Limits to Arraying

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Arraying has been used, tested, analyzed, and generally poked about for the past decade or so. We have used one technique, or block diagram, for arraying for telemetry in the DSN, analyzed a few others, and seen still others used in radio astronomy applications. Looking to the future, there are a number of applications where our present technique will not work as well as we would like, and other techniques must be tried. This article develops (approximately) the feasibility limit to various possible array configurations. It is not intended as a "last-word" analysis, but rather as a guide to which areas would be the most fruitful for future analysis and development.

I. Introduction

Arraying has been used, tested, analyzed, and generally poked about for the past decade or so. We have in hand a large collection of answers to questions we have asked, but do not seem to have a generalized description of where arraying can be applied, and where it cannot. Can we expand an array to an arbitrarily large number of arbitrarily small antennas? The answer, of course, is "No," and the actual limits encountered depend upon the complexity of the signal processing/combining employed.

Arraying of several antennas to combine their effective receiving aperture requires that the narrowband spacecraft signal as it is received at each antenna be precisely aligned in delay and phase with the corresponding signal as received at each of the other antennas so that they can be coherently added. This alignment can be mechanized in a variety of ways. The mechanization used in the DSN today is illustrated in Fig. 1, where a phase locked loop is associated with each individual antenna to determine the phase of the received carrier component of the signal, and thus provide a coherent reference for translating the received carrier to DC (zero-phase). This reference is also used for coherently detecting the modulation, which consists of subcarrier times data. With the carrier phase thus eliminated from further concern, the baseband signal is filtered to a bandwidth consistent with reasonably precise representation of the modulated square wave subcarrier, sampled, delay-adjusted into agreement with the corresponding signals from the other antennas, and weighted by its respective strength prior to being added to the other signals. The combined result is then subjected to subcarrier and symbol detection.

The current DSN arraying is subject to a variety of losses, including that from noise in the carrier reference used to detect the signals at each of the antennas. Two methods of decreasing the noise in the carrier reference signals have been proposed and studied – these are carrier arraying and data-aided carrier tracking. In carrier arraying, the remnant carrier signal is isolated from the modulation sidebands, and the phase of the received carrier in each of the antennas is tracked relative to that in one reference antenna (or possibly to a combined carrier signal), and this relative phase measurement used to align the carrier signals from each antenna so that they can be combined directly. The combined carrier signal is stronger than the signal at each individual antenna, and is thus more easily tracked. Tracking of the relative phase between signals as handled at each of the antennas is also easier because it must follow differential doppler only, at a bandwidth which is on the order of 10^{-3} times the full topocentric doppler. Data-aided carrier tracking improves the ability to develop a carrier reference for each individual antenna by reconstructing a stronger carrier-equivalent signal from the data sidebands.

The Very Large Array (VLA) is a radio astronomy observatory near Socorro, New Mexico, which consists of twenty-seven 25-meter antennas (plus a spare). Its primary product is interferometric maps of various radio-bright objects, but it also happens to be able to produce a combined signal output by a mechanism which is crudely diagrammed in Fig. 2. This is "total-spectrum combining," where both the differential delay and differential phase of the individual antennas are compensated for in one combining step, rather than in two as in other mechanisms.

One additional mechanism which deserves consideration here incorporates the subcarrier demodulation and symbol stream detection into the processing which is done on individual antennas before the signals are transferred to a central point for combining. This adds somewhat to the complexity of the equipment on individual antennas, but makes the final combining process extremely simple – a matter of aligning, weighting, and adding the discrete-time symbol stream. Symbol detection on individual antennas also facilitates the measurement of the signal strength as received at each antenna, which is needed for weighting the combined inputs, and may also facilitate monitoring the health of the combining process.

A word of warning to the reader is in order here – the analysis which follows is at an architectural conceptual level, fraught with approximations and use of nonrealizeable square passband filters, etc. Exact implementation-specific analyses have been done elsewhere for some, but not all, of the configurations discussed. Choice of filters, etc., can easily make the exact results differ by a dB (or a little more) from the approximate results to be presented here.

