A Near-Optimum Receiver Structure for the Detection of *M*-ary Optical PPM Signals

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A class of receivers called "conditionally nulling receivers" is defined for quantum noise limited optical communications. These receivers have the ability to decide at each moment in time whether or not to coherently combine a predetermined local oscillator field with the received optical field, prior to performing an energy measurement (photodetection) on the combined field. Conditionally nulling receivers are applicable to pulse position modulation (PPM) and related modulation schemes, which have the property that, at each moment in time, the transmitted signal is in one of only two states, ON or OFF. The local oscillator field which may or may not be added by the receiver is an exact replica of the negative of the received ON field; hence, the receiver can exactly null the ON signal if the ON signal is present and the receiver chooses to use the local field.

An ideal conditionally nulling receiver achieves very nearly the same error probability (within a multiplicative factor varying from 1 to 2.15) as the optimum quantum measurement for quantum noise limited detection of M-ary *PPM signals. In contrast, other known receiving methods, such as direct, heterodyne, and homodyne detection, are exponentially suboptimum.*

The performance of receivers which are only approximately conditionally nulling receivers, due to imperfect nulling, is also investigated for the quantum limited PPM detection problem. Imperfect nulling is assumed to be caused by a phase discrepancy between the received ON field and the local nulling field. The performance of the imperfect conditionally nulling receiver is found to degrade rapidly to that of a direct detection receiver in the presence of nonzero phase error.

I. Introduction

In many optical communications applications, the predominant "noise" that limits system performance is the fundamental uncertainty of measurement predicted by quantum theory. However, explicit calculation of the ultimate communication inaccuracy legislated by quantum noise has been accomplished for very few problems (Refs. 1-4), and the specification of physical receiving structures for achieving optimum performance is virtually nonexistent (except, see Ref. 5). Conventional receiver structures based on familiar physical devices exhibit peculiar quantum noise limited behavior of their own, but their performance falls short of the fundamental quantum limit.

In this article a new receiver structure is defined for quantum limited optical communications. Receivers with this structure have the ability to decide at each moment in time whether or not to coherently combine a predetermined local oscillator field with the received optical field, prior to performing an energy measurement (photodetection) on the combined field. These receivers are termed "conditionally nulling receivers," because the local oscillator field which may or may not be added by the receiver is an exact replica of the negative of one of the possible received fields. An ideal conditionally nulling receiver achieves very nearly the same error probability (within a multiplicative factor varying from 1 to 2.15) as the optimum quantum measurement for quantum noise limited detection of M-ary PPM signals. In contrast, other known receiving methods, such as direct, heterodyne, and homodyne detection, are exponentially suboptimum.

The purposes of this article are threefold: (1) to set forth a general definition of the class of conditionally nulling receivers; (2) to broaden the definition to include receivers which are only approximately conditionally nulling receivers; and (3) to compare the performance of ideal and imperfect conditionally nulling receivers applied to the quantum noise limited M-ary PPM detection problem.

II. Applicable Modulation Schemes

The useful application of conditionally nulling receivers is restricted to PPM and related modulation schemes, which have the following characteristic: at each moment in time, the transmitted signal is in one of only two states, ON or OFF. The OFF state is assumed to correspond to a field of amplitude zero, and the ON state corresponds to a single nonzero field. At any given instant in time, there may be m possible signals in the ON state and m_o possible signals in the OFF state, and m, m_o may vary with time. Multiple amplitude levels to help distinguish among the m ON signals are not permitted, but the common ON state amplitude level may vary with time. Any receiver attempting to decipher this type of signaling can be viewed as an assembler of *binary* information, favoring either the m ON signals or the m_o OFF signals collectively, extracted from the received signal at each instant in time.

M-ary PPM signaling fits nearly in this category, because at every moment in time $(M - 1)$ of the M possible signals are OFF and only one possible signal is ON. PPM is presented as the prototypical modulation scheme for conditionally nulling receivers, and all of the performance analysis in this article is based on PPM.

Three examples of applicable modulation schemes are shown in Fig. 1a, b, c. Figure 1a shows standard 4-ary PPM, and Fig. 1b shows a PPM-complementary modulation scheme, which is discussed at length in the next section. Figure 1c shows a 4-slot coding scheme in which 6 possible messages are represented by 6 different pulse doublets.

III. An Unconditional Role Reversal Strategy

It is well known that direct detection does not distinguish symmetrically between ON and OFF signals. For example, under quantum noise limited conditions, the detection of a single quantum of energy is sufficient to positively identify the ON signal, but equally conclusive confirmation of the OFF signal is not possible. It is reasonable to ask whether simple direct detection optimally utilizes this inherent asymmetry as it attempts to distinguish at each moment in time between m ON signals and m_o OFF signals. Perhaps more useful information could be obtained if the m signals were OFF and the m_o signals were ON, rather than vice-versa.

A simple, complete reversal of the roles of the ON and OFF signals does not necessarily lead to improvement. For example, consider quantum limited minimum error probability detection of the "role-reversed" M-ary PPM signal set shown in Fig. 1b. With standard PPM signaling, if the jth hypothesis H_j is true, the signal is ON during the jth signaling slot and OFF during the remaining $(M - 1)$ slots. On the other hand, with "role-reversed" PPM, the signal is OFF during the jth slot and ON during the remaining $(M - 1)$ slots, under hypothesis H_j. A direct detection receiver is certain to record a zero energy measurement during at least one slot (the OFF period of the true signal), and it errs only if additional zero energy measurements are also obtained. A performance expression for this problem is easy to write down, if equal a priori probabilities and equal ON signal slot energies are assumed.

$$P_e' = \sum_{m=0}^{M-1} \frac{m}{m+1} \binom{M-1}{m} p^m (1-p)^{M-1-m} \tag{1}$$

