k_{out}, m) - \sum_{k_{in}} E(k_{in}, m) = 0 \qquad (9)$$

E. Supply/Demand End Nodes

The end storage-distribution nodes representing an energy resource (supply) or a building load (demand) are treated similarly to Eq. (9), but with a minor modification. For an end node j out of N_{rd} nodes representing an energy resource distribution with intensity $S(j, m)$, Eq. (9) is changed to:

$$S(j, m) \overline{A}(j) - \sum_{k_{in}} E(k_{in}, m) = 0 \qquad (10)$$

where $j = 1, 2, \cdots N_{rd}$.

Also, for a demand-distribution node j out of N_{dd} nodes representing each given type of building demand, $D(j,m)$, Eq. (9) is changed to:

$$\sum_{k_{out}} T(k_{out}, m) E(k_{out}, m) = D(j, m) \qquad (11)$$

where $j = 1, 2, \cdots N_{dd}$.

Note that either the product $[S(j,m) \overline{A}(j)]$ or the characteristic area $\overline{A}(j)$ will be the decision variable. For instance, in an energy system consisting of a field of solar collectors or a set of wind turbines, the energy intensities $S(j, m)$, will be known quantities and the decision variable in this case will be the characteristic projected area $\overline{A}(j)$ of either the solar collector field, or the blade area of wind turbines, respectively. If, however, energy is purchased from a supplier (such as a utility), the unknown product $[S(j, m) \overline{A}(j)]$, which is the flux, becomes the decision variable. Equations (8), (9), (10), and (11) represent a total of N_o equations for N_o nodes for each m period.

F. Storage Capacity Limit

There is a physical limitation in charging or discharging during any period $\Delta t(m)$ that constrains the changes in storage content to be always less than or equal to the design capacity $V(j)$. For additional cost saving, decisions are made to build or no-build storage media to match optimally the transient supply and demand curves without changing their daily sum of fluxes. Two decisions need to be made: the first is whether or not storage nodes of a new assembly need to be built and the second is to know which capacity should be designed for lowest cost. Since four time steps are assumed for each representative day of each month, it is appropriate to select the optimum storage capacity $V(j)$ that handles supply and demand matching only for a 24-hour time span. This means that at the beginning of the first day-period (where $m = 0, 4, 8, \cdots$, etc.), the storage content $SC(j, m)$ should be taken as zero (i.e., empty storage). The storage content must also be zero at the end of the fourth day period, which is also the beginning of the first day period of the next day representing the next month. Hence,

$$\left.\begin{array}{l} SC(j, 0) = 0 \\ \\ SC(j, 4v) = 0 \end{array}\right\} \qquad (12)$$

where v is monthly time step, $(v = 1, 2, \cdots, 12 N_y)$

This leaves us with the determination of the storage content only at the three intermediate times within a day, where $m = 4v - 3$, $4v - 2$ and $4v - 1$. Some storage nodes should be allowed, when fully charged, to "dump" the excess supply. This is the case, for instance, when a solar collector designed primarily for winter use is used for heating in a summer period. Another example is to intentionally waste available wind energy when both the supply exceeds the demand and the storage nodes are at full charged capacity. Excess energy (or material) flux should be treated in these cases as "slack" variables. During the study period, the storage content profile fluctuates from positive to negative regions as shown in Fig. 10. Although the storage node is designed to provide matching between supply and demand for one day only, different matching schemes over all the days of the study period need to be examined.

To allow for both negative and nonnegative values of the variable SC, it is commonly represented, for later use in the Simplex Algorithm, by the difference between two nonnegative variables:

$$\left.\begin{array}{l} SC(j, m) = SC'(j, m) - SC''(j, m) \\ \\ SC'(j, m) \geqslant 0 \\ \\ SC''(j, m) \geqslant 0 \end{array}\right\} \qquad (13)$$

Although Eq. (13) introduces an additional set of variables, it eliminates the need for nonnegativity constraints. In the final solution, the set $SC'(j, m)$ is reduced to $SC(j, m)$ if $SC(j, m) \geqslant 0$, thus making $SC''(j, m) = 0$. Also, the set $SC''(j, m)$ is reduced to $|SC(j, m)|$ if $SC(j, m) \leqslant 0$ thus making $SC'(j, m) = 0$. In other words,

$$SC'(j, m) = \begin{cases} SC(j, m) & \text{if } SC(j, m) \geqslant 0 \\ \\ 0 & \text{otherwise} \end{cases} \qquad (14)$$

and

$$SC''(j, m) = \begin{cases} |SC(j, m)| & \text{if } SC(j, m) \leqslant 0 \\ \\ 0 & \text{otherwise} \end{cases} \qquad (15)$$

Further, the optimum capacity of the storage node $V(j)$ could be constrained to be either less than or equal to (i.e., undersized) or larger than or equal to (i.e., oversized) the maximum swing of the variable SC.

Defining the maxima of the sets $SC'(j, m)$ and $SC''(j, m)$ over all the N_t time periods as $MSC'(j)$ and $MSC''(j)$, respectively, one may write the two possible constraint equations for $V(j)$ as either:

$$MSC'(j) + MSC''(j) - V(j) \geqslant 0 \qquad (16)$$

or

$$MSC'(j) + MSC''(j) - V(j) \leqslant 0 \qquad (17)$$

Equations (16) and (17) represent two different decisions: Eq. (16) constrains the storage capacity to be toward undersizing, and Eq. (17) toward oversizing. These two decisions differ in their impact on installation cost (due to the size of $V(j)$) and on the operation cost (due to the impact on the matching role between supply and demand fluxes). Only one out of the two constraints of Eqs. (16) and (17) must hold for any single node j. To solve this either/or constraints problem, two additional sets of dichotomous variables $\beta_1(j)$ and $\beta_2(j)$ should be introduced in conjunction with the "big M" method, (Ref. 2). M is a very large number, arbitrarily selected to be larger than any feasible MSC' or MSC''. Equations (16) and (17) are then rewritten as:

$$
\left.
\begin{aligned}
MSC'(j) + MSC''(j) &\geqslant V(j) + M\,\beta_1(j) \\[4pt]
MSC'(j) + MSC''(j) &\leqslant V(j) + M\,\beta_2(j) \\[4pt]
\beta_1(j) + \beta_2(j) &= 1 \\[4pt]
\beta_1(j), \beta_2(j) &\leqslant 1 \\[4pt]
\beta_1(j), \beta_2(j) \quad &\text{integers}
\end{aligned}
\right\} \qquad (18)
$$

Note that adding M to the right-hand side of a constraint equation has the effect of eliminating it. The formulation of Eq. (18) guarantees that one out of the two original constraints must hold. In addition, the maxima $MSC'(j)$ and $MSC''(j)$ could be expressed by the following linear set of constraints:

$$SC'(j,q) - MSC'(j) \leqslant 0 \qquad (q = 1, 2, \cdots 36\,N_y) \qquad (19)$$

and

$$SC''(j,q) - MSC''(j) \leqslant 0 \qquad (q = 1, 2, \cdots 36\,N_y) \qquad (20)$$

where q is the total number of intermediate times of day excluding the start and end times. For three intermediate times per day, 12 days per year, and N_y years of the study period, q ranges from 1 to $36\,N_y$.

G. Beginning of Construction Time

For each assembly, ℓ, there will be associated a set of conversion processes, storage-distribution nodes and links. For each construction time period n, there will be assigned a dichotomous decision variable $\lambda(\ell, n)$. The variable λ is a binary integer that can be either 0 or 1. A zero value for $\lambda(\ell, n)$ means that assembly ℓ is not to be constructed at the beginning of construction-time interval n. A value of one assigned for $\lambda(\ell, n)$ means that for assembly ℓ, construction starts at the beginning of construction time interval n. Construction of any assembly, no matter how large its size, is assumed to take place only once during the total period under study. The present model does not consider the possibility of adding capacity to any assembly sequentially, in multiple sizes and during multiple time steps. Therefore,

$$\sum_n \lambda(\ell, n) \leqslant 1, \qquad \ell = 1, 2, \cdots N_a \qquad (21)$$

where all $\lambda(\ell, n)$ elements are subject to the nonnegativity and integer conditions:

$$
\left.
\begin{aligned}
\lambda(\ell, n) &\geqslant 0 \\[4pt]
\lambda(\ell, n) &\leqslant 1 \\[4pt]
\lambda(\ell, n) \quad &\text{is an integer}
\end{aligned}
\right\} \qquad (22)
$$

H. Construction Cost of New Assemblies

The introduction of the dichotomous variables $\lambda(\ell, n)$ makes it feasible to reduce the construction and implementation cost of an assembly built at a decision time $\bar{n}(\ell)$ to the present worth at the base year. Let the construction cost of an element (e) in an assembly $\ell(e \epsilon \ell)$ to be represented by a linear relationship with its size $P(e)$ in current dollars at the beginning of n

$$CC(e) = CCI(e) + CCS(e)\,P(e) \qquad (23)$$

where $CC(e)$ is the construction cost of an element e, $CCI(e)$ and $CCS(e)$ are the constant cost and the variable cost, respectively, for element e $(e = 1, 2, \cdots N_{el})$. Discounting the current costs to the base year worth and summing over all elements of assembly gives present worth of totals construction cost (TCC):

$$TCC \equiv \left[\sum_{n=1}^{N_{ct}} \sum_e \lambda(\ell, n)\, CCI(e) \right.$$

$$\left. + \sum_{n=1}^{N_{ct}} \sum_e P(e,n)\, CCS(e) \right] \Big/ [1 + i'(n)]^{n-1}$$

$$\qquad (24)$$

The first term in the right-hand of Eq. (24) will only be nonzero when $\lambda(\ell, \bar{n}) = 1$, and is zero otherwise. The second term in the right-hand side is written in terms of "periodical ratings" of the eth element, $P(e, n)$, which are subject to additional constraints as described below, such that $P(e, n)$ are constrained to be only nonzero at a single $n = \bar{n}$ and are zero otherwise.

