Noise Temperature and Noise Figure Concepts: DC to Light

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Deep Space communication systems require extremely sensitive receiving systems. The NASA Deep Space Network is investigating the use of higher operational frequencies for improved performance. Noise temperature and noise figure concepts are used to describe the noise performance of these receiving systems. It is proposed to modify present noise temperature definitions for linear amplifiers so they will be valid over the range $(hf/kT) \ll 1 \ll (hf/kT)$. This is important for systems operating at high frequencies and low noise temperatures, or systems requiring very accurate calibrations. The suggested definitions are such that for an "ideal" amplifier, $T_e = (hf/k) = T_q$ and $F = 1$. These definitions revert to the present definition for $(hf/kT) \ll 1$. Noise temperature calibrations are illustrated with a detailed example. These concepts are applied to system signal-to-noise analysis. The fundamental limit to a receiving system sensitivity is determined by the thermal noise of the source and the quantum noise limit of the receiver. The sensitivity of a receiving system consisting of an "ideal" linear amplifier with a 2.7 K source, (-194.3 dBm/Hz assuming $(hf/kT) \ll 1$) degrades significantly at higher frequencies.

I. Introduction

Deep space communication systems require extremely sensitive receiving systems (Ref. 1). The Deep Space Network is investigating the use of higher operational frequencies for improved performance. Noise temperature and noise figure concepts are used to describe the noise performance of these receiving systems (Refs. 2, 3, 4). These concepts are reviewed for application to higher frequencies by removal of the low-frequency restriction, $(hf/kT) \ll 1$.

II. Theory

The available thermal (Refs. 5, 6, 7) noise power from a source at the amplifier output ($G \gg 1$) is given by

$$P_n = kTBG\left(\frac{hf/kT}{e^{hf/kT}-1}\right) \tag{1}$$

where

T = source temperature, K

h = Planck's constant = 6.6256×10^{-34} J-S

k = Boltzman's constant = 1.3805×10^{-23} J/K

f = operating frequency, Hz

$B = \frac{1}{G} \int_0^{\infty} G(f) df$ = noise bandwidth, Hz

$G(f)$ = available power gain, ratio

G = maximum available power gain, ratio

The amplifier output is approximately (disregarding the contribution of the amplifier: $hf \ll kT$)[1]

$$P_n = kTBG \tag{2}$$

Eqs. (1) and (2) are shown plotted in Fig. 1 for a large range of hf/kT values. Most microwave applications with $(hf/kT) \ll 1$ are restricted to the region near the origin.

It is computationally convenient to define a temperature T' such that

$$P_n = kT'BG = kTBG \left(\frac{hf/kT}{e^{hf/kT} - 1} \right)$$

T' can be conveniently found by subtracting a correction T_c (Ref. 8) from T

$$(P_n/kBG) = T' = T - T_c \tag{3}$$

where

$$T_c = T \left(1 - \frac{hf/kT}{e^{hf/kT} - 1} \right)$$

or conveniently

$$T_c \cong 0.024 f\,(\mathrm{GHz}) - 0.000192\,[f(\mathrm{GHz})]^2/T + \cdots \tag{4}$$

These correction terms are shown plotted in Fig. 2. For example, at 32 GHz, the cosmic background temperature (Refs. 9, 10) of ≅ 2.7 K, correctly defined for use with Eq. (1) is "corrected" to ≅ 2.0 K for use with Eq. (2).

The quantum noise limit (Ref. 7, 11) of a linear amplifier (where both phase and amplitude information is retained), a manifestation of the quantum mechanics uncertainty principle, is given by[2]

$$P_n = hfBG \tag{5}$$

Fundamental limits of an ideal receiving system sensitivity are determined by the sum of the source thermal noise and the quantum noise limit of an ideal amplifier (since these noise sources are uncorrelated (Ref. 7)).

$$P_n = kTBG \left(\frac{hf/kT}{e^{hf/kT} - 1} \right) + hfBG = k(T' + T_q) BG \tag{6}$$

or

$$(P_n/kBG) = T' + T_q$$

This is plotted if Figs. 1, 3 and 4 as functions of (hf/kT), f and T. In Fig. 4, the value of P_n/kBG for $(hf/kT \gg 1)$ is given by hf/k (or T_q) and for $(hf/kT \ll 1)$, by $T + hf/2k$ (or $T + T_q/2$). It is easily shown (Fig. 1) that the quantum noise limit and thermal noise are equal when $(hf/kT) = \ln 2 \cong 0.69$.