Here, P_e' is the error probability, p is the probability of measuring zero energy from the ON signal during a single slot ($p = e^{-E}$, where E is the mean detected ON signal slot energy in units of photons), and m indexes the number of slots in addition to the true signal slot in which no photons are observed. The sum in (1) may be collapsed to the form

$$P_e' = \frac{1}{Mp} \left[(1-p)^M - 1 + Mp\right] \tag{2}$$

The corresponding performance expression for straightforward PPM detection is well known,

$$P_\epsilon^o = \frac{M-1}{M}\, p \tag{3}$$

and it can be shown that

$$P_\epsilon^o \leqslant P_\epsilon' \ , \ \text{for all } p \epsilon\, [0, 1] \tag{4}$$

Thus, "role-reversed" PPM appears to be inferior to straightforward PPM, even with no consideration given to its apparently increased requirements for average transmitter power (requiring $(M - 1)E$ detected photons per symbol instead of E).

IV. The Ideal Conditionally Nulling Receiver

A. General Description

It turns out that the conclusion of the previous section is no longer valid if the ON-OFF role reversal can take place at the receiver and if the receiver is able to continually choose whether or not to perform the reversal, based on its prior observations. Note that the receiver can accomplish the role reversal on uncorrupted ON-OFF signals by coherently adding the negative of the ON signal prior to detection, i.e., by nulling the ON signal. Exact role reversal at the receiver may prove impossible under two nonideal conditions: (1) if the received signal has been corrupted by a noisy channel; or (2) if the receiver cannot produce an exact replica of the ON signal.

Any receiver which has the power to precisely reverse the roles of the ON and OFF signals and, in addition, is able to decide at each instant in time t whether or not to perform this operation, based on its own observations prior to t, is termed an ideal conditionally nulling receiver, or ideal conditional nuller for short. A receiver which tries to act like a conditional nuller, but falls short due to an inability to perform the ON-OFF role reversal exactly, is called a conditionally nulling receiver with imperfect nulling.

A block diagram of the conditionally nulling receiver structure is shown in Fig. 2. The received field at any given time t is either the ON signal, denoted $S(t)$, or else 0. At the receiver a replica of the negative of $S(t)$ is generated. Provision is made to coherently combine this field with the received field, but the combining is subject to an on-off switch. The combined field (or received field only) impinges upon a photodetector, and the output of the photodetector (with assumed conditionally Poisson statistics) is used both to control the position of the on-off switch and eventually to infer which message was sent.

B. The Quantum Limited PPM Detection Problem

It has been shown elsewhere (Ref. 6) that there exists a conditionally nulling receiver for quantum limited detection of M-ary PPM signals which approaches the error probability performance of the optimum quantum measurement within a *multiplicative* factor no greater than 2.23, over all ranges of M and slot energy E. This result is significant because the performance of the direct detector in (3) is *exponentially* inferior to the optimum. In this section, the structure of the ideal conditional nuller and its comparative performance are briefly described.

1. Nulling strategy and decision algorithm. The ideal conditionally nulling receiver for the quantum limited PPM problem is defined as follows. During the first signaling slot, the ON signal is nulled and an energy measurement is performed. If no photons are detected, this is considered a partial confirmation of the first hypothesis relative to the remaining $(M - 1)$, and the receiver will continue to believe this hypothesis unless it subsequently obtains sufficiently conclusive evidence in favor of another. Given this state of affairs at the end of the first signaling slot, the receiver decides to forgo nulling the ON signal for the remaining $(M - 1)$ slots and henceforth to simply direct detect, so that any future evidence impeaching the first hypothesis will automatically confirm with certainty one of the others. If, on the other hand, one or more photons are detected during the first signaling slot, the first hypothesis is completely contradicted and the receiver's task is reduced to the $(M - 1)$-ary version of the same problem, since no information discriminating among the remaining $(M - 1)$ hypotheses has yet been obtained. In this case, the receiver proceeds to the second signaling slot and again nulls the ON signal, this time looking for partial confirmation of the second hypothesis if no photons are detected or complete denial of it otherwise. In the former case the receiver discontinues nulling, while in the latter it proceeds to use nulling to test for the third hypothesis, and so forth. The receiver simply continues to null the ON signal until such time as it obtains partial confirmation of a specific hypothesis by the measurement of zero energy throughout the corresponding PPM slot, and afterward it direct detects through all remaining time slots.

The nulling strategy and decision algorithm for the ideal conditional nuller can be represented by a binary tree diagram, as illustrated in Fig. 3 for the case $M = 4$. The four levels of branches in the tree correspond to the four PPM slots. Each node in the tree is labeled "N" or "D" to denote whether nulling mode or direct detection mode is used during the next slot. The branches are labeled "$\geqslant 1$" or "0" to designate whether or not at least one photon is detected during the corresponding slot. Some paths (such as those containing two "$\geqslant 1$" branches from "D" nodes) have probability zero under the assumed ideal conditions, and these are marked "impossible". The end nodes in the tree which are reached with nonzero probability are all marked with the corresponding optimum decision.

2. **Performance.** The performance of the ideal conditional nuller in quantum limited conditions is easy to calculate recursively, and a simple closed form expression is obtained for the case of equal a priori probabilities and equal slot energies E; for details, see Ref. 6.

$$P_\epsilon = \frac{1}{M} [(1-p)^M - 1 + Mp] \tag{5}$$

where $p = e^{-E}$ as before. Note that the error probability in (5) is smaller by a factor of p than the error probability obtained for the unconditional nulling strategy in (2).

