Life-Cycle Cost Analysis of Projects Using a Polynomial Cash Flow Model for Nonuniform Maintenance and Operations Costs

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A mathematical model is developed for calculating the life-cycle costs for a project where the maintenance and operations (M&O) costs change in a nonlinear manner with time. Closed-form solutions are presented for computing the present worth of projects with periodic cash flow profiles that can be approximated by polynomial functions. The results show that the life-cycle cost for a project can be grossly underestimated (or overestimated) if the M&O costs increase or decrease nonuniformly over time rather than being constant or linear as is often assumed in project economic evaluations. The following range of variables is examined: (1) project life from 2 to 15 years, (2) interest rate from 0 to 30 percent per year, and (3) polynomials of order 0 to 5. Simplified solutions for the present worth are presented for two limiting cases: extended project lifetime and negligible interest rate. Also a simplified expression is provided for accurate present worth M&O estimates for DSN projects. In addition, a sensitivity analysis of the model based on graphical results and a numerical example plus tables and graphs are given to help the reader calculate M&O life-cycle costs over a wide range of variables.

I. Introduction

In the last few years, there has been an increasing emphasis on economic evaluations for comparing projects in the DSN. The total expected costs of systems or modifications, over their lifetimes, has become an important consideration to managers and engineers in TDA and the DSN. This interest has recently culminated in the release of the Deep Space Network Life-Cycle Cost (LCC) Analysis Handbook and the Tracking and Data Acquisition Standard Practice for Life-Cycle Cost Analysis.

The TDA Standard Practice states when an LCC analysis is required and what the analysis will contain, but does not state *how* to do the analysis. The DSN LCC Analysis Handbook complements the Standard Practice by providing managers and engineers with the background information and guidance to conduct an LCC analysis. However, one of the major problems a user faces is modeling the maintenance and operations cost for the life of the system or systems modification. Usually one makes a zeroth-order assumption that the M&O costs are constant over time. This is an unrealistic assumption and has extremely limited use.

A first-order model was developed several years ago for M&O costs that vary in a linear manner with time (Refs. 1 and 2). The purpose of this paper is to develop closed-form solutions for the present worth of the maintenance and operations cost profile, based on an M^{th}-degree polynomial approximating the cash flows. For $M = 0$ and $M = 1$, this general model reduces to the usual models where one assumes that the M&O costs are constant over time ($M = 0$) or where the M&O costs increase or decrease in a linear manner with time ($M = 1$). Also, two limiting cases of special interest for quick engineering project estimates are presented. These are the cases of infinite project life and zero interest rate. The infinite life model approximates many extended life projects that are estimated to last well over 10 years. The zero-interest model may be used to approximate situations where the cost of capital is close to the inflation rate. The validity of this approximation was developed in Ref. 3 where it was shown that inflation and discounting largely cancel each other over long periods.

Application of the general polynomial model is discussed and results are presented graphically. A brief sensitivity analysis is made to show the relative importance of the costs of capital and the degree M of the cash flow polynomial. Finally, a simplified model for evaluating DSN projects is presented.

II. Previous LCC Mathematical Models

A. Zeroth-Order Model

The zeroth-order approximation that is often made in economic analysis is to assume that the M&O costs are constant each year. For this simple model, the present worth factor $(P/G_0, i, n)$ of the life-cycle M&O costs for an n-year project and an interest rate i for a unit cash flow of \$1 per year is shown below and developed in Refs. 1 and 2.

$$(P/G_0, i, n) = \frac{(1+i)^{n-1} - 1}{i(1+i)^n}, n > 1$$

The notation $(P/G_M, i, n)$ will be used throughout the paper. P represents the present worth of the life-cycle costs. G_M represents the order of the cash flow profile where $M = 0$ is a constant cash flow, $M = 1$ is a linear cash flow, etc. The i represents the interest rate used for the present worth calculation and n denotes the life of the project.

Needless to say the zeroth-order model has very limited application to most real problems where costs are rarely constant over time. In the next section we will consider a first-order model.

B. First Order Model

A mathematical model for calculating the life-cycle costs of a project where the M&O costs increase or decrease in a linear manner with time has been treated in detail in Refs. 1 and 2.