III. Noise Temperature and Noise Figure: $(hf/kT) \ll 1$

A receiving system noise performance is characterized by the operating noise temperature (Refs. 2, 3, 4, 13) defined by (linear amplifier, single channel, matched source, $G \gg 1$)

$$T_{op} = N_{T_o}/kBG \tag{7}$$

[1]Note that $(hf/kT) \approx 0.048\, f\,(\mathrm{GHz})/T(k) \approx 0.00048$ at 1 GHz and 100 K and ≈ 1 at 208 GHz and 10 K indicates that Eq. (2) is an extremely good approximation for most microwave applications.

[2]Eq. (5) is appropriate only for linear amplification. At optical frequencies, using discrete photons, techniques may exist (Ref. 7) to circumvent this limitation. Equating kTB to quantum noise hfB results in an equivalent quantum noise temperature, $T_q = (hf/k)$. Although T_q is a fictitious temperature, it is useful for computational analysis and can be used with Eq. 2 to compute P_n.

where

N_{T_o} = receiver system total output noise power, within the frequency band B, excluding output load noise, W

Also

$$T_{op} = T_i + T_e \tag{8}$$

where

T_i = input source noise temperature, K

T_e = effective input noise temperature of the receiver, K

The definition (Refs. 2, 3, 4) of an amplifier noise figure is

$$F = N_{T_o}(T_i = T_o)/kT_oBG \tag{9}$$

where

$N_{T_o}(T_i = T_o)$ = receiver total output noise power with input termination at temperature T_o, W

T_o = 290 K

From Eqs. (7), (8) and (9)

$$F = 1 + (T_e/T_o) \tag{10}$$

or

$$T_e = (F - 1)\,T_o$$

For an "ideal" amplifier, $T_e = 0$, $F = 1$, and

$$T_{op} = T_i$$

IV. Noise Temperature and Noise Figure: $0 \leq (hf/kT) \leq \infty$

As before,

$$T_{op} = N_{T_o}/kBG \tag{11}$$

Substituting T_i' for T_i in Eq. (8)[3]

$$T_{op} = T_i' + T_e \tag{12}$$

where

$$T_i' = T_i\left(\frac{hf/kT_i}{e^{hf/kT_i}-1}\right)$$

Following the spirit ($F = 1$ for an "ideal" amplifier) of Eq. (9), define

$$F = N_{T_o}(T_i = T_o')/k(T_o' + T_q)BG \tag{13}$$

where

$N_{T_o}(T_i = T_o')$ = receiver total output noise power with input termination at temperature T_o', W

$$T_o' = T_o\left(\frac{hf/kT_o}{e^{hf/kT_o}-1}\right)$$

T_o = 290 K

From Eqs. (11), (12) and (13)

$$F = (1 + T_e/T_o')/(1 + T_q/T_o') \tag{14}$$

or

$$T_e = (F - 1)\,T_o' + FT_q$$

For an "ideal" amplifier $T_e = T_q$, $F = 1$, and

$$T_{op} = T_i' + T_q$$

These equations all revert to present definitions, Section III, when $(hf/kT) \ll 1$; i.e., $T_q = 0$, $T_i' = T_i$, $T_o' = T_o$.

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[3]T_e is obtained by analysis or measurement as shown in Appendix A. For an ideal amplifier, $T_e = hf/k = T_q$.

V. Noise Temperature and Noise Figure: $(hf/kT) >> 1$

As before,

$$T_{op} = N_{T_o}/kBG \tag{15}$$

At these very high frequencies, thermal noise is negligible compared to T_q, so that $T_i' = T_o' \to 0$, and from Eq. (12)

$$T_{op} = T_e \tag{16}$$

and from Eq. (13),

$$\begin{aligned} F &= N_{T_o}/kT_qBG \\ &= N_{T_o}/hfBG \end{aligned} \tag{17}$$