I. Lower and Higher Bounds of Element Rating

If an element e, in an assembly $\ell(e\epsilon\ell)$ is to be constructed at the beginning of interval $\bar{n}$ (i.e., $\lambda(\ell, n) = 1$) then its rated capacity must be greater than or equal to some given lower limit $P_L(e)$ and below or equal to some given upper limit $P_H(e)$. On the other hand, if the rating is below the lower bound or above the higher bound, as illustrated in Fig. 11, then the element must not be constructed [$\lambda(\ell, n) = 0$] and the element rating should be forced to equal zero, otherwise its construction cost is included in Eq. (24). Therefore, for the lower bound:

$$P(e, n) \geqslant \lambda(\ell, n) P_L(e) \quad \text{for all } e\epsilon\ell, \text{ all } n$$

or

$$P(e, n) - \lambda(\ell, n) P_L(e) \geqslant 0 \qquad (25)$$

and for the upper bound:

$$P(e, n) \leqslant \lambda(\ell, n) P_H(e) \quad \text{for all } e\epsilon\ell, \text{ all } n$$

or

$$P(e, n) - \lambda(\ell, n) P_H(e) \leqslant 0 \qquad (26)$$

Note that if an element is to be constructed, only one $\lambda(\ell, n)$ at $n = \bar{n}$ will be equal to 1, which means that the present worth of element construction cost from Eq. (24) will be nonzero only at the period $\bar{n}$, and zero otherwise.

Another way of forcing the periodical rating $P(e, n)$ to zero if the element is not to be constructed (i.e., if $\lambda(\ell, n) = 0$) is to use only a lower bound together with a "big M" control instead of the above lower and upper bounds. In this case, the upper bound constraint, Eq. (26), is replaced by:

$$P(e, n) \leqslant M \lambda(\ell, n)$$

or

$$P(e, n) - M \lambda(\ell, n) \leqslant 0 \qquad (27)$$

where M exceeds the maximum feasible value of any $P(e, n)$. Equation (27) ensures that when $\lambda(\ell, n)$ is zero, $P(e, n)$ must be zero, and when $P(e, n) > 0$, the variable $\lambda(\ell, n)$ will be 1 (and not zero). The optimal solution that minimizes the objective function will always choose $\lambda(\ell, n) = 0$ when $P(e, n) = 0$.

Equation (27) must be used if neither upper nor lower bounds are imposed on $P(e, n)$, or combined with Eq. (25) or (26) as appropriate.

J. Periodical Flux in Links

Even if the start-of-construction decision is taken for assembly ℓ at an optimum time period $\bar{n}(\ell)$ where $\bar{n} = 1, 2, \cdots N_{ct}$, the energy (or material) interactions in all the assembly links, conversion processes and storage-distribution nodes will not start until construction is complete, i.e., after a given time delay $\theta(\ell)$ is elapsed. This construction time delay $\theta(\ell)$ could take, in general, any value. However, in this model it is rounded off to the nearest month, hence constrained to be a multiple of months. Therefore, the benefit of constructing a new assembly will commence or be accounted for at the beginning of the month immediately following the end of the construction period. Additional constraints are needed to ensure that the energy (or material) flux carried in the links of each new assembly must commence after the construction is complete at $t(\bar{m})$:

$$SC(j, m), E(k, m) \left.\begin{cases} = 0 & 0 \leqslant t(m) \leqslant t(\bar{m}) \\ \geqslant 0 & t(N_t) \geqslant t(m) > t(\bar{m}) \end{cases}\right. \qquad (28)$$

where

$$\lambda(\ell, \bar{n}) = 1, \quad n = \bar{n}$$
$$\lambda(\ell, n) = 0, \quad n \neq \bar{n}$$
$$m = 1, 2, \cdots N_t$$

Two sets of dichotomous variables ν and η are further derived from the original set λ as illustrated in Fig. 12. The first set $\nu(\ell, m)$ are equivalent to the set $\lambda(\ell, n)$ but displaced $\theta(\ell)$ months due to the construction delay. Hence, for $n = 1, 2, \cdots$ $[(N_t - 4\theta)/q' + 1]$:

$$\nu(\ell, m) = \begin{cases} 0 & 1 \leqslant m \leqslant 4\theta(\ell) \\ 0 & m \neq 4\theta + (n-1) q', m > 4\theta \\ \lambda(\ell, n) & m = 4\theta + (n-1) q' \end{cases} \qquad (29)$$

where $q' = 4$, 12, 24, and 48 for monthly, quarterly, semiannual, and annual construction decisions, respectively. The set $\eta(\ell, m)$, on the other hand, is the cumulative sum of $\nu(\ell, m)$ where:

$$\eta(\ell, m) = \sum_{r=1}^{m} \nu(\ell, r) \qquad (30)$$

Equations (29) and (30) guarantee that the set η will be zero before the time $t(\bar{m})$, and unity thereafter. Accordingly, the

energy (or material) flux in the various links will be subject to the following constraints:

$$E(k, m) \leqslant \eta(\ell, m) M$$

or

$$E(k, m) - \eta(\ell, m) M \leqslant 0 \tag{31}$$

Equation (31) forces $E(k, m)$ to be zero if $\eta(\ell, m)$ is zero, and eliminates the constraint equation as $\eta = 1$.

K. Rating of Elements

The selection of the proper design capacity for an element (e) in an assembly (ℓ) is subject to the peak flux value experienced during the element operation. Note that the daily time step, Δt, which is selected for this model to be in the order of 4 to 10 hours, makes the peak flux an "apparent peak" and not a "true peak." A "true peak" flux is commonly measured during a 15-minute sampling; therefore, two additional design allowances should be considered for a "safe" optimum design. The first design allowance, as shown in Fig. 13, is to make the element rated capacity $P(e)$ slightly larger than the "true peak" flux. The second allowance relates the "true peak" flux to the "apparent peak" flux. Assuming that the ratio between the element rating $P(e)$ to the "apparent peak" flux is 1.58 as determined from Fig. 13, then

$$1.58 \, E(k, m) \leqslant P(e)$$

or

$$1.58 \, E(k, m) - P(e) \leqslant 0 \quad (e, k) \epsilon \ell \tag{32}$$

Equation (32), which represents m equations for each link k, constrains the ceiling value of the link flux to be less than the element rated capacity. Furthermore, to limit the periodical element rating $P(e, n)$ to be less than or equal to the element rating $P(e)$, we write for all $n = 1, 2, \cdots N_{ct}$ and all $e \epsilon \ell$

$$P(e, n) - P(e) \leqslant 0 \tag{33}$$

L. Utility Costs

the costs of purchased energy (or material) will be summed over the study period on a monthly basis, where for each month the costs are discounted to the base year. The present worth of total operation cost (TOC) is:

$$TOC = \sum_{m} \sum_{k} E(k, m) \frac{1 + e'(v)}{[1 + i'(v)]^v} \, Cu\,[k, (m-1)] \, \Delta t(m) \tag{34}$$

where Cu is the unit cost of purchased energy (or material) flux transmitted by link k, calculated at the beginning of the month corresponding to the period m. Unit cost and escalation rates are assumed to change only each month (i.e., every 4 periods).

M. Maintenance and Sustaining Costs

Similar to the construction cost of an element e of an assembly ℓ in Eq. (24), the maintenance and sustaining costs are assumed to be composed of two parts:

(1) A constant, uniformly recurring maintenance cost, $MCI(e, v)$, for the element e, at the end of the month v. Cost accrual starts only at the end of the month following the construction completion, and the start of assembly operation $(m > \overline{m})$. The dichotomous variable $\eta(\ell, 4v)$, given by Eq. (30), must be introduced.

(2) A variable maintenance cost, uniformly recurring, and proportional to the size or rated capacity of the constructed element. Also cost accrual should start only at the end of the month following the construction completion. The proportionality cost is $MCS(e)$.

Discount of both parts (a) and (b) should be made to a base-year worth, using a monthly maintenance escalation rate, $m'(v)$, which is counted above general inflation. Hence, present worth of total maintenance cost (TMC),

$$TMC = \frac{\displaystyle\sum_{e} \sum_{v=1}^{12\,N_y} [\eta(\ell, 4v)\,MCI(e, v) + MCS(e)\,\overline{P}(e, v)]}{[1 + m'(v)]^v} \tag{35}$$

where $\overline{P}(e, n)$ are "modified periodical rating" of the element capacity, which are derived for maintenance cost calculations from $P(e, n)$. Note that $\overline{P}(e, v)$ are nonzero at $v \geqslant \overline{v}$ if an assembly ℓ is built (i.e., if $\lambda(\ell, \overline{n}) = 1$); otherwise, the rest of the periodical ratings at $v < \overline{v}$ will be zero. This is different from the variable construction cost $[P(e, n)\,CCS(e)]$, which is nonzero only once for the whole study period at $n = \overline{n}$.

The variable maintenance cost $[\overline{P}(e, n)\,MCS(e)]$, must be calculated for all time periods immediately after the construction is completed. The modified periodical ratings are also identical to the decision rating $P(e)$ for each month v, but must vanish at $v < \overline{v}$, and be nonzero as $v \geqslant \overline{v}$, hence,

$$\overline{P}(e, v) - M\eta(\ell, 4v) \leqslant 0 \tag{36}$$

$$\overline{P}(e, v) - P(e) \leqslant 0 \tag{37}$$

Equation (36) guarantees that $\bar{P}(e, v)$ will be zero as $\eta(\ell, 4v)$ is zero (i.e., at $v \leqslant \bar{v}$) and eliminates any constraint on $\bar{P}(e, v)$ as $\eta(\ell, 4v)$ is one. Equation (37), however, adds the additional constraint that each nonzero $\bar{P}(e, v)$ must be equal to $P(e)$.

N. Total Life-Cycle Cost

The total life-cycle cost (TLCC) of the energy network will be the summation over all the elements and all the time periods of: (1) the total construction cost, TCC, from Eq. (24), (2) the total operation cost, TOC, from Eq. (34), and (3) the total maintenance and sustaining cost, TMC, from Eq. (35). The objective function TLCC is written as:

$$TLCC - (TCC + TOC + TMC) = 0 \qquad (38)$$

where minimum TLCC is sought.

O. Summary of Constraints

The constraints equations described above are listed as follows:

(1) Process efficiency:

$$E(k_{out}, m) - F^{(m)}(k_{in}, k_{out}) \, T(k_{in}, m) \, E(k_{in}, m) = 0$$
$$(5a)$$

This represents N_c equations for each month.