II. Arraying Mechanisms and Limiting Conditions

The ensemble of conditions pertinent to arraying is large, and hence some arbitrariness will limit the set of conditions first considered to symmetric arrays only, such as the VLA. Nonsymmetric arrays will be briefly discussed later. The performance boundaries are a function of two parameters – the data rate and the individual antenna size relative to that of the total array. We assume that the overall signal into the array is just adequate to support the desired data rate. Thus, as the data rate decreases, the signal power as received by either the total array or the individual antennas decreases, and the task of synchronizing the signals from them becomes increasingly difficult. Similarly, as the size of the individual antennas decreases, the signal power in each of them decreases correspondingly, and the task of arraying them likewise becomes more difficult. We will *assume*, for a concrete example, that adequate synchronizing of the signals being arrayed requires a loop margin of +15 dB in the phase-lock/delay-lock circuitry. This choice of +15 dB for a nominal threshold is somewhat arbitrary, and could be pushed upward (or downward) by a few dB if we asked for an optimum balance between losses from synchronization errors and other factors. In Fig. 3, configurations of data rate times antenna size which are above or to the right of the indicated threshold lines will provide synchronization loop margin of greater than +15 dB for the indicated mechanization, and are thus acceptable to that mechanization. We recall that arraying requires alignment of both delay and phase, so that mechanisms which treat only delay (cases E or F) or only phase (cases A, B, D, or G) must be combined with a complementary mechanism to construct a complete system. The various mechanizations will be discussed in the following.

Case A

This is the carrier sync step of the standard DSN configuration. We use separate phase locked loops at each antenna of the array to perform carrier tracking and demodulation prior to combining at baseband. Assume that the spacecraft utilizes the approximate optimum modulation index as found by Greenhall (Ref. 1). Assume also that the topocentric doppler requires use of a 30 Hz tracking loop. For each data rate, the smallest acceptable single arrayable antenna is one which will just provide a +15 dB margin in the 30 Hz phase locked loop bandwidth.

Curve A also grants some insight into the carrier loop margin requirements: at data rates above about 3×10^3 bps, the whole array (single antenna) carrier margin is above the +15 dB margin level, while at lower data rates it is below, implying that the criterion of +15 dB loop margin for all synchronizing circuitry may be too stringent at low data rates, and too easy at high rates.

Case B

This is again the mechanism for carrier sync in the standard DSN arraying configuration, but in this case the spacecraft has accommodated the fact of arraying by lowering its modulation index to provide a stronger carrier signal to be phase-locked at the individual antennas. The penalty, of course, is lowered data power. Suppose that the total array consists of several large antennas plus one small one. The smallest antenna's carrier margin will dominate the selection of mod index if we require that its margin be +15 dB or more. That small antenna would be *too small* to consider under the +15 dB carrier margin constraint if lowering the mod index to achieve that margin level also lowered the aggregate data power into the overall array by more than would be lost by simply abandoning the small antenna. This size limit to the smallest acceptable antenna is shown as Case B on Fig. 3.

Case C

Total spectrum combining. The combiner must track both differential carrier phase and envelope delay. Of these, the carrier phase is the more critical, and is assumed to be tracked in a bandwidth of 0.03 Hz. Use of this bandwidth for a 20 km baseline is approximately consistent with use of a 10 Hz bandwidth for the earth-rotation component of the full topocentric doppler. Both could potentially be reduced by a small amount. The envelope delay varies significantly more slowly, could be tracked with a loop of 0.003 Hz or smaller bandwidth, and thus would have a much higher loop SNR than would the phase tracking loop for the same conditions.

Assume that the array consists of M equal antennas. The input to the phase tracking loop is the time series of cross products of signal-plus-noise from each two of these M antennas. Let the data rate of the signal be B bits per second. With convolutional coding, like that of the Voyager, the channel rate is expanded to $2B$ symbols per second. If this symbol stream is modulated onto a subcarrier with N subcarrier cycles per symbol, and the resultant modulated onto a carrier, the spacecraft signal occupies a bandwidth of $2 \times (N + 1) \times 2B$ when only the first harmonics of the subcarrier are included, and several times that if higher harmonics are needed. For the purposes of estimating differential delay, we assert that the right signal bandwidth to use is one which just encloses the first harmonics. This filtering "gives away" about 1 dB of signal power in the subcarrier harmonics, plus another 1/2 dB in the upper-sideband data spectrum harmonics, while eliminating a much larger proportion of noise. The estimation process is a close analog of the Costas or squaring loop, and the performance is usually limited by noise times noise terms. Let the energy signal-to-noise ratios of the detected bit stream be R (actually 2.6 dB, or so) at threshold. Then the signal-to-noise ratio of the *bandpass* signal (first harmonic zone) from each of the M antennas is approximately:

$$\frac{R \times B}{M \times 2\,(N+1) \times 2B} \qquad \text{(less 1.5 dB)}$$

In forming differential phase/delay estimates, this bandpass signal is sampled, cross-products taken, and the resultant signal filtered to approximately a 0.03 Hz bandwidth. Assuming that the bandpass signal is dominated by noise, we find that the loop SNR of the phase tracking process is:

$$Loop\ SNR = \left(\frac{R}{M} \times \frac{B}{4\,(N+1)\,B}\right)^2 \frac{4\,(N+1) \times B}{0.03} \qquad \text{(less 3 dB)}$$

$$= \frac{R^2}{M^2} \times \frac{B}{4\,(N+1) \times 0.03} \qquad \text{(less 3 dB)}$$

which we are insisting should be +15 dB. For $R = 1.8$ (2.6 dB), the phase tracking loop SNR is above +15 dB whenever

$$B \geqslant 2.2 \times (N+1) \times M^2$$

The case C line on Fig. 3 corresponds to $N = 10$ (subcarrier cycles per symbol), which is approximately the expected Voyager configuration at its Uranus or Neptune encounters.

Case D

Carrier combining. As in Case A, assume that the modulation index corresponds to that identified by Greenhall for the optimum for the aggregate array. As in Case C, the differential phase tracking is performed via filtered cross products of the signal as received at two antennas. Here, however, the predetection filter needs only to be wide enough to encompass the carrier, e.g., about 300 Hz. This 300 Hz bandwidth should be workable, but would probably require use of an ephemeris-tuned local oscillator. The tracking loop filter is again 0.03 Hz, providing a +40 dB increase in SNR between the raw cross products and the resultant phase-estimate. As we are demanding a +15 dB tracking loop SNR, we can accept a cross-product SNR of -25 dB, or a filtered signal SNR of -12.5 dB prior to the cross-product operation.

The available SNR for the 300 Hz prefilter for the whole array has been worked backwards assuming that the bit SNR is 2.6 dB, and using the approximate optimum mod index as shown in Table 1. The smallest acceptable single antenna size for carrier combining is that which will provide at least a -12.5 dB SNR in 300 Hz bandwidth for the individual antenna.

As an example, a data rate of 3×10^4 bps gives a carrier SNR in the 300 Hz predetection filter of +10.5 dB in the total array. An individual antenna with collecting aperture 23 dB below that of the total array would just provide a -12.5 dB SNR individually in that bandwidth, and thus be the smallest antenna acceptable. Other values are exhibited in the case D line of Fig. 3. We see that carrier combining can handle very large arrays indeed (about 500 equal-sized antennas in the example case!).

Case E

Baseband combining. This is the second step of the current DSN arraying operations. This case is very similar to the total spectrum combining, Case C, but differs in two important considerations. First of all, the carrier phase information has been removed from the signal and does not need to be tracked – the remaining differential envelope delay is more slowly varying, and can be tracked in a narrower bandwidth, say 0.003 Hz. Secondly, the signal has been coherently folded about the carrier so that its bandwidth occupancy is lessened by a factor of two. Following the notation of Case C, the SNR of the pre-cross-product signal is

$$\frac{R}{M} \times \frac{B}{(N+1) \times 2B} \qquad \text{(less 1.5 dB)}$$

for which the loop SNR becomes

$$\frac{R^2}{M^2} \times \frac{B}{2\,(N+1) \times 0.003} \qquad \text{(less 3 dB)}$$

which should be +15 dB for satisfactory operation. The delay tracking loop is above +15 dB whenever

$$B \geqslant 0.11 \times (N+1) \times M^2$$

The case E line on Fig. 3 corresponds to $N = 10$.