From Eqs. (15), (16), and (17), or directly from Eq. (14),

$$F = T_e/T_q \tag{18}$$

or

$$T_e = FT_q$$

For an "ideal" amplifier, $F = 1$, $T_e = T_q$, and

$$T_{op} = T_q$$

VI. System Performance

The output signal-to-noise power ratio for a receiving system is given by $((hf/kT) << 1)$,

$$\begin{aligned} (S_o/N_o) &= \left(S_iG/N_{T_o}\right) \\ &= S_i/kT_{op}B \\ &= S_i/k\,(T_i + T_e)\,B \\ &= S_i/k\,(T_i + (F-1)\,T_o)\,B \end{aligned} \tag{19}$$

where

S_i = input signal power, W

For $(0 \leqslant (hf/kT) \leqslant \infty)$,

$$\begin{aligned} (S_o/N_o) &= S_i/kT_{op}B \\ &= S_i/k\,(T_i' + T_e)\,B \\ &= S_i/k\,(T_i' + (F-1)\,T_o' + FT_q)\,B \end{aligned} \tag{20}$$

and for $(hf/kT) >> 1$,

$$\begin{aligned} (S_o/N_o) &= S_i/(kT_{op}B) \\ &= S_i/kT_eB \\ &= S_i/FkT_qB \\ &= S_i/FhfB \end{aligned} \tag{21}$$

The performance of a receiving system composed of an "ideal" amplifier ($T_e = T_q$) and a source at the cosmic background temperature ($T_i = 2.7$ K), is shown in Table 1 and Fig. 5. S_i is the input signal power required for $(S_o/N_o) = 1$. Table 1 and Fig. 5 demonstrate the loss in sensitivity at very high frequencies relative to low frequencies for a conventional receiving system with an "ideal" linear receiver.

VII. Conclusion

The equations developed in the previous sections are tabulated in Table 2. The use of noise temperature and noise figure concepts require special consideration when $(hf/kT) \neq << 1$. This is usually important for systems operating at high frequencies with low noise temperatures, or systems requiring precise calibrations.

Fundamental limits to a receiving system are determined by the thermal noise of the source and the quantum noise limit of the amplifier. The sensitivity of a receiving system consisting of an "ideal" linear amplifier with a 2.7 K source (−194.3 dBm/Hz assuming $(hf/kT) << 1$) degrades significantly at higher frequencies. Of course, there are tremendous advantages at higher frequencies, such as greater potential bandwidth, less spectrum crowding, link security, smaller antennas, photon counting schemes, etc.

A frequent end goal of using the more complicated equations of Section IV is to obtain the most accurate value of T_{op} for use with system performance evaluation such as in Section VI. T_{op} consists of the sum of T_i' and T_e. The decrease of T_i' and increase of T_e with frequency tends to cancel, minimizing the error in T_{op}. Therefore, the techniques of Section III can be used to very good accuracy to extremely high frequencies (see Table A-1 and Fig. A-2) for calculation of T_{op}.

However, for precise calibrations, the proposed equations of Section IV should be used.

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Table 1. Tabulation of the sensitivity of an "ideal" receiver ($T_e = T_q$) with an input source temperature of 2.7 K as a function of frequency

Parameter	Equations (8), (19)	Equations (12), (20)				Equations (16), (21)
	Frequency, GHz					
	0.0	32	200	400	2000	20,000 (λ = 15 μm)
T_i', K	2.7	2.0	0.3	0.0	0.0	0.0
$T_e = T_q$, K	0.0	1.5	9.6	19.2	96.0	960.0
T_{op}, K	2.7	3.5	9.9	19.2	96.0	960.0
S_i, dBm/Hz	−194.3	−193.1	−189.0	−185.8	−178.8	−168.8

Table 2. Summary of noise temperature and noise figure concepts: dc to light (Linear amplifier, single channel, matched source, G >> 1)