From Refs. 1 and 2, the present worth factor for the life-cycle M&O costs with a linear increasing cash flow profile for an n-year project and an interest rate i was shown to be

$$(P/G_1, i, n) = \frac{(1+i)^n - ni - 1}{i^2(1+i)^n}, n > 1$$

The results in Refs. 1 and 2 show that the life-cycle cost for a project can be grossly underestimated (or overestimated) if the operating costs increase (or decrease) uniformly over time rather than being constant as is often assumed in project economic evaluations. This model is a good first step forward for analyzing life-cycle M&O costs, but a more general model is really needed to supplement Appendix E in the DSN Life-Cycle Cost Analysis Handbook. This appendix is entitled, "Tools for Calculating a Life-Cycle Cost." The following theoretical development will substantially improve the tools available to do a life-cycle cost analysis for most cash flow profiles that we will probably encounter in the DSN. It will no longer be necessary to make such simplifying assumptions as uniform annual M&O costs or linear M&O costs. The following model, which is based on the recent developments in Ref. 4, will allow us to calculate the life-cycle M&O costs for a generalized cash flow profile.

III. A General Life-Cycle Cost Mathematical Model

A. Formulation of the Model

The cash flow profile for an n-year project can often be approximated by some M^{th} degree polynomial of the form:

$$y = C_M(x-1)^M + C_{M-1}(x-1)^{M-1} + \cdots + C_1(x-1) + C_0 \tag{1}$$

where $C_0, C_1, \cdots, C_M$ are constants and y is the amount of cash flow in year x. We assume that this profile can be represented as a series of discrete annual payments (see Fig. 1). Throughout this paper, discrete compounding of interest is used and, for simplicity, cash flows are assumed to be in dollars. We use $(x - 1)$ instead of x in Eq. (1) to be consistent with standard engineering economic texts (Refs. 5 and 6) where the cash flows for a linear gradient have their first costs occurring at the end of year 2.

All results are based on annual cash flows and an annual interest rate. However, the results may be extended to any type of periodic cash flow if an appropriate interest rate is used. For example, the above polynomial may be used to describe cash flows on a monthly basis, where n represents the number of months in the project lifetime. The subsequent analysis would then require a monthly interest rate.

We will first consider the most basic form of Eq. (1). In this case, the cash flow y in year x of a specific n-year project is determined by the equation

$$y = (x - 1)^M \quad 1 < x < n \tag{2}$$

for a suitable choice of integer $M \geqslant 1$ (refer to Fig. 2). The present worth in year zero of the total cash flow of this project at an annual interest rate i is:

$$(P/G_M, i, n) = \sum_{x=2}^{n} (x - 1)^M (1 + i)^{-x} \quad (M \geqslant 1) \tag{3}$$

We will now construct a more tractable equation for the present worth of the M&O life-cycle costs. Equation (3) reduces to the following after algebraic manipulation

$$(P/G_M, i, n) = \frac{1}{i} \sum_{x=1} [\nabla(x^M)] (1 + i)^{-x} \frac{n^M}{i(1 + i)^n} \quad (M \geqslant 1) \tag{4}$$

where ∇ is the backward difference operator:

$$\nabla(x^M) = x^M - (x - 1)^M = \sum_{k=1}^{M} \binom{M}{k} x^{M-k} (-1)^{k+1} \tag{5}$$

and $\binom{M}{k}$ is the binomial coefficient

$$\frac{M!}{k!(M - k)!}$$

By substituting Eq. (5) into Eq. (4), we have

$$(P/G_m, i, n) = \frac{1}{i} \sum_{x=1}^{n} \sum_{k=1}^{M} \binom{M}{k} x^{M-k} (-1)^{k+1} (1 + i)^{-x} - \frac{n^M}{i(1 + i)^n}$$

which reduces to the recursive formula

$$(P/G_M, i, n) = \frac{1 + i}{i} \sum_{k=0}^{M-1} \binom{M}{k} (-1)^{M-k-1} (P/G_k, i, n) + \frac{(-1)^M}{i} - \frac{(n - 1)^M}{i(1 + i)^n}, \quad M \geqslant 1 \tag{6}$$

Thus, $(P/G_M, i, n)$ is a linear combination $(P/G_0, i, n)$ $(P/G_1, i, n), \cdots, (P/G_{M-1}, i, n)$.