(2) Conservation laws for nodes. Each is taking-in one of the four forms below.

for N_{sd} storage-distribution nodes:

$$\Delta t(m) \sum_{k_{out}} T(k_{out}, m) \, E(k_{out}, m) - \Delta t(m) \sum_{k_{in}}$$

$$- SC'(j, m) + SC''(j, m) + SC'[j, (m-1)]$$

$$- SC''[j, (m-1)] = 0 \qquad (8)$$

for N_d distribution nodes:

$$\sum_{k_{out}} T(k_{out}, m) \, E(k_{out}, m) - \sum_{k_{in}} E(k_{in}, m) = 0$$
$$(9)$$

for N_{rd} resource-distribution nodes:

$$S(j, m) \, \bar{A}(j) - \sum_{k_{in}} E(k_{in}, m) = 0 \qquad (10)$$

for N_{dd} demand distribution nodes:

$$\sum_{k_{out}} T(k_{out}, m) \, E(k_{out}, m) = D(j, m) \qquad (11)$$

Equations (8) through (11) represent N_o equations for each m period in N_t.

(3) Storage content at beginning and end of each day:

$$SC'(j, 0), SC''(j, 0) = 0$$
$$SC'(j, 4v), SC''(j, 4v) = 0 \quad (v = 1, 2, \cdots 12 N_y)$$
$$(12)$$

Therefore, we need only to compute the intermediate values of $SC'(j, m)$ and $SC''(j, m)$ where $m = 4v - 3$, $4v - 2$, $4v - 1$ for $v = 1, 2, 3, \cdots 12 N_y$, and $j = 1, 2, \cdots N_{sd}$.

(4) Maximum swing of storage content vs size:

$$MSC'(j) + MSC''(j) - V(j) - M \beta_1(j) \geqslant 0$$
$$MSC'(j) + MSC''(j) - V(j) - M \beta_2(j) \leqslant 0$$
$$\beta_1(j) + \beta_2(j) = 1 \qquad (18)$$
$$\beta_1(j), \beta_1(j) \leqslant 1$$
$$\beta_1(j), \beta_2(j) \text{ integers (0 or 1)}$$

where $j = 1, 2, \cdots N_{sd}$. Equation (18) represents five equations for each of the N_{sd} storage-distribution nodes.

(5) Maxima of storage content:

$$SC'(j, q) - MSC''(j) \leqslant 0 \qquad (19)$$

$$SC''(j, q) - MSC''(j) \leqslant 0 \qquad (20)$$

where $q = 4v - 3$, $4v - 2$, $4v - 1$, and $v = 1, 2, \cdots 12 N_y$, $j = 1, 2, \cdots N_{sd}$. Equation (19) or (20) represents $36 N_y N_{sd}$ equations.

(6) Construction dichotomous variable, λ:

$$\sum_{n} \lambda(\ell, n) \leqslant 1 \qquad (21)$$

$$\lambda(\ell, n) \leqslant 1$$
$$\lambda(\ell, n) \text{ integer (0 or 1)}$$
$$(22)$$

where $n = 1, 2, \cdots N_{ct}$, and $\ell = 1, 2, \cdots N_a$. Equation (21) represents N_a equations for N_a assemblies. Equation (22), however, represents N_a equations for each period n.

(7) Lower and higher bounds of rated capacity:

For each n $(= 1, 2, \cdots N_{ct})$ and $e\epsilon\ell$, where $e = 1, 2, \cdots N_{el}$, one may write

$$P(e, n) - \lambda(\ell, n) P_L(e) \geqslant 0 \qquad (25)$$

$$P(e, n) - \lambda(\ell, n) P_H(e) \leqslant 0 \qquad (26)$$

if no bound is imposed, Eq. (25), (26), or both could be replaced by:

$$P(e, n) - M \lambda(\ell, n) \leqslant 0 \qquad (27)$$

Equations (25), (26), or (27) represents $N_{e\ell}$ equations for each time period n.

(8) Flux dichotomous variables (ν, η):

$$\nu(\ell, m) = 0 \quad 0 \leqslant m < 4\theta(\ell)$$

$$\nu(\ell, m) - \lambda(\ell, n) = 0 \quad \left\{ \begin{array}{l} m = 4\theta + (n - 1) q' \\ \\ n = 1, 2, \cdots, \left(\dfrac{N_t - 4\theta}{q'} + 1 \right) \end{array} \right.$$

$$\nu(\ell, m) = 0 \quad \left\{ \begin{array}{l} m > 4\theta \\ \\ m \neq 4\theta + (n - 1) q' \end{array} \right.$$

$$(29)$$

Equation (29) represents N_{el} equations for N_{el} elements in N_a assemblies. Also,

$$\eta(\ell, m) - \sum_{r=1}^{m} \nu(\ell, r) = 0 \qquad (30)$$

Equation (30) represents $(48 N_y \cdot N_{el})$ equations.

(9) Flux in links after construction completion:

$$E(k, m) - \eta(\ell, m) \; M \leqslant 0 \qquad (31)$$

Equation (31) represents N_{ℓ}· equations for ℓ links during each m time step.

(10) Rated capacity of elements:

$$1.58 E(k, m) - P(e) \leqslant 0 \qquad (32)$$

where Eq. (32) represents N_ℓ equations for each m time step. Also,

$$P(e, n) - P(e) \leqslant 0 \qquad (33)$$

Equation (33) represents $N_{e\ell}$ equations for each n time period.

(11) Modified periodical rating:

$$\bar{P}(e, \nu) - M \eta(\ell, 4\nu) \leqslant 0 \qquad (36)$$

$$\bar{P}(e, \nu) - P(e) \leqslant 0 \qquad (37)$$

Equations (36) and (37) represent N_{el} equations for each monthly period ν.

VII. Numerical Solution

Upon defining a block diagram of a "preferred" system configuration, the user will provide only some input data tabulated and grouped as listed in Appendix A. Currently, a computer program (OMEGA) for the Optimization Model of Energy Generation and Distribution is being written using the objective function of Eq. (38) and the above-mentioned constraints. The data entered by the user are grouped as follows:

(1) *Configuration data* include the number of system elements (processes, storage-distribution nodes and links), the coupling between different elements and whether or not the capacity of a certain element is known or treated as an unknown decision variable, and the number of assemblies.

(2) *Conversion efficiency* could be assumed constant for the period under study or else given for some selected time periods to reflect changes due to aging, wear, partial loading, and increased maintenance.

(3) *Facility loads* include the electrical-connected loads, space heating load, space cooling load, domestic hot water heating loads, and process steam. The data are entered for each time step, and should account for changes in load profile due to the facility growth or decrease in activity.

(4) *Energy resource intensity S* of an available resource for the energy supply end nodes should be entered at the appropriate time steps. The data include: (1) the solar radiance for different collector orientations, tracking mechanisms, site latitude, and ground reflectivity, and

(2) the site's wind velocities, wind-turbine characteristics, and cut-in, rated, and cut-off velocities.

(5) *Time data* include the number of years for a total life-cycle cost study, N_y, delay time due to implementation θ, on-peak, midpeak and off-peak schedule.

(6) *Economics data* include constant and variable construction costs of elements, constant and variable maintenance costs, escalation rates of purchased energy units, e', maintenance costs, m', money discount rates, i', and reliability costs.

The printed output of the OMEGA program will include an echo of input data for cross checking, the minimum value of TLCC (objective function) and the following set of optimum decision variables:

(1) The energy (or material) flux $E(k,m)$ carried by each link k during each time $t(m)$, whether the housing assembly is built or not.

(2) The construction decision variables $\lambda(\ell, n)$ giving the "build" or "no-build" decisions for new assemblies, and storage capacities. Also, the dichotomous variables ν and η will be printed.

(3) The decision time $(\bar{n})$ at which construction should begin if construction is decided.

(4) The optimum storage capacity, $V(j)$ and the periodical storage content $SC(j,m)$ of storage-distribution nodes in an assembly if construction of the housing assembly and storage are decided.

(5) The optimum rating $P(e)$ of new conversion processes, and links in an assembly if construction of the assembly is decided.

(6) The optimum characteristic area $A(j)$ for solar-powered or wind-powered end nodes if construction of the housing assembly is decided.

(7) Dynamic representation of the optimum system energy (or material) flux and cost display, categorized by elements during every time step $\Delta t(m)$, and summed monthly, quarterly, semiannually or annually. This gives a detailed picture of the optimum path of the dynamic facility for future sensitivity analysis.

(8) Diagnostics and error messages: in addition, the programming will print out the problem title supplied, control parameters, problem size, and number of integer variables, bounds on the integer variables, the constraint types, and the matrix format type as part of the initial data. Error messages are printed for abnormal terminations, and suggest the reason and give the iteration number.

The user will assign consecutive numbers for conversion processes/components from 1 to N_c, for storage-distribution nodes from 1 to N_o, for connecting links from 1 to N_ℓ, and for years of study period from 1 to N_y. Four time steps, not necessarily equal in time, for each representative day of the month will be assumed. This choice is made for better refinement of consumer loads and purchased energy costs during on-peak, midpeak, and off-peak periods. A preprocessor subroutine is envisioned to read a few input data elements, to configure both the objective function coefficients and the nonzero coefficients of the constraint equations, and put them in a "conventional" matrix form ready for execution.

Slack and artificial variables are introduced as appropriate to change inequality constraints to equality forms. Since the decision variables are divided into "continuous" variables (which can take in any value ≥ 0) and integer variables (which are restricted in this model to binary values), the problem is a mixed-integer programming type. Both the objective function and the constraint equations are in linear form, which makes the problem very suitable to branch-and-bound mixed integer linear programming (Refs. 1 and 2). A dual, revised Simplex Algorithm will be imbedded in the program instead of the pivot-Simplex Algorithm for less memory storage, execution time, and cost. Due to the large number of variables that are expected to be dealt with, the diagonalized form of the constraint coefficients and the Dantzig-Wolfe Decomposition Principle (Refs. 1 and 2) will be employed. The set of unknowns or decision variables, x are divided into two types: *Type a* are decision variables that are independent of time-of-day, such as $\bar{A}(j)$, $\beta_1(j)$, $\beta_2(j)$, $V(j)$, $MSC'(j)$ $MSC''(j)$ and $P(e)$; *type b* are decision variables that are time dependent, such as $E(k,m)$, $SC'(j,m)$, $SC''(j,m)$, $P(e,n)$, $\bar{P}(e,n)$, $\lambda(\ell, n)$, $\nu(\ell, m)$ and $\eta(\ell, m)$. Let the constraint equations be written in the matrix format

$$\mathbf{A} \, \mathbf{x} = \mathbf{b} \tag{39}$$

where the form of the vector $\mathbf{x}$ is composed of variables type (a) first, followed by type (b) variables. Further, if all type (b) variables are grouped such that month 1 and year 1 variables appear first, followed by month 2, and year 1 variables, and so on, the matrix $\mathbf{A}$ will be transformed to the preferred shape shown in Figs. 14 and 15. Monthly submatrices (or subproblems) will be placed diagonally together with a top "row" matrix, and a side "column" matrix. Standard Simplex methods (Refs. 1 and 2) using the decomposition principle and dual-Simplex programming could then be adopted. The rest of the matrix $\mathbf{A}$ is full of zeros. No attempt is made at this point to reduce further the monthly submatrix, though filled with many zeros, into smaller submatrices. The compact methods available for storing large matrices will be employed.