The error probability achievable by the optimum quantum measurement is also known for this problem, and it is given by (Ref. 3).

$$P_\epsilon^* = \frac{M-1}{M^2} [\sqrt{1+(M-1)p} - \sqrt{1-p}]^2 \tag{6}$$

The three error probabilities P_ϵ, P_ϵ^*, P_ϵ^o are plotted in Fig. 4a, b, c as a function of the average detected slot energy E, for $M = 2, 16, 256$. The conditional nuller's performance is seen to track the performance of the optimum quantum measurement very closely in all situations. In fact, it can be demonstrated numerically that the deviation is never more than a multiplicative factor of 2.15; i.e.,

$$1 \leqslant P_\epsilon / P_\epsilon^* \leqslant 2.15 \qquad \text{for all } E, M \tag{7}$$

On the other hand, the performance of the direct detection receiver is exponentially inferior to that of the optimum measurement. This inferiority is most apparent in the $M = 2$ graph, in which the (logarithmic) slope of the near-optimum P_ϵ curve is exactly double the (logarithmic) slope of the P_ϵ^o curve. As M increases, larger values of E are needed before the direct detection curve begins to diverge from the other two.

V. Conditionally Nulling Receivers with Imperfect Nulling

The potential performance advantage of conditionally nulling receivers for quantum limited PPM detection has been demonstrated, but there remains a delicate practical problem in implementing the nulling operation. Perfect amplitude and phase coherence is required in order to exactly null the ON signal, and such precision is probably impossible for the receiver to achieve. Thus, it is important to investigate how the performance of a conditional nuller degrades with imperfect nulling.

A. The Effect of Imperfect Nulling

A general definition of the class of conditional nullers with imperfect nulling will not be given here. As stated earlier, imperfect nulling at the receiver might be inevitable in principle due to random channel disturbances imparted to the transmitted signal, or else it may result simply from the receiver's inability to produce an exact replica of a deterministic ON signal. Only the second case is considered in this article.

A conditionally nulling receiver with imperfect nulling differs from an ideal conditional nuller in one important respect. In the absence of nulling, both receivers continually obtain by direct detection information distinguishing an ON signal of mean slot energy[1] E from an OFF signal of slot energy 0. After perfect nulling, the ON signal is converted to slot energy 0 and the OFF signal to mean slot energy E, a perfect reversal of roles. After imperfect nulling, on the other hand, the ON signal is reduced to some nonzero slot energy E_o', and the OFF signal is imparted slot energy E' not necessarily equal to E. It is assumed that the nulling operation is at least accurate enough that $E_o' < E'$. However, the slot energies E', E_o' will always be less distinguishable via direct detection than the slot energies E, 0, and so the efficacy of attempting an imperfect nulling operation is drawn into question.

B. The Quantum Limited PPM Detection Problem

The remainder of the article evaluates the impact of imperfect nulling on the performance of the ideal conditionally nulling receiver found to be near-optimum for the quantum limited PPM detection problem.

1. **A reasonable nulling strategy.** A slight generalization of the ideal conditional nuller's operation is required in order

[1]For simplicity, it is assumed that the mean slot energies E, E_o', E' are derived from mean conditional Poisson intensity functions which are constant over a slot time. This assumption eliminates the need to worry about count data records over time intervals finer than the slot interval.

to adapt it to imperfect nulling. With imperfect nulling, the nulled signal can never be entirely contradicted, and in fact may actually be corroborated relative to the opposite signal by the detection of a small, nonzero number of photons. Thus, a natural generalization of the conditionally nulling receiver described earlier involves setting a threshold θ to determine whether to continue or discontinue nulling after observations during each signaling slot. The generalized receiver starts off in the nulling mode as before, only now the nulling operation is imperfect. At the end of any signaling slot in which nulling was used, a decision is made to continue nulling if the number of photons detected during the slot exceeds θ and to discontinue nulling if the number does not exceed θ. As with the ideal conditional nuller, nulling, once discontinued, is never resumed. On the other hand, a downgraded hypothesis, corresponding to a signaling slot in which nulling was used and counts exceeded threshold, could later be revived and chosen as most likely by the receiver if more damaging negative evidence is ultimately obtained against all the other $(M - 1)$ hypotheses.

The conditional nuller with imperfect nulling just described is not necessarily the best conditional nuller subject to the same nulling inaccuracy. It is simply a straightforward adaptation of the ideal conditional nuller with perfect nulling that was found to be near-optimum for the given detection problem.

The description in this section has detailed only the basic *structure* of the imperfect conditional nuller, the rules by which it prescribes what type of measurement to perform, i.e., to null or not to null prior to detection. It still remains to specify the algorithm by which the receiver optimally combines the outcomes of all its measurements to arrive at a decision. For the ideal conditional nuller, the decision rules and nulling rules went hand-in-hand, and no additional analysis was needed.

2. The MAP decision rule. For the nonideal conditional nuller, the derivation of the optimum decision rule is still somewhat simplified (for the quantum limited problem) by the hard decisions that take place if photons are ever detected in direct detection mode. The receiver must combine the always inconclusive information obtained in nulling mode with the possibly conclusive evidence obtained in direct detection mode. To minimize error probability, the receiver uses a maximum a posteriori probability (MAP) decision rule.

The MAP rule for the nonideal conditional nuller is derived in the Appendix, under the assumptions of equal a priori probabilities, equal slot energies, constant nulling inaccuracy from slot to slot, and constant (but selectable) threshold level θ. A slightly suboptimum version of the rule is used here. It can be stated as follows:

(1) If no switchover from nulling to direct detection occurs, choose the slot with the fewest counts. (In case of a tie for fewest counts, select randomly from the tied slots.)

(2) If a switchover occurs after slot k, and nonzero energy is later detected in slot i, $i > k$, choose H_i.

(3) If a switchover occurs after slot k, and no further counts are recorded, choose H_k.