In Table 1, we present closed-form solutions for $(P/G_M, i, n)$, when $M = 0, 1, 2, 3, 4$, or 5.

We feel that using $M \leqslant 3$ will provide sufficient accuracy for most LCC project evaluations in the DSN. In addition, this model will be useful for doing sensitivity and risk analyses for LCC studies.

B. Special Cases

There are two cases of special engineering interest. The first concerns the limiting behavior of the present worth as the project life n becomes infinite. This behavior is important when evaluating projects that have long lifetimes of more than, say, 20 years, like a DSN antenna. The second case is when the interest rate approaches zero. This can be used for doing quick engineering calculations where the interest and inflation rates nearly cancel. This was shown to be the case for the DSN (Ref. 3).

1. Infinite project life. The effect of discounting causes the present worth $(P/G_M, i, n)$ of a project to converge as the lifetime n becomes infinite. This can be verified by applying the ratio test to the terms of the infinite series generated from Eq. (3). Thus, $(P/G_M, i, n)$ exhibits asymptotic behavior as

$n \to \infty$. If the interest rate is not zero, a recursive formula for this asymptotic level is:

$$F(M, i) = \lim_{n\to\infty} (P/G_M, i, n) = \frac{1+i}{i} \sum_{j=1}^{M-1} \binom{M}{j} (-1)^{M-j-1} F(j, i) + \frac{(-1)^M}{1}, \quad M \geq 1$$

where $F(0, i) = 1/i$. This formula was derived by taking limits in Eq. (6). In Table 2, the limiting values are summarized for $M = 0, 1, 2, 3, 4,$ and 5.

These results are very useful for evaluating facilities that are expected to have a long lifetime, like DSN antennas.

2. Zero interest rate. The second case of special interest in the DSN concerns the limiting behavior of the present worth as the interest rate i approaches zero. We will now develop closed-form solutions for these limiting cases. We can avoid taking limits of the closed-form expressions in Table 1 by considering the simpler case of Eq. (3). Using this equation, the present worth of the zero-interest form of the basic model for $M \geq 1$ is

$$(P/G_n, 0\%, n) = \sum_{x=2}^{n} (x-1)^M = \sum_{x=1}^{n-1} x^M$$

In this special case, the present worth is just the sum of the M^{th} powers of the first $n - 1$ positive integers; closed-form expressions for this sum are widely available (Ref. 7). Based on our definition of $(P/G_0, i, n)$, this present worth is n when $M = 0$. In Table 2, the limiting values are summarized for $M = 0, 1, 2, 3, 4,$ and 5.

C. Application of the LCC Model

We will now discuss how one applies the model and then give an example. As described in subsection III-A, the predicted cash flow profile of an n-year project can be approximated, in general, by some M^{th}-degree polynomial of the form given in Eq. (1). Consequently, the cash flow y in year x has $M + 1$ components; a constant cost of size C_0, a linear cost of size C_1, $\cdots$, and a M^{th}-degree cost of size C_M. Hence, we can determine the total present worth PW_{LCC} of a project having this cash flow profile by finding the corresponding linear combination of basic present worths $(P/G_0, i, n)$, $(P/G_1, i, n), \cdots, (P/G_M, i, n)$:

$$PW_{LCC} = C_M(P/G_M, i, n) + \cdots + C_1(P/G_1, i, n) + C_0(P/G_0, i, n).$$

This method will be described in more detail in subsection III-D.

In most real-life situations, the future cash flow profile is determined by an equation of the form

$$y = B_M x^M + B_{M-1} x^{M-1} + \cdots + B_1 x + B_0 \qquad (7)$$

where y is the cash flow in year x. Accordingly, Eq. (7) is converted to the required form of Eq. (1) by using the identity

$$C_j = \sum_{k=j}^{M} B_k \binom{k}{j}, \quad j = 0, 1, 2, \cdots, M$$

It should be noted that repeated applications of this model may be necessary to evaluate the present worth of the total LCC. For example, suppose we found polynomials y_1 and y_2 corresponding to the cash flow profiles for initial investment costs and for annual maintenance and operations costs, respectively, as shown in Fig. 3. Since the initial investment costs begin in year 1, we may directly apply the general model to the cash flow polynomial y_1 to get R_1, the present worth in year zero of the initial investment costs. Next, we apply the general model to y_2 to get the present worth in year n' of the annual operating costs. We then apply the present worth factor $(P/F, i, n') = 1/(1+i)^{n'}$ to the result to get R_2, the present worth in year zero of the annual operating costs. Finally, the present worth in year zero of the total LCC is the sum of R_1 and R_2.