VIII. Summary

In this first part of the optimization study, an analytical model is outlined using linear programming for the optimum generation and distribution of facility demands among competing resources at different design and economic criteria. The model will be used as an in-house engineering tool in the analysis of the Deep Space Network ground facilities and its energy systems and subsystems. The model will provide conceptual as well as design-oriented "optimum" decisions to complement the current practice of designing "working" systems. Some of the decisions included are: the optimum time to build an assembly of elements, the inclusion of a storage medium of some type, the size or capacity of the basic components that will minimize the total life-cycle cost over a given number of years. The broad-class model is structured in piecewise linear, multitime divisions to smooth out nonlinear load effects. Since the number of integer variables, noninteger variables, and constraint equations are found large, it becomes convenient to use the decomposition principle for large-size problems, the Branch-and-Bound Method in mixed-integer programming and the Revised Simplex technique for efficient and economic computer use.

Upon defining a block diagram of a "preferred" system configuration, the user will provide a few input data, partly described in Appendix A, which include the number and type of components, conversion efficiencies, consumer loads, resource intensities, time steps, and various cost and economics data.

A summary of all constraint equations is made in Subsection VI-N with a monthly submatrix form as shown in Fig. 15. For each month, the maximum number of decision variables will be $(6 N_{sd} + 10 N_{el} + 4 N_{\varrho} + N_a)$ and the number of constraint equations will be $(4 N_c + 4 N_o + N_a + 3 N_{el} + 9 N_{\varrho})$. For the left-hand common column matrix, the number of variables is $(N_{rd} + 5 N_{sd} + N_{el})$. Also, the number of constraint equations for the top common row matrix is $(72 N_y N_{sd} + 5 N_{sd} + N_a + 96 N_y N_{el})$. Further reduction in the size of the $E(k)$ unknown where k 1, 2, $\cdots N_{\varrho}$ is possible using the equalities in Eq. (5) such that fewer links are considered. Details of the computer program (OMEGA) that is being written, the testing and application of the model to several case studies, and the sensitivity of the results to changing design or economical factors will be addressed in a future progress report.

Acknowledgment

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List of Symbols

A coefficient matrix of constraint equations

$\overline{A}$ characteristic surface area

b constant coefficient vector

C coefficient of the objective function

Cu unit cost of purchased energy (or material)

CCI construction cost intercept

CCS construction cost slope

D consumer energy (or material) demand rate

e element index in an assembly ℓ

e' energy escalation rate above general inflation

E energy (or material) flux at a sending end of a link

$\overline{E}$ energy (or material) flux at a receiving end of a link

F process efficiency or yield

i' money interest rate (or discount) above general inflation

i conversion process index $(1, 2, \cdots N_c)$

j storage-distribution node index

k, k_{in}, k_{out} link index $(1, 2, \cdots N_\varrho)$

ℓ assembly index $(1, 2, \cdots N_a)$

m time index for flux calculations $(1, 2, \cdots N_t)$

m' maintenance cost escalation rate above general inflation

$\overline{m}$ time at which energy (or material) flux is computed for a new assembly