This statement of the decision rule is exactly optimum if the nulling strategy threshold parameter θ satisfies

$$\theta \leqslant \theta_o \equiv \frac{E + E' - E'_o}{\ln (E'/E'_o)} \tag{8}$$

If θ exceeds θ_o, the true MAP decision rule consists of rules 1 and 2 exactly as stated, together with a refined version of rule 3:

(3*) If switchover occurs after slot k, and no further counts are recorded, choose H_k, unless $k < M$ and the number of counts during slot k exceeds θ_o. In the latter case, randomly select from $H_{k+1}, \cdots, H_M$.

The difference between rules 3 and 3* is minor, and it does not significantly affect the numerical results. Rule 3 is used in this paper because it allows a much clearer presentation of the performance analysis than rule 3*.

A tree diagram depicting the behavior of the nonideal conditional nuller is given in Fig. 5 for the case $M = 4$. The notation conventions are the same as in Fig. 3. The branches leaving the direct detection nodes are the same as before, but the branches leaving the nulling nodes are now marked "$> \theta$" and "$\leqslant \theta$" to indicate the nonzero threshold used to determine when to discontinue nulling. The decisions taken at the end nodes are the same as those in Fig. 3, with the exception that the top node on the tree is now reachable with nonzero probability and an optimum decision rule at that node is now well-defined.

3. Performance. The error probability $\tilde{P}_\epsilon$ achieved by the nonideal conditional nuller can now be calculated. Use as a conditioning variable k, the slot which triggers a switchover from nulling to direct detection, and let n_i, $1 \leqslant i \leqslant M$, denote the number of counts recorded during the ith slot. With the assumed nulling strategy, slot k triggers the switchover if and only if $n_k \leqslant \theta$ and $n_i > \theta$ for $1 \leqslant i \leqslant k - 1$. Note that the possibility that $k = M$ is included in this definition, even though in this case the switchover occurs too late to actually be implemented. There is one remaining possibility, that of no switchover at all (i.e., $n_i > \theta$ for $1 \leqslant i \leqslant M$), and this case is treated separately below.

The error probability $\tilde{P}_\epsilon$ is expanded in the form

$$1 - \tilde{P}_\epsilon = \gamma + \frac{1}{M} \sum_{j=1}^{M} \sum_{k=1}^{M} Pr(k|H_j)\, Pr(C|k,H_j) \qquad (9)$$

where

$$\gamma = \frac{1}{M} \sum_{j=1}^{M} Pr(\text{correct detection and no switchover}|H_j) \qquad (10)$$

and $Pr(C|k,H_j)$ is the conditional probability of correct detection, given H_j and switchover after slot k. The latter probability is easily calculated from decision rules 2 and 3.

$$Pr(C|k,H_j) = \begin{cases} 1 - e^{-E} & , k<j \\ 1 & , k=j \\ 0 & , k>j \end{cases} \qquad (11)$$

The conditional probability for the switchover location, given H_j, is evaluated as

$$Pr(k|H_j) = \begin{cases} [1 - P(\theta,E')]^{k-1}\, P(\theta,E') & , k<j \\ [1 - P(\theta,E')]^{k-1}\, P\ \theta,E_o') & , k=j \\ [1 - P(\theta,E')]^{k-2}\, [1 - P(\theta,E_o')]\, P(\theta,E') & , k>j \end{cases} \qquad (12)$$

where $P(N,x)$ denotes the cumulative Poisson distribution function with mean x.

$$P(N,x) = \sum_{n=0}^{N} p(n,x) \qquad (13)$$

where

$$p(n,x) = \frac{x^n}{n!}\, e^{-x} \qquad (14)$$

After insertion of (11) and (12) into (9) and considerable algebraic manipulation, the expression for $\tilde{P}_\epsilon$ collapses to

$$\tilde{P}_\epsilon = (1 - P(\theta,E_o') - e^{-E})\, \frac{1 - [1 - P(\theta,E')]^M}{MP(\theta,E')} + e^{-E} - \gamma \qquad (15)$$

The final item needed for $\tilde{P}_\epsilon$ is the evaluation of the term γ. Expand this probability by conditioning on n_j, then apply decision rule 1. For an exact calculation, a second conditioning variable is also needed. This variable, m, denotes the number of slot counts n_i, $i \neq j$, which exactly tie n_j for lowest count.

$$\gamma = \frac{1}{M} \sum_{j=1}^{M} \sum_{n_j=\theta+1}^{\infty} p(n_j,E_o') \sum_{m=0}^{M-1} \binom{M-1}{m} \frac{1}{m+1}$$

$$\times\ [p(n_j,E')]^m\ [1 - P(n_j,E')]^{M-1-m} \qquad (16)$$

The sum over m is evaluated in the same manner as the one in (1) and the sum over n_j is independent of j. Thus, (16) reduces to

$$\gamma = \frac{1}{M} \sum_{n=\theta+1}^{\infty} \frac{p(n,E_o')}{p(n,E')} \{[1 - P(n-1,E')]^M - [1 - P(n,E')]^M\} \qquad (17)$$

Note that, for the ideal conditional nuller, $\theta = 0, P(\theta,E_o') = 1$, $P(\theta,E') = e^{-E}$, and $p(n,E_o') = 0$ for $n \geqslant \theta + 1$. Thus, $\gamma = 0$ and (15) reduces to (5).

4. Numerical results for imperfect nulling due to phase error only. The error probability expression (15) is a function of the number of PPM slots M, the mean slot energy E, the two nulling mode slot energies E_o', E', and the nulling strategy threshold parameter θ. In this section, the dimensionality of this problem is reduced from five to three by (numerically) optimizing the nulling strategy threshold and by assuming that the nulling field is subject to phase inaccuracy only. In this case, the nulling mode slot energies are related to the signal slot energy by

$$E_o' = E\,|1 - e^{j\phi}|^2 = 2E\,(1 - \cos\phi)$$
$$E' = E \qquad (18)$$

where ϕ is the phase discrepancy (assumed constant) between the nulling field generated at the receiver and the negative of the received ON field.