The polynomial form that one uses to represent the yearly cash flows can be developed either by curve-fitting or from a forecasting model.

Equivalent uniform annual cost (EUAC) computations can easily be performed by converting the previous present worth calculations to a level annuity:

$$EUAC = (P/G_M, i, n)(P/G_0, i, n)^{-1}$$

where

$$(P/G_0, i, n)^{-1} = i(1+i)^n/[(1+i)^n - 1]$$

is the capital recovery factor at interest rate i.

D. Example

This section gives an example of how one might apply the above model in an economic analysis of a proposed project in the DSN. Let's consider, as an illustration, the introduction of increased automation at a Deep Space Station, to reduce operating costs (Refs. 8 through 11). A strong effort has been made since 1969 to reduce operating costs by reducing the number of people at the station. For example, the crew size at a station has been reduced from 26 people in 1969 to 4 today. However, this automation requires large capital investments that must be justified using an LCC analysis. The capital expenditures required must be compared to the reduction in future operating and maintenance costs. Our following example will look at the present value of the projected M&O cash flows for an existing system during its entire life.

Suppose, for example, that the future operating costs of one part of the Deep Space Station are expected to vary according to the second-degree model

$$y = 6x^2 - 11x + 10$$

where y is the operating costs (in units of \$10,000 for this example) at year x, and year zero is taken to be the startup time of the automation project. This polynomial has the form of Eq. (7) and thus is rewritten as

$$y = 6(x - 1)^2 + (x - 1) + 5$$

to coincide with the form of Eq. (1). The present worth of the operating costs at 10 years after startup is

$$PW_{LCC} = 6(P/G_2, 10\%, 10) + (P/G_1, 10\%, 10)$$

$$+ 5(P/G_0, 10\%, 10)$$

Using the formulas in Table 1 with $i = 0.10$ and $n = 10$, we have

$$(P/G_0, 10\%, 10) = 6.1446$$

$$(P/G_1, 10\%, 10) = 22.8913$$

$$(P/G_2, 10\%, 10) = 133.7292$$

Therefore,

$$PW_{LCC} = 6(133.7292) + 22.8913 + 5(6.1446)$$

$$= 8.56 \text{ million dollars}$$

Therefore, if our implementation cost for automation is less than 8.56 million dollars, the expenditure is justified.

Compared to conventional discounting of individual cash flows, this model is very helpful when different polynomials for cash flow prediction exist in several subsystems of one complex system. For example, in a Deep Space Station there will be different cash flow polynomials for the antenna, receivers, and transmitters.

IV. Graphical Review of Results

We examined the behavior of the general present-worth $(P/G_M, i, n)$ as a function of three parameters: the order M of the cash flow model, the prevailing cost of capital i, and the project life n.

First, using the formulas in Table 1, $(P/G_M, 10\%, n)$ was calculated for values of M between 0 and 5 and for project lifetimes of from 2 to 15 years. These results are shown in Fig. 4. For $M = 0$ and 1, the graphs will quickly approach the asymptotic limits described in Table 2.

Next, we performed a graphical sensitivity analysis of the relative effects of the parameters i and M on the present-worth function $(P/G_M, i, n)$, $M = 0, 1, 2, 3$. For each of these values of M, semilogarithmic plots similar to those in Fig. 4 were made for interest rates of 5%, 10%, and 15%. These results are presented in Fig. 5 and show four families ($M = 0, 1, 2, 3$) of present-worth curves at three interest rates (5%, 10%, 15%). We can see from Fig. 5 that M has a more significant impact on the LCC present value $(P/G_M, i, n)$ than does the interest rate i. For example, increasing M from 2 to 3 increases the present worth by a factor of about 6, for interest rates of 5% or 10%. However, decreasing i from 10% to 5% increases the present worth only by a factor of about 1.4, for $M = 2$, or 3.