M very large number

MCI maintenance cost intercept

MCS maintenance cost slope

$MSC'\,MSC''$ maximum and minimum storage content of a node

n construction-time index $(1, 2, \cdots N_{ct})$

$\overline{n}$ construction time for a new assembly

N number of constituents

p process-node coupling identifier

$P, \overline{P}$ power or rating of an element

PW present worth

q, q' parameter

S energy resource rate per unit area

SC storage content of node

t time

T transmission efficiency of a link

TCC total construction cost

$TLCC$ total life-cycle cost

TMC total maintenance cost

TOC total operation cost

V storage capacity of a node

v monthly index

X unknown (decision) variable

α rate of energy (or material) losses across a link

β_1, β_2 dichotomous variables for storage size decisions

λ, ν, η dichotomous variables for construction decisions

θ months to complete construction

Subscripts

a	assembly
c	conversion process
ct	construction time step
d	distribution node (no storage)
dd	demand-distribution end node
el	elements in an assembly
H	higher bound
g	constraint equations
ℓ	link
L	lower bound
o	all nodes
rd	resource-distribution end node
sd	storage-distribution node
t	time-element index
u	unknown (decision) variables
y	total years of study
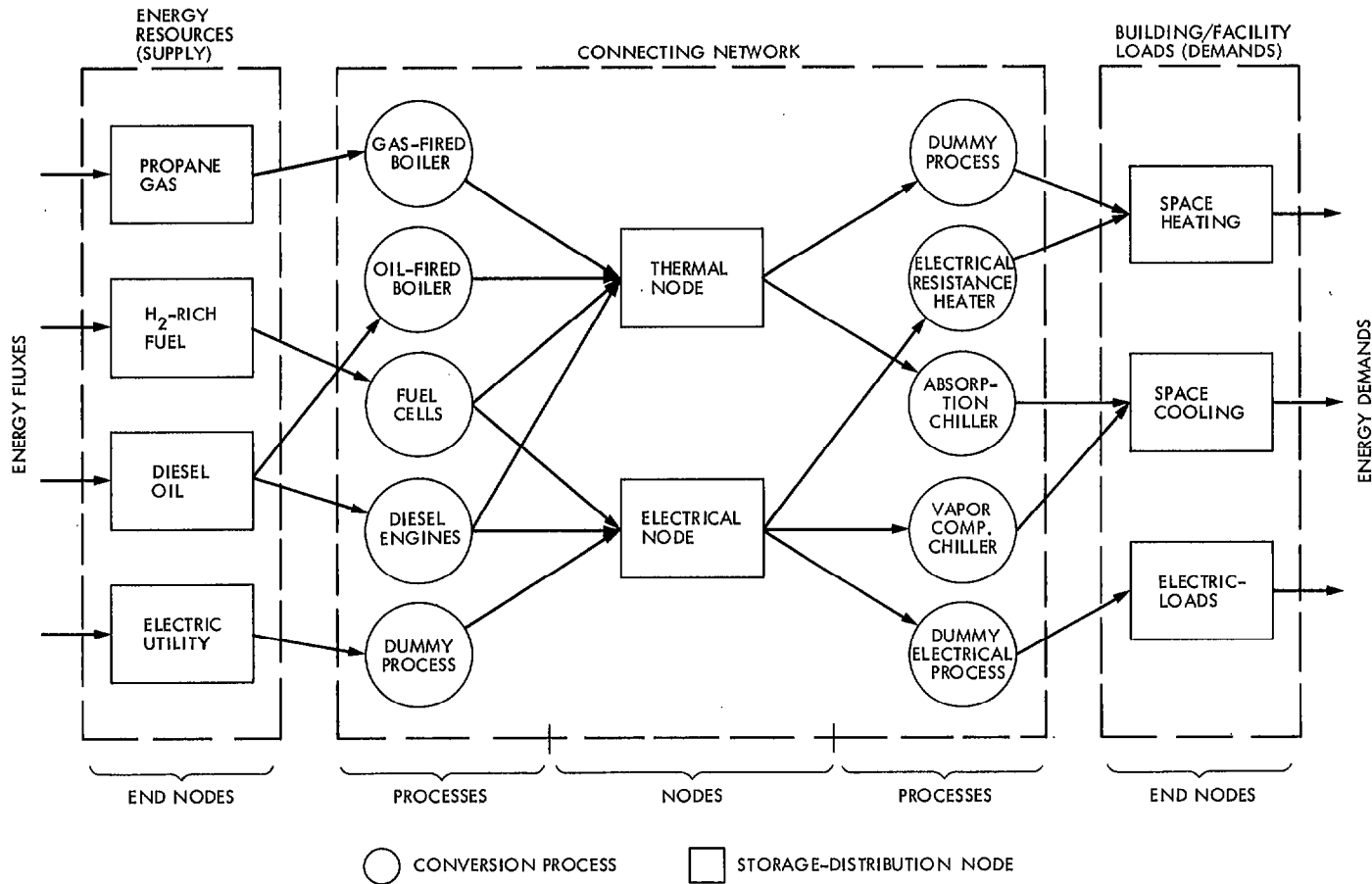
Fig. 1. Combined fuel cells and oil-fired diesel engines as prime mover with waste heat recovery
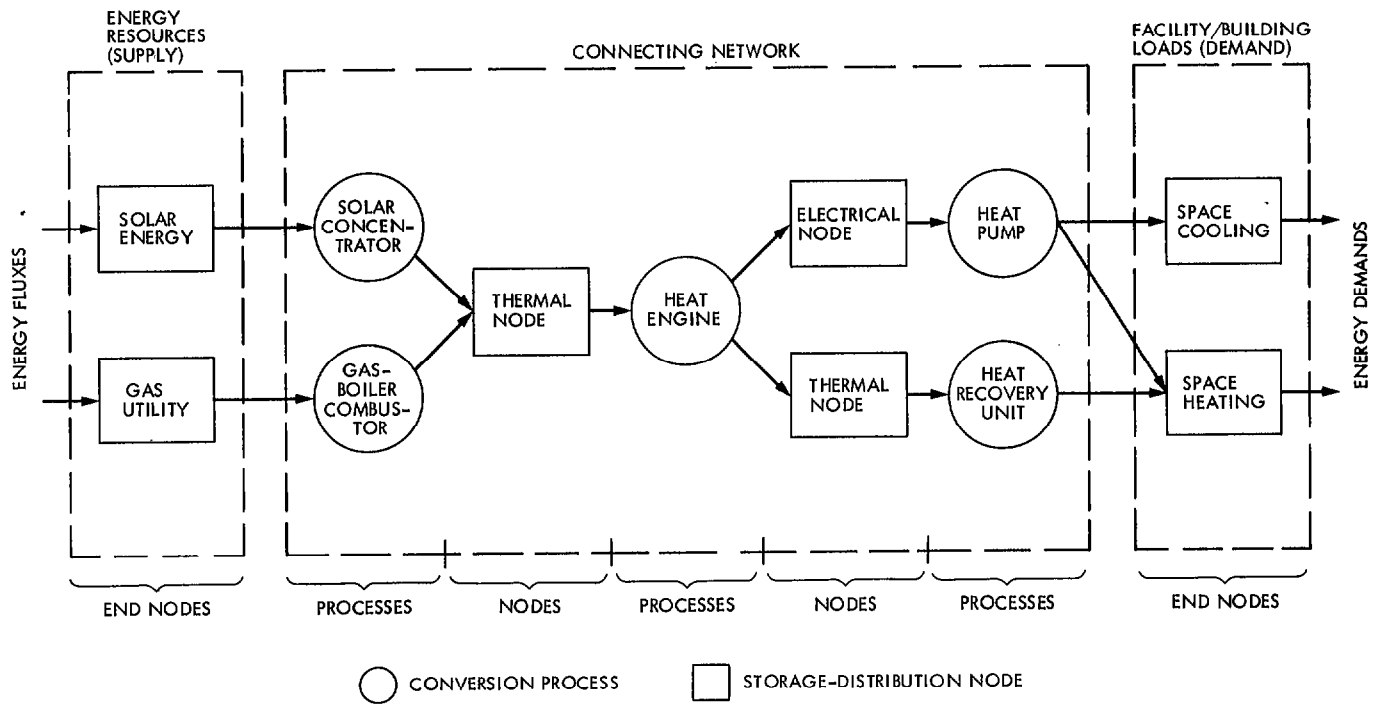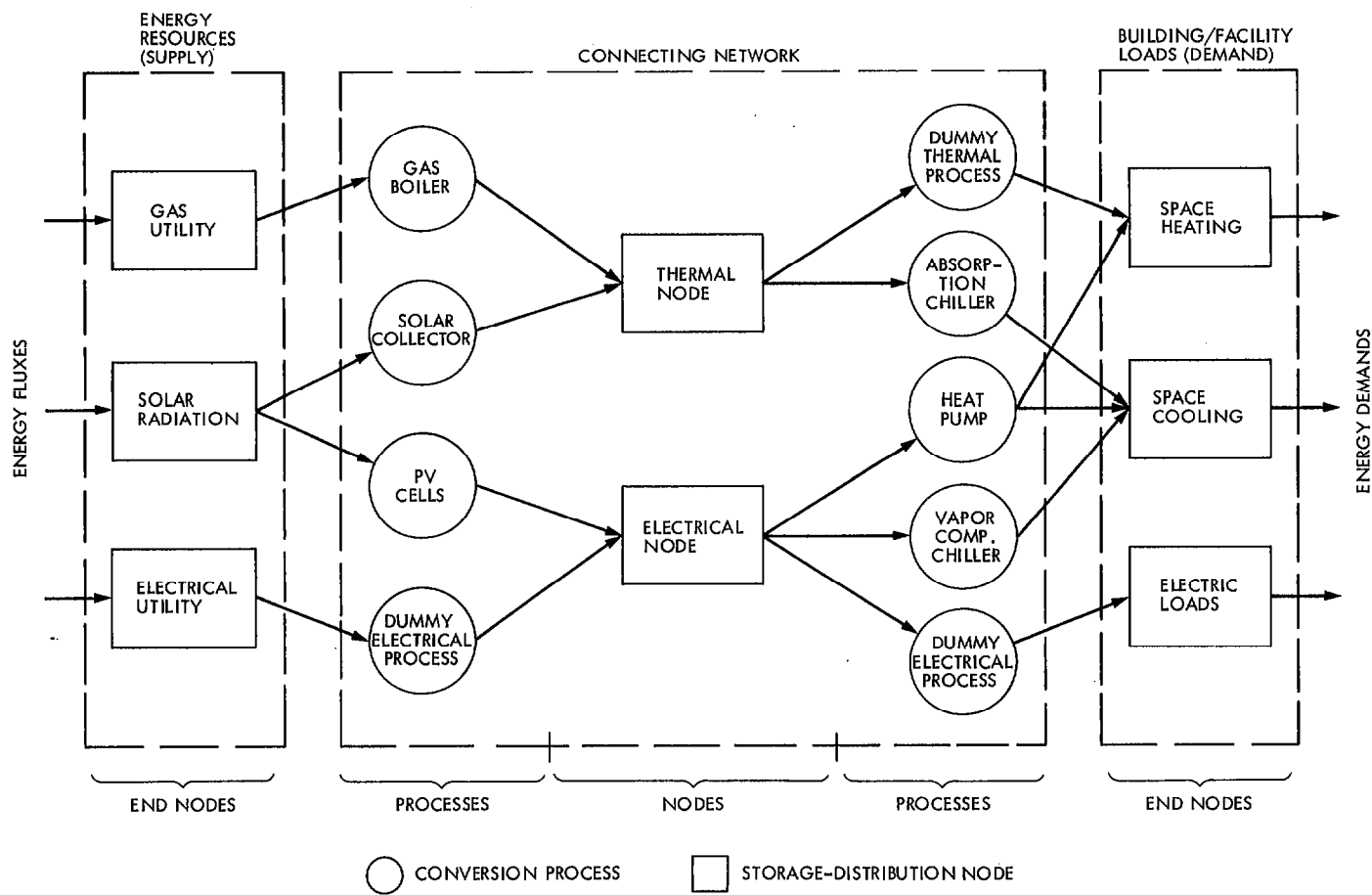
Fig. 2. Solar-assisted gas-fired heat-pump system

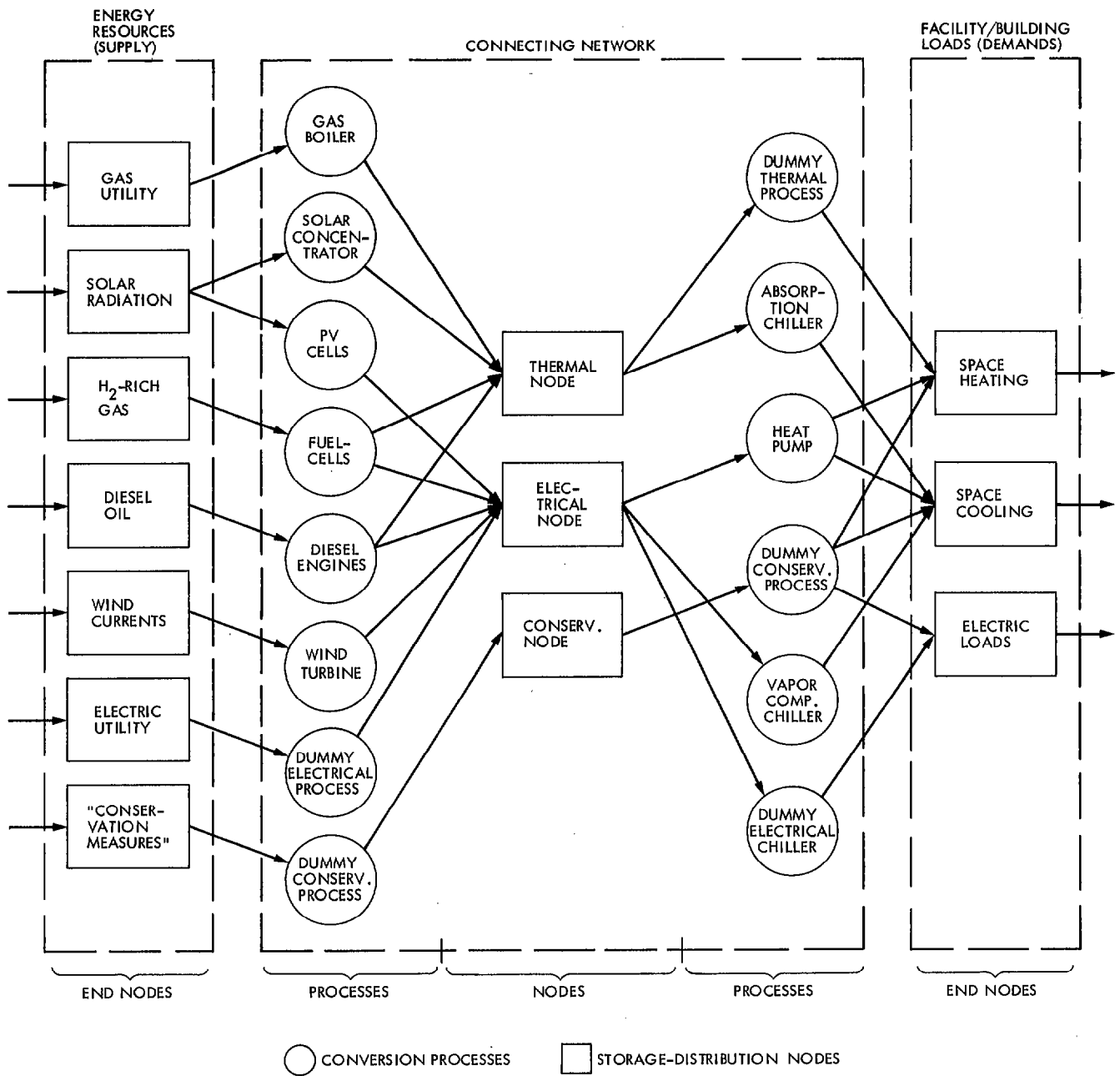Fig. 3. Superimposed solar-PV and solar-thermal elements on a conventional gas-electricity building

ENERGY
RESOURCES
(SUPPLY)

CONNECTING NETWORK

FACILITY/BUILDING
LOADS (DEMANDS)