Error probabilities were numerically evaluated for various values of phase error ϕ, with the nulling strategy threshold θ optimized. Some representative curves are shown in Fig. 6a, b, c, for $M = 2, 16, 256$, respectively. The cusps evident in these curves occur at transitions between discrete values of the optimum threshold.

Superimposed on the phase error performance curves in Fig. 6a, b, c are the corresponding curves from Fig. 4a, b, c for the direct detection receiver and the ideal conditionally nulling receiver. It is seen from Fig. 6a, b, c that small values of phase error are sufficient to cause much of the performance advantage of the ideal conditional nuller to disappear. For example, approximately half the performance advantage (measured logarithmically) of the ideal conditional nuller is erased by phase errors of $\phi = 4°, 2°, 1/2°$, for $M = 2, 16, 256$, respectively.

It is interesting to note that the error probability curves for the imperfect conditional nuller (with optimized threshold) are upper bounded by the error probability curves for the direct detection receiver. This is a general characteristic of the assumed nulling strategy that also holds for larger amounts of nulling inaccuracy than those assumed in Fig. 6a, b, c, even though for large nulling error the information extracted during nulling mode is very small. This property results from the fact that the error probability performance of the direct detection receiver for the quantum limited PPM detection problem is unaffected if data from the first slot is unavailable. An imperfect conditional nuller starting in nulling mode can always set its nulling strategy threshold high enough that direct detection mode is essentially guaranteed for the final $(M - 1)$ slots. Therefore, it can always at least match the performance of the direct detection receiver; with optimized threshold, there will be some improvement. Thus, it always pays, however slightly, for the imperfect conditional nuller to start out in nulling mode.

VI. Conclusions

In this article a general class of optical receivers called "conditionally nulling receivers" was postulated for PPM and related modulation schemes to take advantage of the inherent asymmetry in the information obtained by direct detection of ON and OFF signals. An ideal conditional nuller achieves essentially the same error probability as the optimum quantum measurement for quantum noise limited PPM detection. Analysis of the conditionally nulling receiver structure was extended to assess the effects of imperfections in the nulling process. It was found that the near-optimum performance of the conditionally nulling receiver degrades rapidly, albeit gracefully, to the performance of a direct detection receiver in the presence of small nulling field phase errors.

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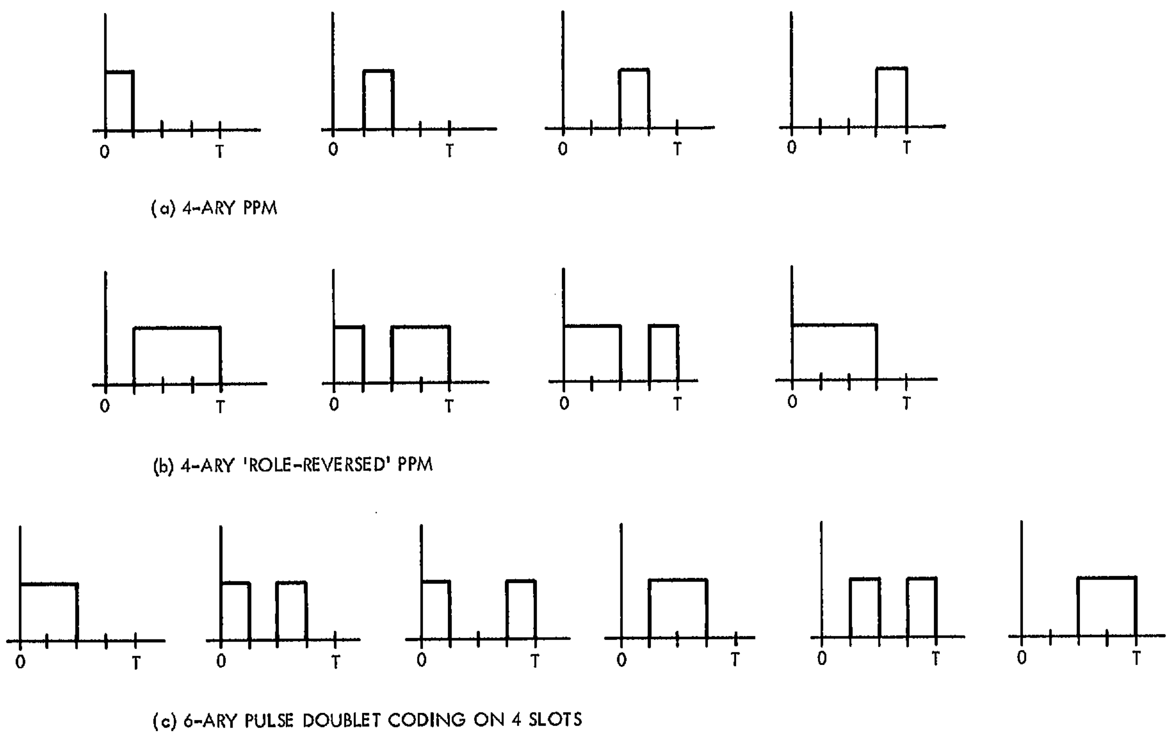