The next section deals with approximating the logarithm of these curves by linear functions, for a 10-year project life. The approach that will be presented is valid for any project life, but a 10-year project life is of interest to the DSN and several U.S. government agencies, where it is used as a benchmark in evaluating many projects.

V. Simplified Model for a Project With a Ten-Year Life Cycle

Figure 6 is a semilogarithmic graph of $(P/G_M, i, 10)$ as a function of the interest rate i (1% to 15%). Since the six plots in this figure are nearly straight lines, $\log_{10} (P/G_M, i, 10)$ may be approximated by a linear function for each value of M. Consequently, the present worth may be approximated by much simpler formulas than those given in Table 1. Six linear regressions on $\log_{10} (P/G_M, i, 10)$ were performed and the resulting functions $f_M(i)$ that approximate $(P/G_M, i, 10)$ are presented in Table 3. Clearly, $\log_{10} f_M(i)$ produces a straight line for each M.

The functions in Table 3 are very good approximations of the complex formulas in Table 1. The maximum relative error is only 1.7% and the average relative error is only 0.7%. Returning to the example in subsection III-F, we have

$$PW_{LCC} = 6(P/G_2, 10\%, 10) + (P/G_1, 10\%, 10)$$

$$+ 5\,(P/G_0, 10\%, 10)$$

We now use the values given in Table 3 to get

$$PW_{LCC} = 6f_2\,(10\%) + f_1\,(10\%) + 5f_0\,(10\%)$$

$$= 6S_2T_2^{0.1} + S_1T_1^{0.1} + 5S_0T_0^{0.1}$$

$$= 8.62 \text{ million dollars}$$

(vs. 8.56 million dollars as calculated in subsection III-F. using the exact equation). The relative error between these figures is only 0.7%. This is well within the accuracy of most project economic models.

Most U.S. government agencies, such as NASA, the Navy, and the Department of Defense, use a value of $i = 10\%$ in their calculations (Ref. 12). Table 4 gives the value of $f_M(i)$ for $i = 10\%$. Now it is very easy to estimate the life-cycle cost for projects in the DSN by simply using the results in Table 4.

VI. Summary

We have constructed a mathematical model for determining the present worth of the life-cycle costs of a project when the expected future disbursements can be approximated by a general polynomial function, and we have illustrated by means of an example the application of the model. This model is a generalization of a basic model that has the future cash flows predicted by the equation $y = (x - 1)^M$, where y is the expected costs of the proposed project of year x. The model extends the usual engineering economic models of a level annuity ($M = 0$) and a linear gradient ($M = 1$). We then developed closed-form expressions of $(P/G_M, i, n)$ for $M = 0$ through $M = 5$.

Limiting cases of the model were also examined, and we found that in the case of an infinite project life or a negligible interest rate, the present worth calculation could be considerably simplified. A method for computing equivalent uniform annual costs was also presented.

The present worth $(P/G_M, i, n)$ is a function of the interest rate i, the estimated project life n, and the degree M of the predicting equation. A graphical sensitivity analysis of the model showed that the parameter M corresponding to the degree of the predicting equation was more influential in determining the present worth $(P/G_M, i, n)$ than the interest rate i. For a project with a 10-year life, graphical analysis showed that approximations of the present worth expressions could be developed using linear regression. These approximations are much simpler than the original expressions and have the advantage of introducing an average error of only 0.7%.

By using the results developed here for the DSN, the life-cycle cost calculation will be greatly simplified when comparing projects.