The foregoing analysis assumes that the predetection filters indicated in Fig. 1 have been set to the near-ideal bandwidth for the combining process. If this filter bandwidth is made wider, or the filter eliminated entirely, the Case E line will move to the right in Fig. 3. For example, should the predetection filter be eliminated from the block diagram, and the baseband bandwidth left at approximately 3 MHz to cover the the first 9 harmonics of the Voyager subcarrier, the result would be to reduce the pre-cross-product SNR in the control path by a factor of about 8 (9 dB), and to move the Case E line on Fig. 3 *to the right* (larger antennas) by 9 dB.

Case F

Subcarrier demodulation and symbol detection before combining. The limiting conditions will be those imposed by the need for phase lock to the subcarrier. The loop bandwidth must accommodate both the topocentric doppler (scaled to subcarrier frequency) and modest oscillator variations, as well as limits to the human observers' patience in awaiting loop lock. If a 30 Hz loop can handle the X-band doppler, then a subcarrier loop of approximately 0.03 Hz should handle the doppler on the 360 kHz subcarrier of the Voyager. That value is used for both subcarrier tracking loop bandwidth and symbol synchronizer loop bandwidth in the following discussion.

A "squaring loop" provides the example configuration for a subcarrier demodulator. After coherent detection by the local carrier reference, the signal, consisting of data-modulated subcarrier plus noise, is bandpass-filtered about the fundamental of the subcarrier to remove excess noise and then squared to produce a pure unmodulated tone at twice the subcarrier frequency. This can be tracked to develop a coherent subcarrier reference which would be used to demodulate the data stream. The best bandpass filter will just enclose the fundamental zone of the data spectrum, thus giving away about 1 dB of signal power in the subcarrier harmonics, and another 1 dB in the harmonics of the data spectrum. Following the notation of Case C, the bandwidth occupancy of the signal is $4B$, and the SNR of the pre-squaring signal is

$$\frac{R}{M} \times \frac{B}{4B} \qquad \text{(less 2 dB)}$$

for which the SNR of a 0.03 Hz loop becomes

$$\frac{R^2}{M^2} \times \frac{B}{4 \times 0.03} \qquad \text{(less 4 dB)}$$

which should be above +15 dB for satisfactory operation. For $R = 2.6$ dB, the subcarrier tracking loop SNR is above +15 dB whenever

$$B \geqslant 2.9 \times M^2$$

This is shown as the Case F line in Fig. 3.

Case G

Data-aided carrier tracking at the individual antennas. In the best-of-all worlds, the detected data stream from the combined array is fed back to the individual antennas to allow the data modulation in the sidebands to be eliminated and the full power of the signal made available to be tracked as the carrier reference. There are no nonlinearities at low SNR to generate quadratic dependencies, so the loop margin in a 30 Hz loop becomes

$$\frac{R \times B}{M \times 30}$$

which, for $R = 2.6$ dB, is above +15 dB whenever

$$B \geqslant 500\,M$$

This is shown as the case G line on Fig. 3. This mechanism is more capable than tracking the residual carrier on individual antennas, but not as capable as carrier combining. There would also be some difficulty in initial acquisition of references needed to allow the decision feedback.

III. Two Examples

Two examples from potential arraying events should help to illustrate these results. The Voyager spacecraft at its Neptune encounter in 1989 could communicate at a data rate of 14.4 kbps if it is supported by an array consisting of all of Goldstone plus a large part of the VLA with masers for low-noise front-end amplifiers. The total array receiving aperture would be about five times that of the current DSN 64-meter antenna, while the 34-meter antenna at DSS 12 has a receiving aperture which is one-quarter that of the 64-meter: this antenna is thus 13 dB below the total array. The individual VLA antennas are expected to represent an aperture which is 18% of that of the 64-meter and are thus 14 dB below the size of the total array. Referring to Fig. 3, we could not reasonably expect to achieve 15 dB phase lock on the individual carrier signal at either of these antennas. We could, however, successfully involve either of them with total spectrum combining, or carrier combining coupled with baseband combining, and achieve this synchronization at the +15 dB loop SNR.