Fig. 1. Examples of generalized ON – OFF modulation schemes

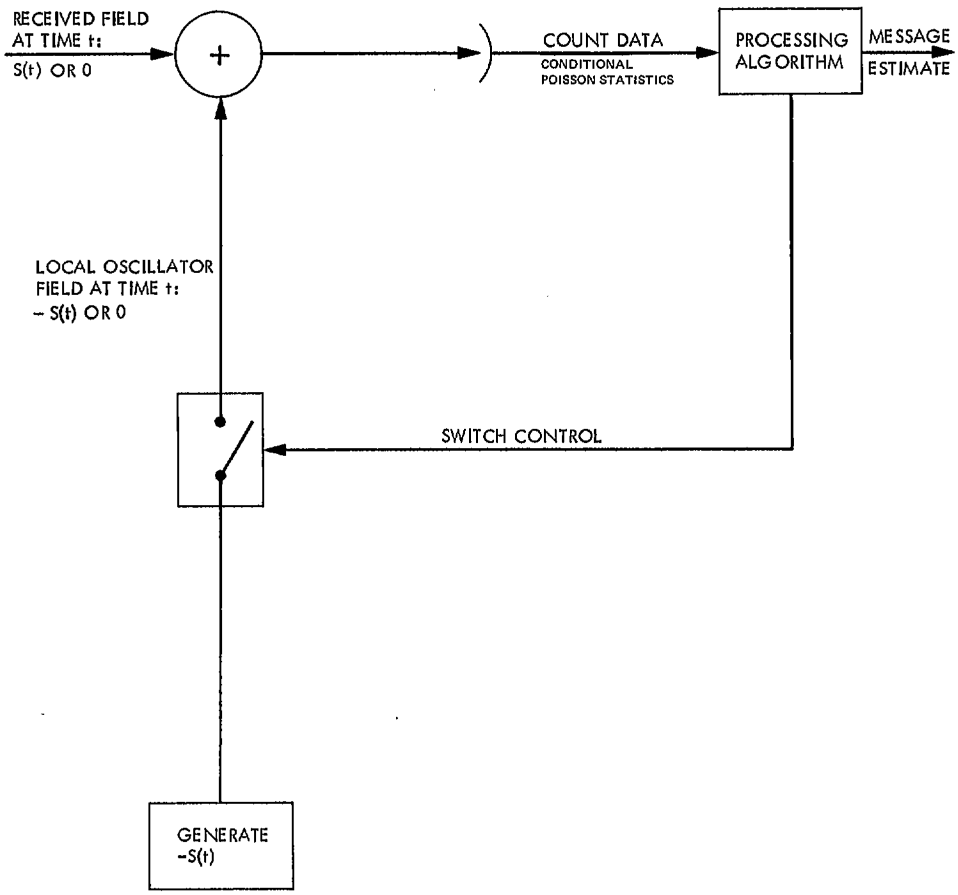

Fig. 2. Conditionally nulling receiver block diagram

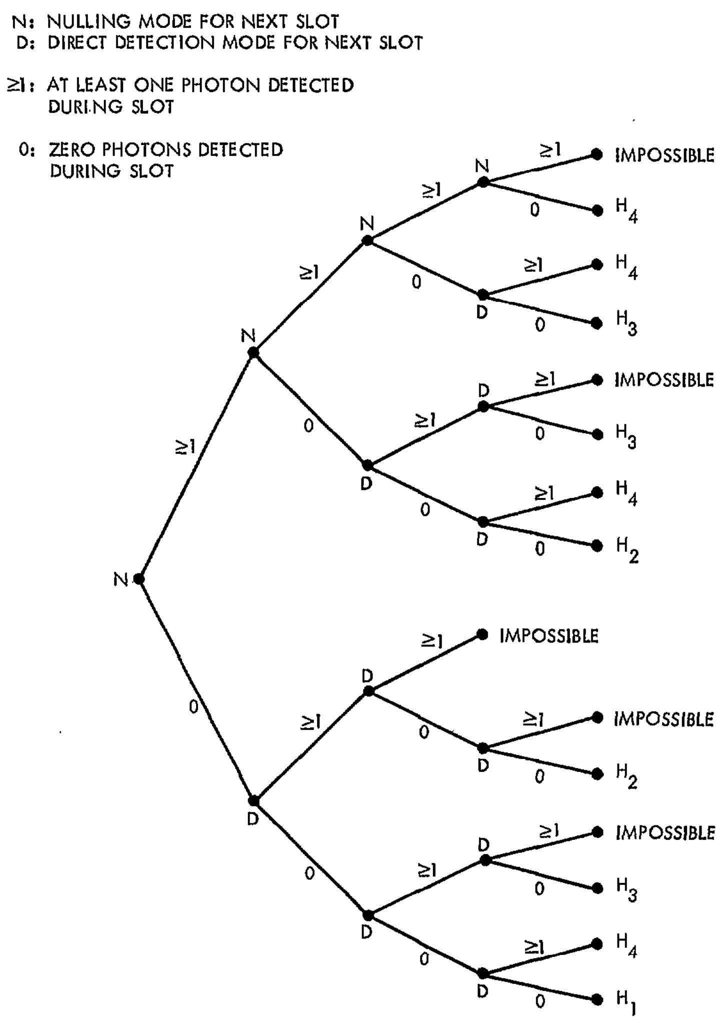

Fig. 3. Nulling strategy/decision algorithm of ideal conditional nuller for quantum limited PPM detection problem ($M = 4$)