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Table 1. Closed-form solutions for the basic model ($n > 1$)

M	$(P/G_M, i, n)$
0	$\dfrac{1}{i\,(1+i)^n}\;[(1+i)^{n-1} - 1]$
1	$\dfrac{1}{i^2\,(1+i)^n}\;[(1+i)^n - ni - 1]$
2	$\dfrac{1}{i^3\,(1+i)^n}\;[(1+i)^{n+1} + (1+i)^n - n^2i^2 - (2n+1)\,i - 2]$
3	$\dfrac{1}{i^4\,(1+i)^n}\;[(1+i)^{n+2} + 4\,(1+i)^{n+1} + (1+i)^n - n^3i^3 - (3n^2+3n+1)i^2 - 6\,(n+1)\,i - 6]$
4	$\dfrac{1}{i^5\,(1+i)^n}\;[(1+i)^{n+3} + 11\,(1+i)^{n+2} + 11\,(1+i)^{n+1} + (1+i)^n - n^4i^4 - (4n^3+6n^2+4n+1)\,i^3 - (12n^2+24n+14)\,i^2 - (24n+36)\,i - 24]$
5	$\dfrac{1}{i^6\,(1+i)^n}\;[(1+i)^{n+4} + 26\,(1+i)^{n+3} + 66\,(1+i)^{n+2} + 26\,(1+i)^{n+1} + (1+i)^n - n^5i^5 - (5n^4+10n^3+10n^2+5n+1)\,i^4 - (20n^3 + 60n^2 + 70n + 30)\,i^3 - (60n^2+180n+150)\,i^2 - (120n+240)\,i - 120]$

Table 2. Asymptotic levels for the basic model

M	Infinite project life, $\lim_{n\to\infty} (P/G_M, i, n)$	Zero interest rate, $\lim_{i\to 0} (P/G_M, i, n)$
0	$1/i\,(1+i)$	$n-1$
1	$1/i^2$	$(n^2-n)/2$
2	$(i+2)/i^3$	$(2n^3-3n^2+n)/6$
3	$(i^2+6i+6)/i^4$	$(n^4-2n^3+n^2)/4$
4	$(i^3+14i^2+36i+24)/i^5$	$(6n^5-15n^4+10n^3-n)/30$
5	$(i^4+30i^3+150i^2+240i+120)/i^6$	$(2n^6-6n^5+5n^4-n^2)/12$

Table 3. Approximating functions $f_M(i)$ for $(P/G_M 1, i, 10)$

M	$(P/G_M, i, 10) \approx f_M(i) = S_M \cdot T_M^i$	
	$S_M/10^M$	$T_M \times 10^{-5}$
0	9.739	108
1	4.390	16.01
2	2.785	7.008
3	1.981	4.228
4	1.501	3.033
5	1.183	2.403

Table 4. Approximations for $(P/G_M, 10\%, 10)$

M	$(P/G_M, 10\%, 10) \approx f_M(10\%)$	
	$f_M(10\%)/10^M$	Relative error, %
0	6.193	0.78
1	2.306	0.75
2	1.347	0.71
3	0.911	0.68
4	0.608	0.66
5	0.514	0.65