The second example is somewhat less encouraging. The Pioneer 10 and 11 are on their way out of the Solar System, and will be straining our ability to communicate with them by the end of this decade. At their extreme range, these spacecraft will be operating at S-band at 16 bps with an E_b/N_0 of 3 dB, and a fixed modulation index of 42°. These parameters represent a P_c/N_0 of 16 dB into the whole array, so that if we ask for a carrier margin on the whole signal (after arraying of the carrier) of +15 dB, we require the use of a 1 Hz phase-locked loop. At S-band, with no planetary bodies nearby to speed up the doppler, this narrow bandwidth could be workable. If we array carriers with a 300 Hz pre-detection filter and a differential loop bandwidth of 0.01 Hz, we get a +45 dB boost in SNR in going from the raw cross-products of the differential phase measurement to the filtered loop estimate of it. Asking for a +15 dB SNR in this differential phase tracking loop means that an SNR of -30 dB in the raw cross products is acceptable, and hence that an SNR of -15 dB in the 300 Hz pre-cross-product bandwidth is also acceptable at the individual antennas. At threshold conditions, the carrier SNR in 300 Hz for the whole array is -10 dB, which implies that the individual antennas should be no smaller than -5 dB, or one-third the total array. We could flex this limit moderately by narrowing the predetection bandwidth, or by accepting an arraying loop SNR of less than +15 dB, but even with these steps, it seems unlikely that an array like the VLA could be readily used to support the Pioneers at their extreme range.

IV. Discussion – Asymmetric Arrays

Although the preceding paragraphs are derived for a symmetric array where all antennas, including the reference element, are the same size, it is relatively routine to state similar results for asymmetric conditions: for Case A, B, and F, the antennas are treated individually, and the *smallest* one must be capable of supporting phase tracking at the specified SNR. For Cases C, D, and E, the phase and delay tracking are all performed using cross products of signals from separate antennas. If one antenna is larger (the reference antenna) but its bandpass is still noise dominated, then the SNR of the cross products is the product of the individual SNR's. For this case, we can interpret the behavior of the asymmetric array as if it were a symmetric array with the size of each apparent antenna being the square root of the product of the sizes of the actual reference and auxiliary antennas. In the limit, if we try to extract the ultimate performance from an array, we could attempt to use the aggregate array as the reference for each of the individual elements (Cases C, D, and E). Assuming that the acquisition problems can be solved, then the apparent performance boundaries for this case correspond merely to an asymmetric array with its reference antenna equal to the whole array.

Acquisition of the initial phase and delay estimates would not be easy, and would impose its own limit to operability of the array. One way to acquire the initial phase/delay estimates is to utilize one of the array elements as the reference antenna, achieving a +3 to +5 dB SNR, and then shift the reference to the aggregate signal to achieve low-loss signal combining and detection. This is a complexity which is probably best avoided, but would allow operating with smaller element antennas (or larger arrays). By accepting a phase acquisition SNR of only +5 dB, we can accept cross-product SNR's which are 10 dB less than those required for the Fig. 3 lines, and hence can accept single antenna SNR's or antenna sizes which are 5 dB smaller than those of Fig. 3.

V. Summary and Conclusions

There are limits to our ability to utilize ever larger arrays of modest-sized antennas to acquire data from distant spacecraft, but, as indicated on Fig. 3, they do not pose serious limits to data acquisition from Voyager or similar-capability spacecraft using the VLA. Trying to follow the Pioneers at a data rate of 16 bps out to the heliopause would be better done with an array of only a few larger antennas.

Acknowledgment

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Reference

1. Greenhall, C. A., "A Compact Presentation of DSN Array Telemetry Performance," *TDA Progress Report 42-71*, Jet Propulsion Laboratory, Pasadena, Calif., pp. 137-142, Nov. 15, 1982.