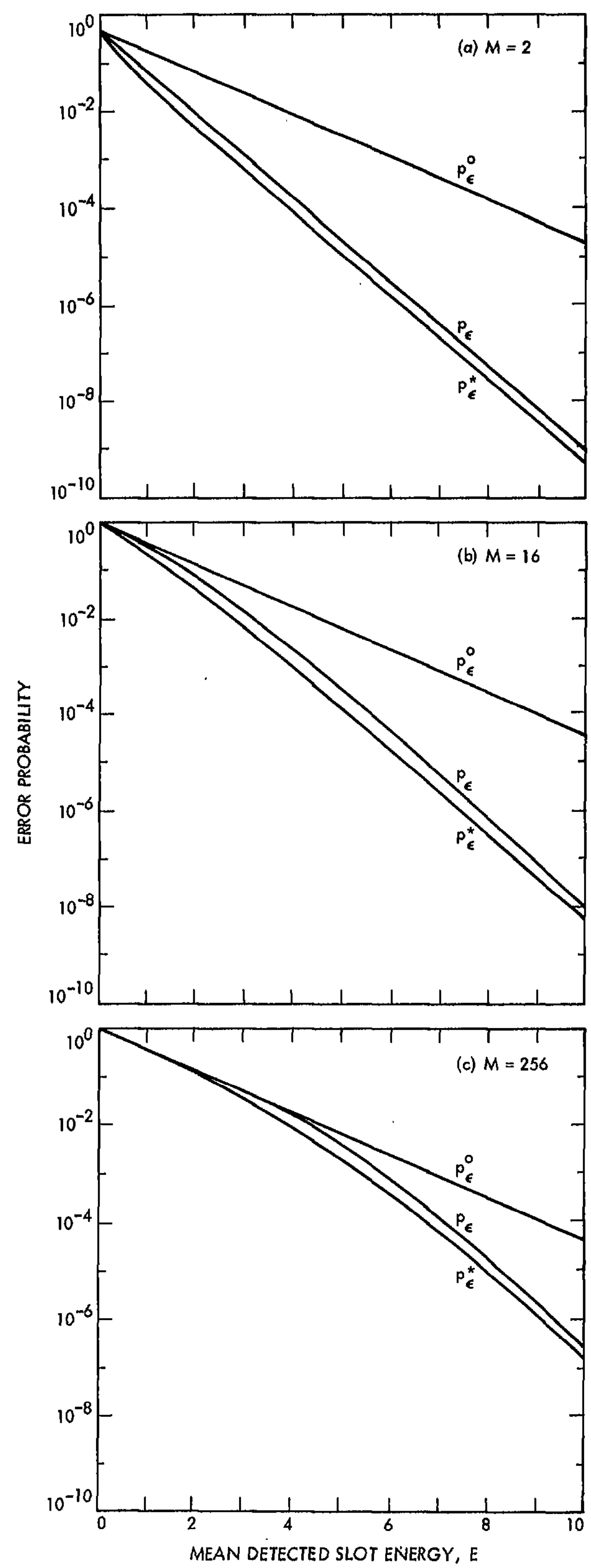

Fig. 4. Comparison of P_ϵ, P_ϵ^*, and P_ϵ^o

N: NULLING MODE FOR NEXT SLOT

D: DIRECT DETECTION MODE FOR NEXT SLOT

n_i: NUMBER OF COUNTS RECORDED DURING SLOT i

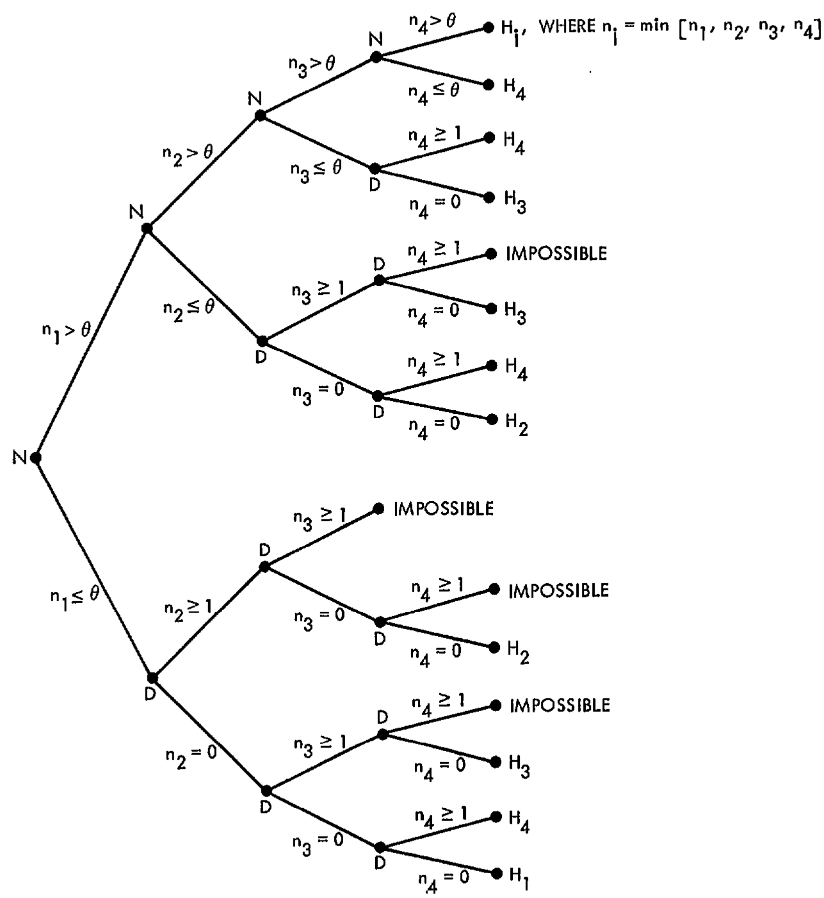

Fig. 5. Nulling strategy/decision algorithm of conditional nuller with imperfect nulling for quantum limited PPM detection problem ($M = 4$)