Parameter	$(hf/kT) = Tq/T$		
	$0 \leqslant (hf/kT) \leqslant \infty$	$(hf/kT) << 1$	$(hf/kT) >> 1$
$T_{op} = N_{T_o}/kBG$	$= T_i' + T_e$	$= T_i + T_e$	$= T_e$
	$= *[T_i' + T_q]$	$= *[T_i]$	$= T_q$
F	$= N_{T_o}\,(T_i = T_o')/k\,(T_o' + T_q)\,BG$	$= N_{T_o}\,(T_i = T_o)/kT_oBG$	$= N_{T_o}/hfBG = N_{T_o}/kT_qBG$
	$= (1 + T_e/T_o')/(1 + T_q/T_o')$	$= 1 + (T_e/T_o)$	$= 1 + (T_e/T_q)$
	$= *[1]$	$= *[1]$	$= *[1]$
(S_o/N_o)	$= S_i/k\,(T_i' + T_e)\,B$	$= S_i/k\,(T_i + T_e)\,B$	$= S_i/kT_eB$
	$= S_i/k(T_i' + (F-1)\,T_o' + FT_q)\,B$	$= S_i/k\,(T_i + (F-1)\,T_o)\,B$	$= S_i/FhfB = S_i/FkT_qB$
	$= *[S_i/k\,(T_i' + T_q)\,B]$	$= *[S_i/k\,T_i B]$	$= *[S_i/hfB = S_i/kT_qB]$

*[ ] "Ideal" amplifier (i.e., $T_e = T_q, F = 1$)

$$T_i' = T_i \left(\frac{T_q/T_i}{e^{T_q/T_i} - 1} \right)$$

S_i = Input signal power, W

N_{T_o} = Total output noise power within bandwidth B, W

$$T_o' = T_o \left(\frac{T_q/T_o}{e^{T_q/T_o} - 1} \right)$$

T_o = 290 K

$T_q = (hf/k) \cong 0.048f$ (GHz)

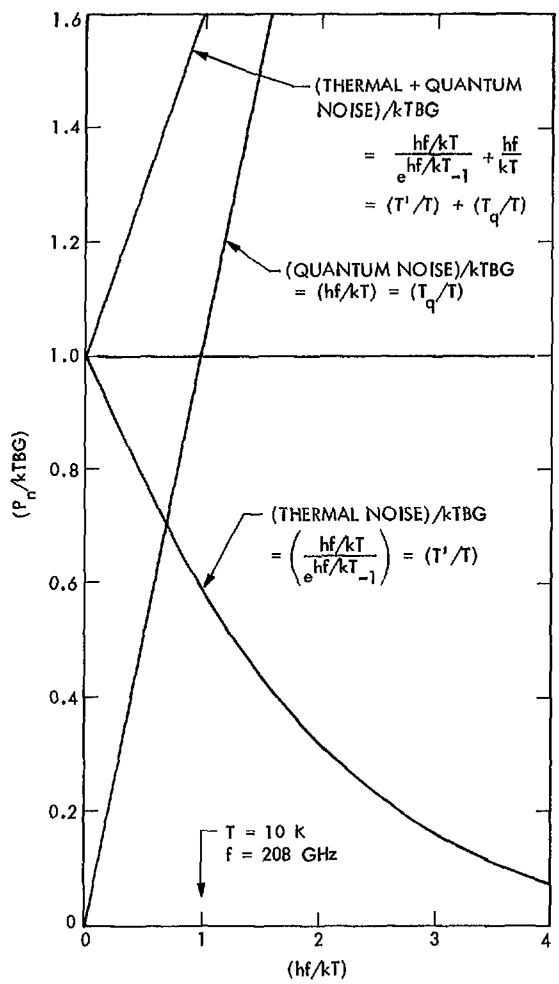

Fig. 1. $(P_n/kTBG)$ vs (hf/kT), showing the thermal and quantum noise contributions

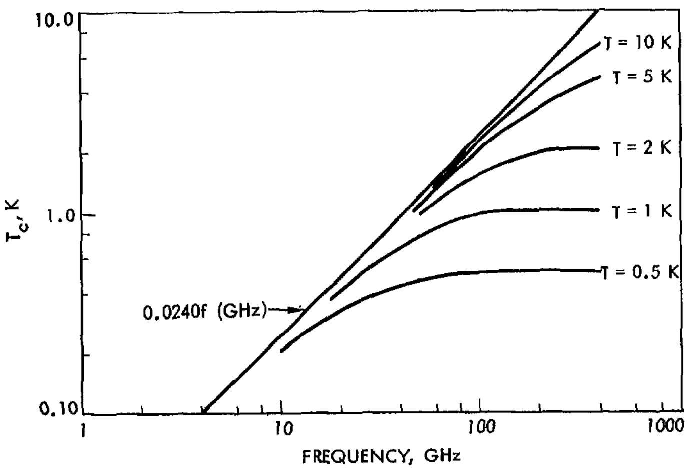

Fig. 2. Thermal noise temperature correction T_c as a function of frequency and temperature