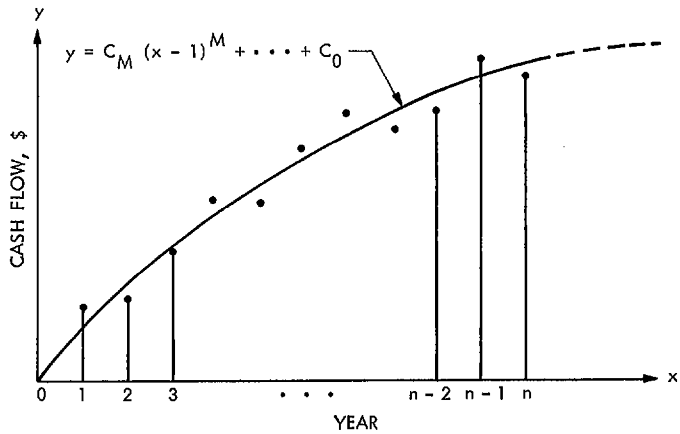

Fig. 1. General approximation of cash flows

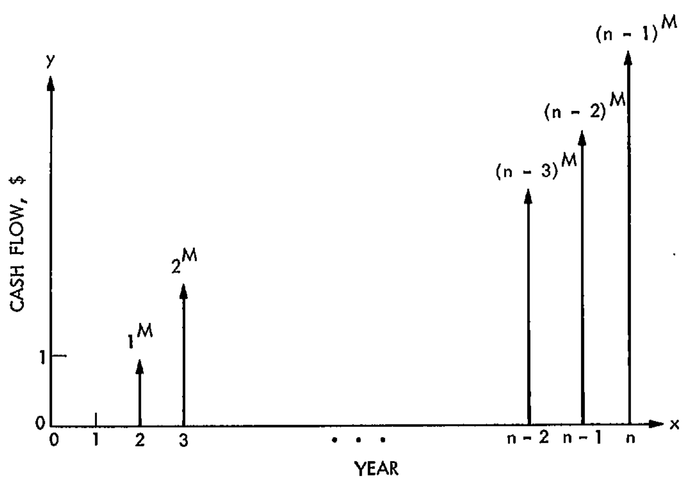

Fig. 2. Cash flow profile for the basic model

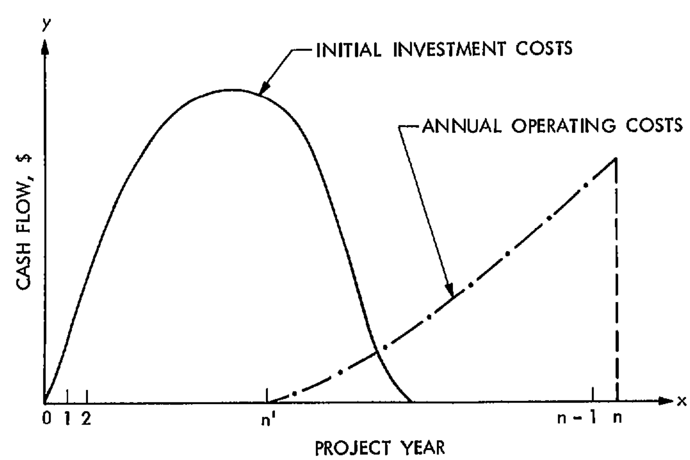

Fig. 3. Two-stage life-cycle cost example

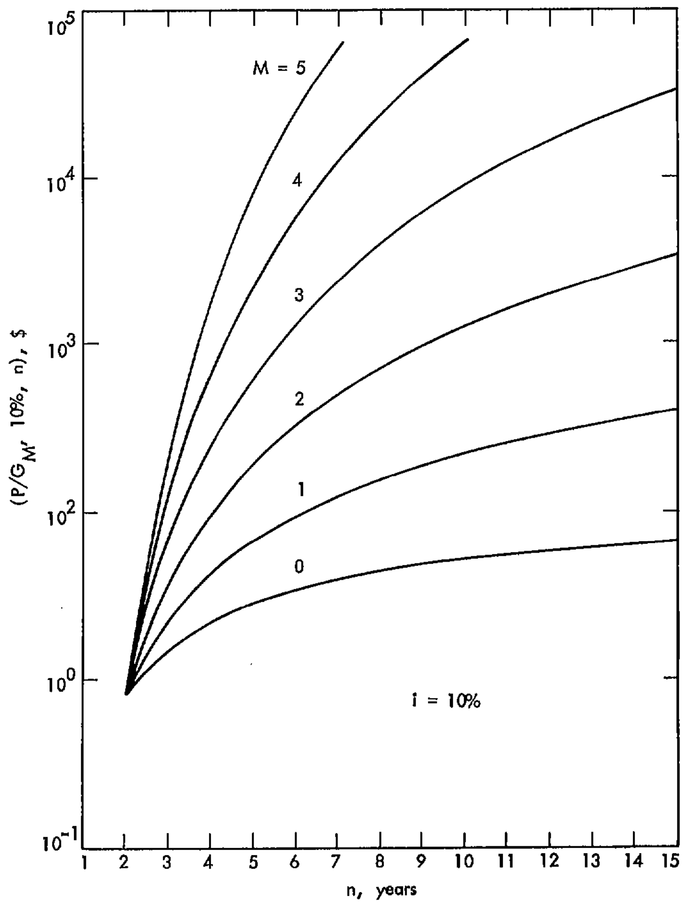