GAS
UTILITY

SOLAR
RADIATION

H₂-RICH
GAS

DIESEL
OIL

WIND
CURRENTS

ELECTRIC
UTILITY

"CONSER-
VATION
MEASURES"

GAS
BOILER

SOLAR
CONCEN-
TRATOR

PV
CELLS

FUEL-
CELLS

DIESEL
ENGINES

WIND
TURBINE

DUMMY
ELECTRICAL
PROCESS

DUMMY
CONSERV.
PROCESS

THERMAL
NODE

ELEC-
TRICAL
NODE

CONSERV.
NODE

DUMMY
THERMAL
PROCESS

ABSORP-
TION
CHILLER

HEAT
PUMP

DUMMY
CONSERV.
PROCESS

VAPOR
COMP.
CHILLER

DUMMY
ELECTRICAL
CHILLER

SPACE
HEATING

SPACE
COOLING

ELECTRIC
LOADS

END NODES   PROCESSES   NODES   PROCESSES   END NODES

◯ CONVERSION PROCESSES   ▢ STORAGE-DISTRIBUTION NODES

Fig. 4. A large network for energy generation and distribution including "conservation measures"
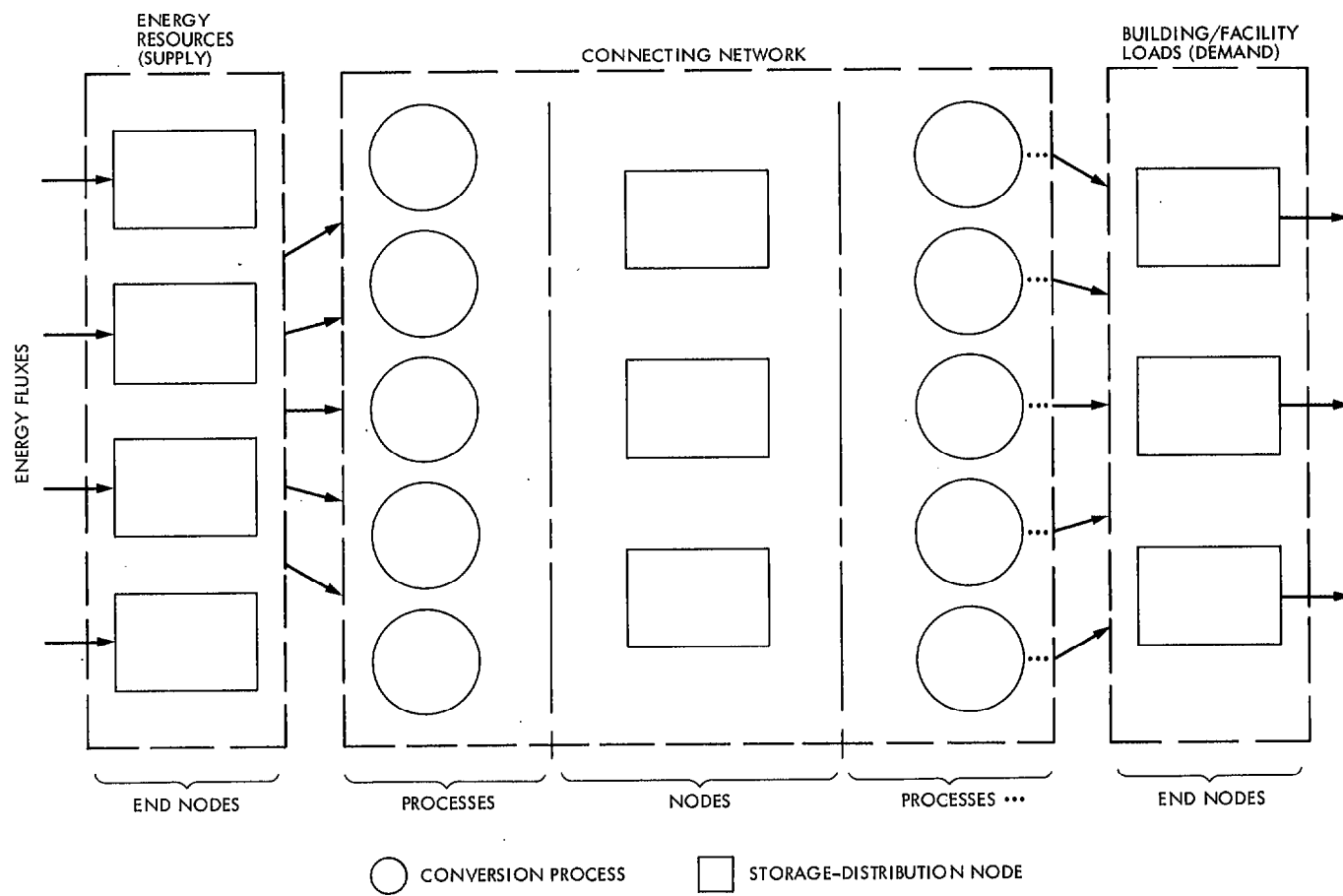
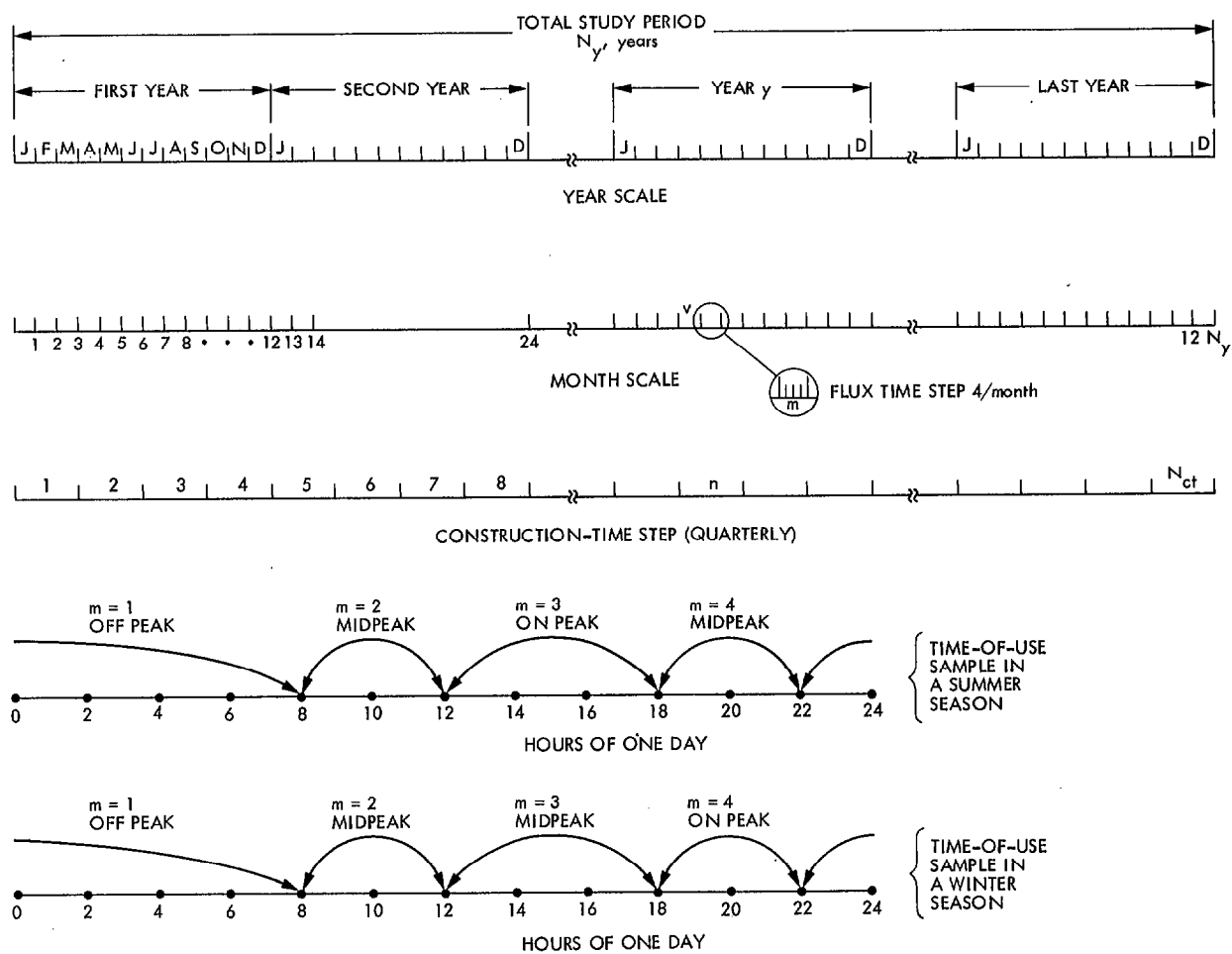Fig. 5. Block diagram for general energy generation and distribution system
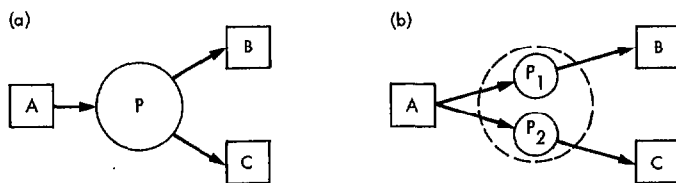
Fig. 6. Time steps

Fig. 7. Decomposition of a compound process into simple processes: (a) compound process P; (b) equivalent simple processes P_1, P_2
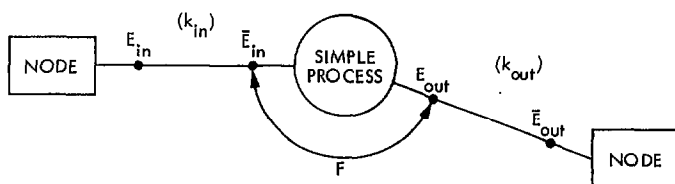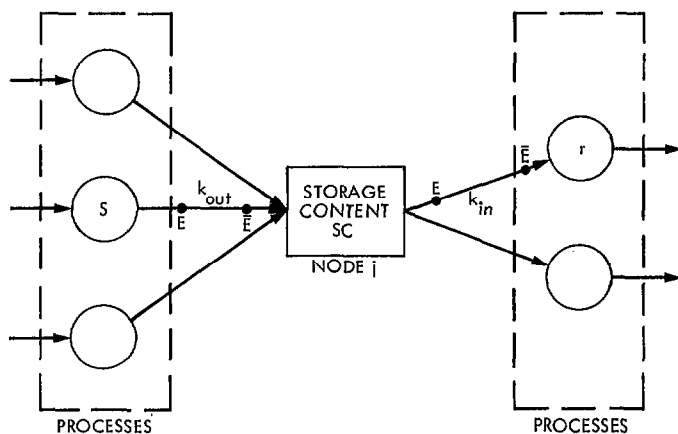