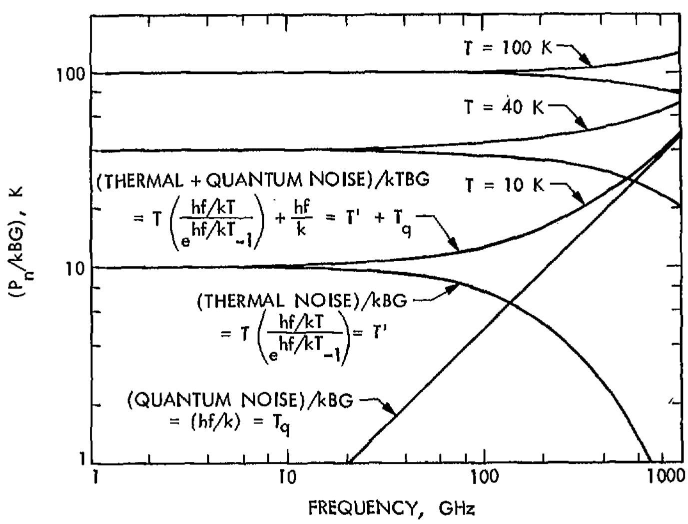

Fig. 3. $(P_n/kTBG)$ vs frequency for various temperatures, showing thermal and quantum noise contributions

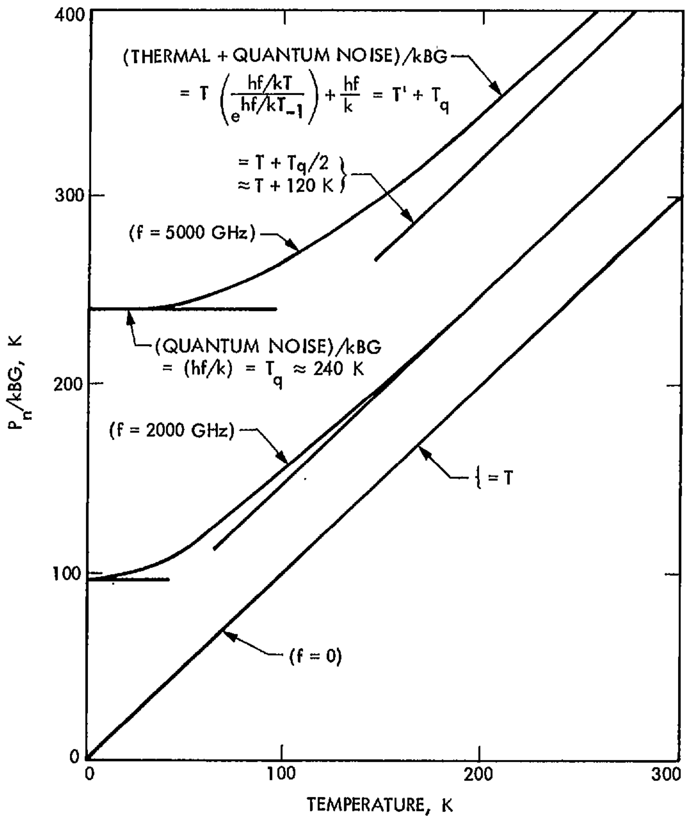

Fig. 4. (P_n/kBG) **vs temperature for various frequencies**

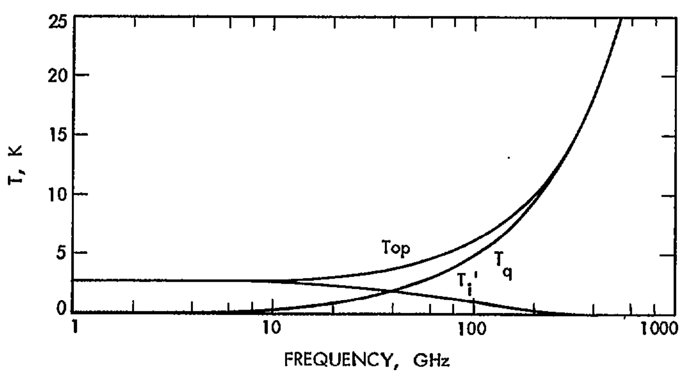

Fig. 5. T'_i, T_q **and** T_{op} **vs frequency using an "ideal" receiver (**$T_e = T_q$**) with an input source temperature of 2.7 K.**

Appendix A

Amplifier Noise Temperature Calibration

Consider the Y factor equations for the amplifier in the test configuration shown in Fig. A-1 using two matched calibration terminations at physical temperatures T_2 and T_1.