Fig. 4. Life-cycle cost present worth (P/G_M, 10%, n) vs project life

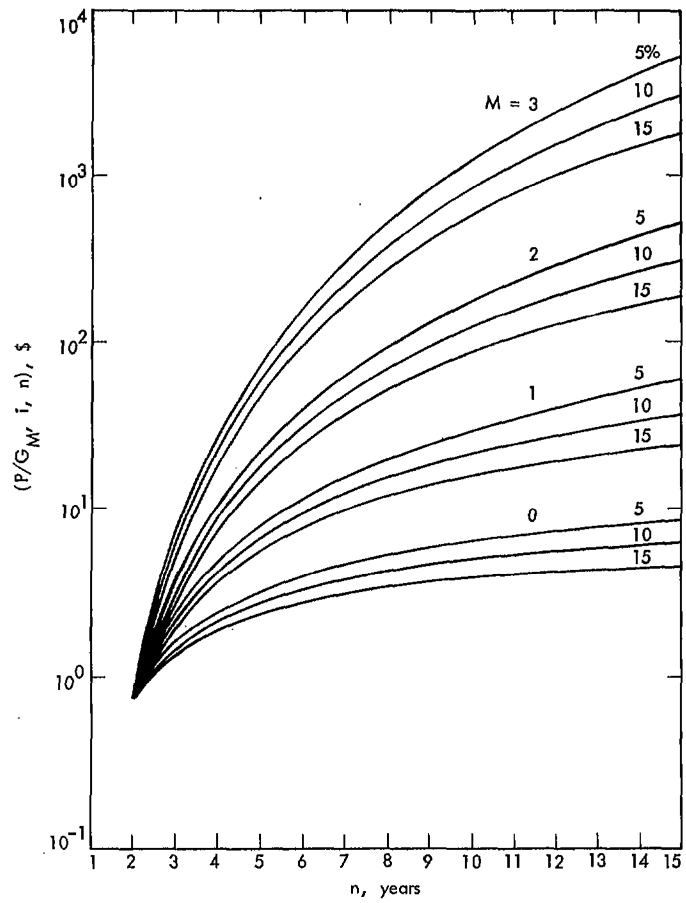

Fig. 5. Life-cycle cost present worth (P/G_M, i, n) vs project life

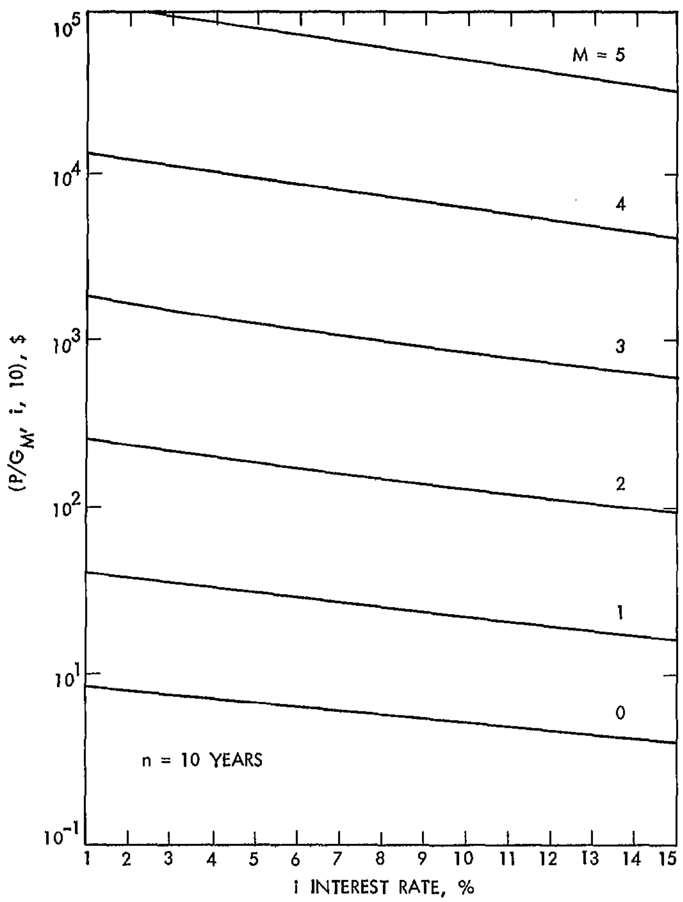

Fig. 6. Life-cycle cost present worth vs interest rate