Fig. 8. Conversion efficiency of a single process

Fig. 10. Storage content at different time steps
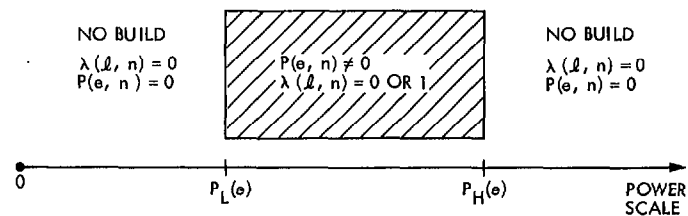

Fig. 9. Balancing fluxes around a storage-distribution node

Fig. 11. Lower and upper bounds for an element e in an assembly at construction time n
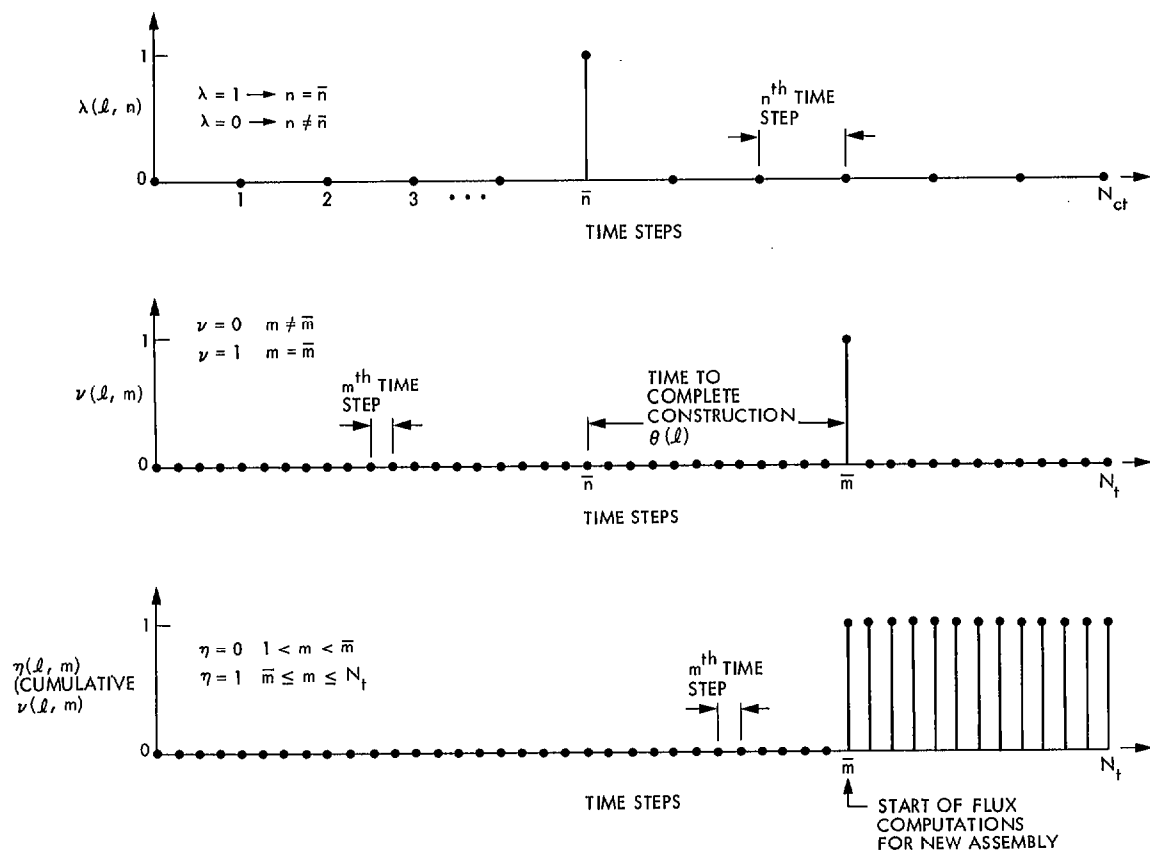
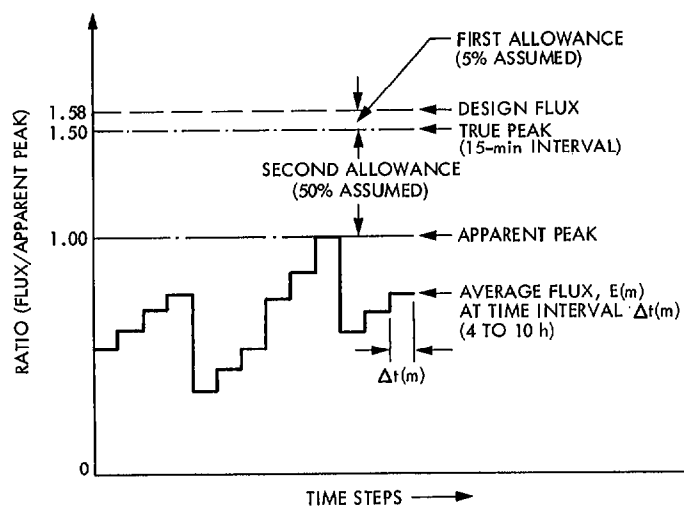Fig. 12. Relationships between dichotomous variables λ, ν, η

Fig. 13. Relationship between flux average, apparent peak,
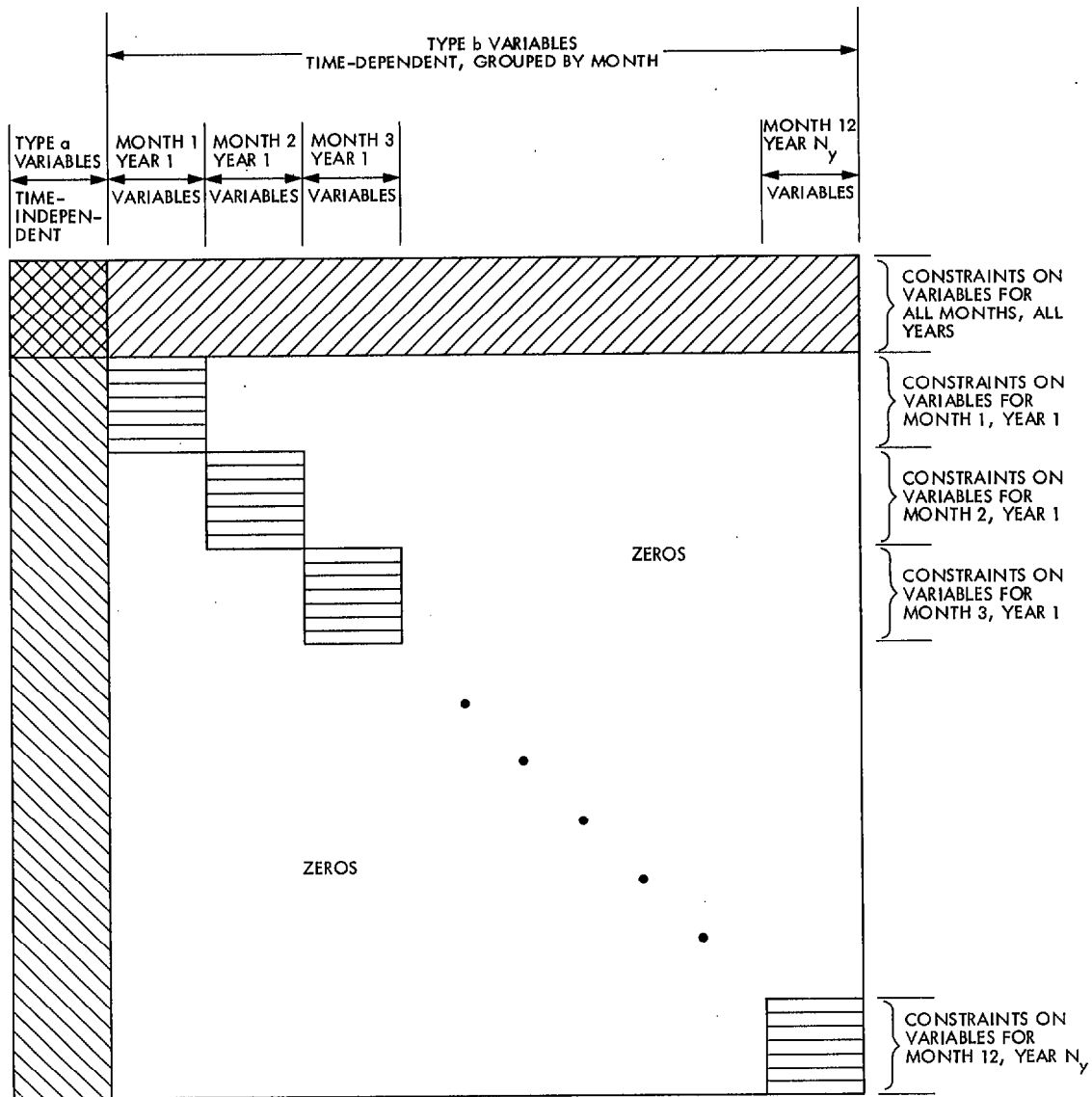true peak, and design capacity