Case 1: $(hf/kT) \ll 1$

$$Y = \frac{T_2 + T_e}{T_1 + T_e} \tag{A-1}$$

Solving

$$T_e = \frac{T_2 - YT_1}{Y - 1} \tag{A-2}$$

Case 2: $0 \leqslant (hf/kT) \leqslant \infty$

$$Y = \frac{T_2' + T_e}{T_2' + T_e} \tag{A-3}$$

Solving

$$T_e = \frac{T_2' + YT_1'}{Y - 1} \tag{A-4}$$

where

$$T_2' = T_2 \left(\frac{hf/kT_2}{e^{hf/kT_2} - 1} \right)$$

$$T_1' = T_1 \left(\frac{hf/kT_1}{e^{hf/kT_1} - 1} \right)$$

Consider the following parameters applicable to Fig. A-1.

$T_1 = 100\text{ K} \qquad T_i = 10\text{ K}$

$T_2 = 400\text{ K} \qquad Y = 3.7$

These values are used with Eqs. (A-2) and (A-4) for a wide range of frequencies. The results are tabulated in Table A-1 and plotted in Fig. A-2. T_i' and T_q can be estimated from Fig. 3. For these parameters, T_i decreases with frequency, while T_e increases. Since T_{op} is the sum of T_i' and T_e, there is a compensatory effect; however, there is a resultant net increase in T_{op} with frequency. For these parameters, the amplifier is nearly "ideal" at 400 GHz.

Table A-1. Tabulation of receiver and system parameters, using test parameters: T_1 = 100 K, T_2 = 400 K, T_i = 10K and Y = 3.7

	Equations (A-1), (8)	Equations (A-2), (12)		
	Frequency GHz			
Parameter, K	0.00	32	200	400
T'_1	100	99.23	95.28	90.71
T'_2	400	399.23	395.22	390.48
T'_i	10	9.25	5.96	3.30
$T_q = (hf/k)$	0.0	1.54	9.60	19.20
T_e	11.11	11.88	15.81	20.32
T_{op}	21.11	21.13	21.77	23.62

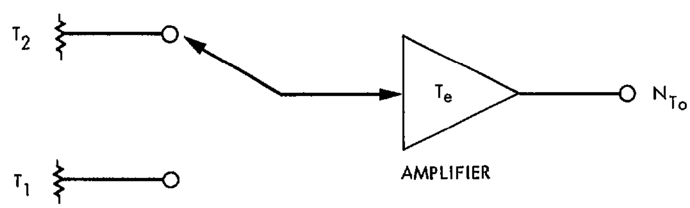

Fig. A-1. Block diagram of amplifier test configuration

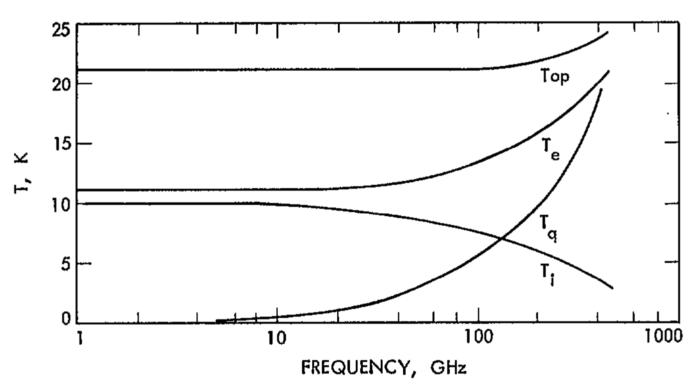

Fig. A-2. T'_i, T_e, T_q and T_{op} vs frequency assuming T_i = 10K and Y = 3.7