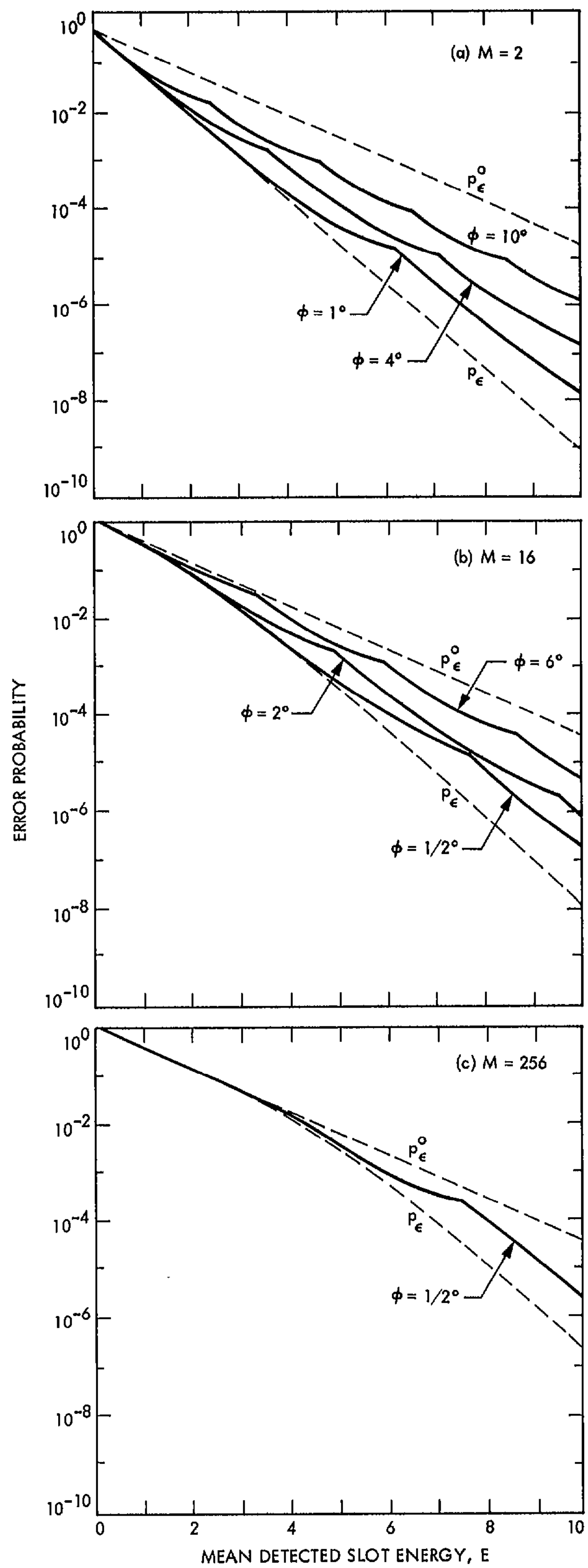

Fig. 6. Deterioration of the performance of the conditionally nulling receiver in the presence of nulling field phase error

Appendix

The following three observations lead directly to the optimum decision rule stated in the text:

(1) If photons in any positive number are detected during a signaling slot in which nulling is *not* used, the corresponding hypothesis has a posteriori probability unity and is automatically selected. If no photons are detected during the signaling slots in which nulling is not used, then all of the hypotheses corresponding to these slots have equal a posteriori probabilities.

(2) Among the hypotheses corresponding to signaling slots in which nulling *is* used, the one with maximum a posteriori probability is the one whose signaling slot yielded the fewest detected photons. In case of ties in count totals, the corresponding a posteriori probabilities are equal. In particular, since the threshold θ is constant, this implies that any slot which triggers a switchover from nulling to direct detecting is "king of the hill" relative to all preceding slots.

(3) If a switchover from nulling to direct detecting occurs after the kth slot and no photons are recorded in direct detection mode during the remaining $(M-k)$ slots, then the decision algorithm depends only on the number of detected photons in the kth slot, n_k. There is a threshold

$$\theta_o = \frac{E + E' - E'_o}{\ln(E'/E'_o)} \qquad \text{(A-1)}$$

such that H_k is selected if $n_k \leqslant \theta_o$, and a random choice among $H_{k+1}, \cdots, H_M$ is made if $n_k > \theta_o$. In particular, this implies that H_k is always chosen in this situation if the switchover threshold θ is no greater than the decision threshold θ_o.

Justification of the above statements is based on pairwise comparison of the a posteriori probabilities of hypotheses within two separate categories, corresponding to signaling slots in which nulling is or is not used. Note that, for the PPM signal set, a pairwise comparison between the a posteriori probabilities of two hypotheses depends only on data obtained during the two corresponding signaling slots. Thus, the most probable hypothesis within each category can be determined by successive application of binary PPM MAP decision principles. The rule for the direct detection category (statement 1) is the usual rule for direct detection of PPM signals under noise-free conditions. The rule for the nulling category (statement 2) is similar to the rule for PPM signals in background noise, except here the true signal slot is characterized by lowest energy instead of highest energy. The final test to complete the MAP decision is to compare the a posteriori probabilities of the leaders from each category. The likelihood ratio Λ_{ki} between hypotheses H_k and H_i, $k+1 \leqslant i \leqslant M$, is based entirely on the data from slots k and i. Under the conditions of statement 3, during slot k nulling is used and n_k counts are observed, while during slot i nulling is not used and 0 counts are observed. Thus

$$\Lambda_{ki} = \frac{Pr(n_k, 0 | H_k)}{Pr(n_k, 0 | H_i)} = \frac{\dfrac{(E'_o)^{n_k}}{n_k!} e^{-E'_o} \cdot 1}{\dfrac{(E')^{n_k}}{n_k!} e^{-E'} \cdot e^{-E}} \qquad \text{(A-2)}$$

This expression leads to the threshold test

$$\Lambda_{ki} \geqslant 1 \quad \text{if and only if} \quad n_k \leqslant \theta_o \qquad \text{(A-3)}$$

with θ_o defined in (A-1).