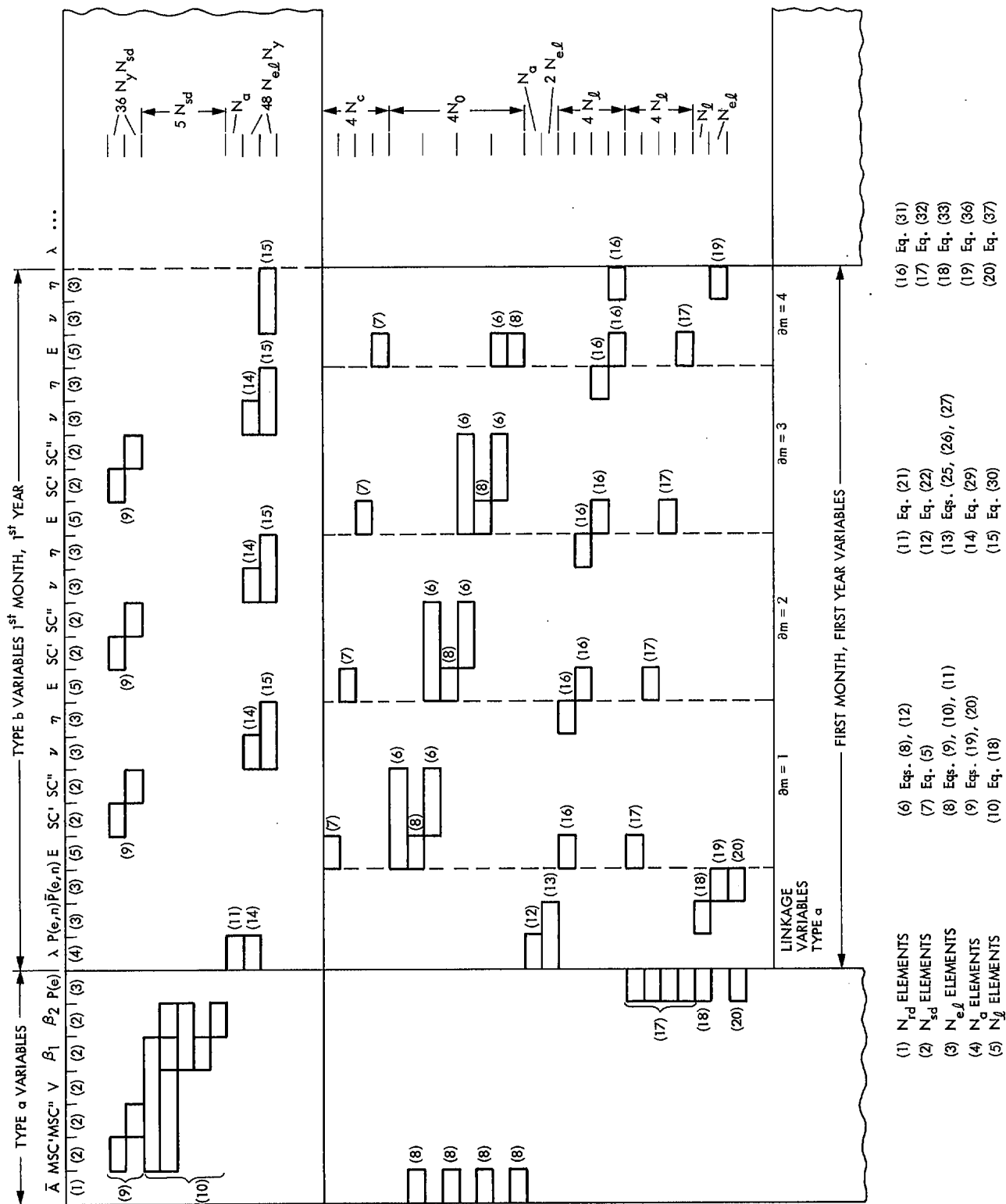Fig. 14. Structure of the nonzero elements of the coefficient matrix A

Fig. 15. Details of the submatrices of A

Appendix A

Selected Input Data

Tables A-1 through A-10 list the requirements to be considered by the user in entering some selected variables. A preprocessor routine will organize the user information and generate the necessary coefficients of both the constraint equations and the objective function. The user must supply the proper index number wherever a dot appears in the table.

Table A-1. Description matrix for conversion processes, $DC(N_c, 2)$

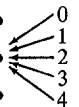| Process No.	Description	Housing Assembly No.
1	•	•
2	•	•
i	•◀$\begin{matrix}0\\1\\2\end{matrix}$	•
N_c	•	•

All conversion processes will be numbered including dummy processes, consecutively starting from 1, in steps of 1 up to the maximum number of processes N_c. The elements of description column will take in 0, 1, or 2 where

0 = a new process under optimization decisions

1 = a dummy process

2 = an existing process

Table A-2. Description matrix for storage-distribution nodes, $DS(N_s, 2)$

Node No.	Description	Housing Assembly No.
1	•	•
2	•$\begin{matrix}0\\1\\2\\3\\4\end{matrix}$	•
j	•	•
N_o	•	•

All storage-distribution nodes will be numbered, including dummy nodes, consecutively starting from 1 in steps of 1, up to the maximum number of nodes N_0. The elements of the description column will take 0, 1, 2, 3 or 4 where

0 = a new node under optimization decisions

1 = a dummy node or an existing node without storage

2 = an existing node with storage

3 = an end node for energy resource (supply)

4 = an end node for facility loads (demand)

Table A-3. Description of assemblies, $DA(N_a, 2)$

Assembly No.	Description	Construction time θ, months
1	•	•
2	•	•
ϱ	•◀$\begin{matrix}0\\1\end{matrix}$	•
N_a	•	•

All conversion processes, storage-distribution nodes, and the associated linkages will be grouped into assemblies that could have some overlap elements. Assemblies are numbered consecutively starting from 1, in steps of 1 up to the maximum number of assemblies N_a. The elements of the description column will take in 0 or 1 where

0 = a new assembly under optimization decision

1 = existing assembly

Table A-4. Coupling matrix between conversion processes and storage-distribution nodes, $P(N_c, N_o)$

Process No.	1	2	j	N_o
1	•	•	•	•
2	•	•	•	•
i	•	•	k	•
N_c	•	•	•	•

The coupling matrix, $p(i,j)$ is an asymmetric matrix that relates the ith process with the jth node and the kth link connecting them. $p(i,j)$ will take in

0 if no link exists between process i and node j

k if flux in link k is from process i into node j

$-k$ if flux in link k is from node j into process i

The elements of the ith row of the matrix p identify all the nodes connected to process i through inlet and outlet links. The elements of the jth column of the matrix p identifies all the processes connected to the jth node.

Table A-5. Conversion efficiency, F

Links at process inlet	1	2	k_{out}	N_ϱ
1	•	•	•	•
2	•	•	•	•
k_{in}	•	•	F	•
N_ϱ	•	•	•	•

The conversion F is an asymmetric matrix that gives the ratio of output to input fluxes $E(k_{out})/\overline{E}(k_{in})$ associated with simple processes, at a given time period. Several matrices for F could be specified for different time periods to allow inefficient operation due to equipment aging and partial loading. Elements of one row represent the products efficiency in converting the input link flux. Elements of one column, however, represent the relative contributions by all inlet links to the selected outlet link flux.

Table A-6. Facility loads for demand-end nodes

Demand node No.	Time step 1	2	m	N_t
1	•	•	•	•
2	•	•	•	•
j	•	•	$D(j, m)$	•
N_{dd}	•	•	•	•

Each node represents one type of facility load such as electrical, space heating, space cooling, domestic water heating, and process steam. Data are entered for each time step during the study period; hence allowance is made for changes due to facility growth or decreases in activity by time.

Table A-7. Energy resource intensities for supply-end nodes

Resource node No.	Time step 1	2	m	N_t
1	•	•	•	•
2	•	•	•	•
j	•	•	$S(j, m)$	•
N_{rd}	•	•	•	•

Each node represents one type of nondepletable energy resource (such as solar and wind) whose intensity S is given for the appropriate time periods. A preprocessor should be

included to obtain the data of Table A-7 from other sources of information.

For the following three tables (A-8, A-9, and A-10), the NBS rules for LCC computations (Ref. 9) are summarized as follows:

(1) All investment costs, nonfuel operation and maintenance costs, repair and replacement costs, salvage values, and energy costs should be accounted for.

(2) All future dollar amounts must be estimated in "constant dollars" (i.e., excluding the effects of general price inflation) and not in "current dollars."

(3) A real interest (or discount) rate, i', also excluding inflation, must be used to adjust all dollar values to a present worth in the base year.

(4) Energy prices and price growth projections provided by The Department of Energy (DOE) may be used unless the actual prices to the facility are higher. Projections are provided (Ref. 9) by region, consuming sector, fuel type, and time period.

(5) The study period should be the lesser of 25 years or: (a) the expected life of the system for a building system retrofit, (b) the period of intended use for a new building design, (c) the effective remaining term of the lease for a leased building, or (d) an equivalent study period where choices are mutually exclusive.[1]

Table A-8. Construction and maintenance costs

Type of element	Description	Construction cost Fixed CCI	Construction cost Variable CCS	Maintenance cost Fixed MCI	Maintenance cost Variable MCS
Conversion processes	Solar collector	•	•	•	•
	PV cells	•	•	•	•
	Wind turbine	•	•	•	•
	Heat pump	•	•	•	•
	Boiler	•	•	•	•
	Electrical resistance	•	•	•	•
Storage– distribution nodes	Cold water tank	•	•	•	•
	Hot water tank	•	•	•	•
	Battery	•	•	•	•
Links	Piping	•	•	•	•
	Electrical line	•	•	•	•

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[1]Mutually exclusive means that choosing one alternative precludes choosing the other. Nonmutually exclusive choices means making one choice does not necessarily preclude making the other.

The common study period for evaluating mutually exclusive choices may be either: (a) the estimated life of the choice having the longest life, or (b) the lowest common multiple of the estimated lives of the alternatives not to exceed 25 years.

Table A-9. Purchased Energy (or material) costs at beginning of base year

| Type of resource node | Unit cost | | | | | |
| | Summer | | | Winter | | |
	Off peak	Mid-peak	On peak	Off peak	Mid-peak	On peak
Electricity	●	●	●	●	●	●
Natural gas	●	●	●	●	●	●
Diesel oil	●	●	●	●	●	●
LPG	●	●	●	●	●	●

Table A-10. Escalation rates above general inflation

| Escalation rate | | Month | | | |
		1	2	ν	$12N_y$
Money i'		●	●	●	●
Energy e'	Electricity	●	●	●	●
	Gas	●	●	●	●
	Oil	●	●	●	●
Maintenance m'		●	●	●	●

Monthly escalation rates are computed from yearly rates by simple (not compound) division. No escalation is considered for time steps less than one month. Costs are incurred at the *end* of each month.