Table 1. Approximate optimum mod index for various data

Bit rate, bps	Mod index, deg	Carrier SNR/300 Hz, dB
1.2×10^5	80	+13.7
3×10^4	76	+10.9
10^4	71	+8.6
3×10^3	65	+6.0
10^3	59	+2.9
300	40	+0.0

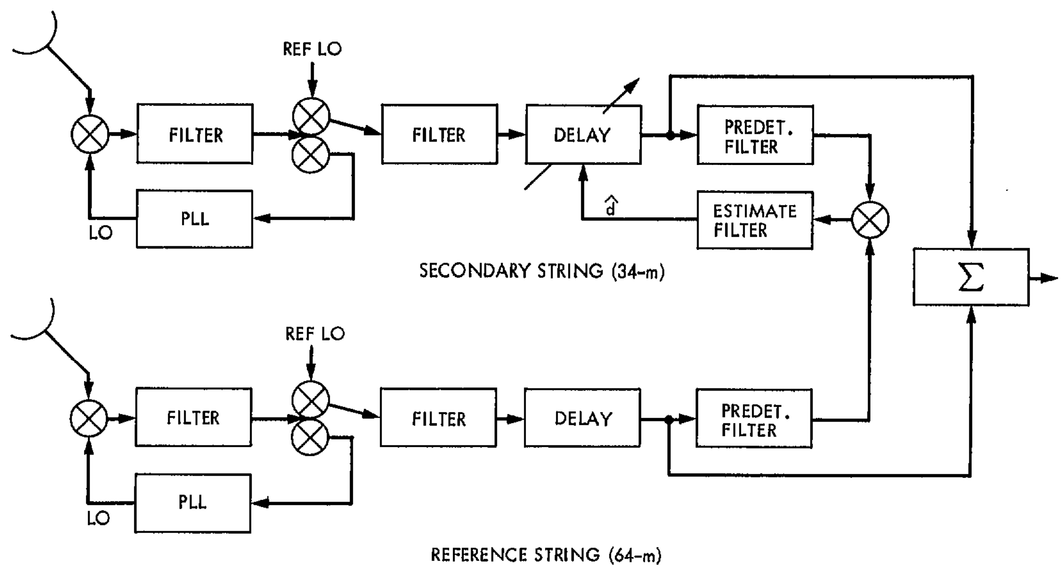

Fig. 1. DSN Combining process

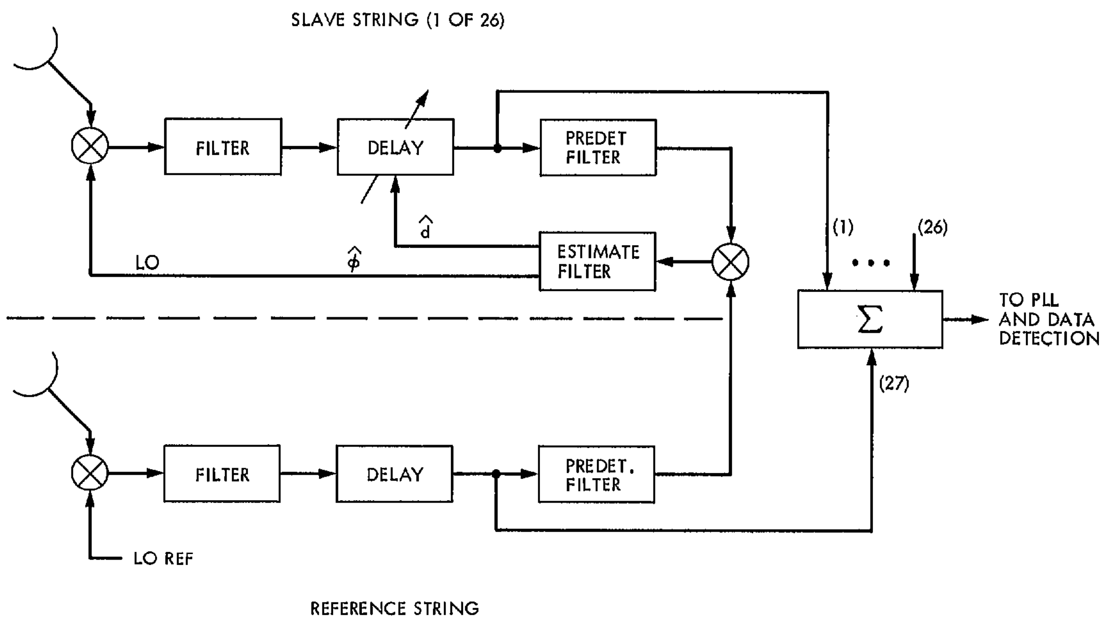

Fig. 2. VLA Combining process

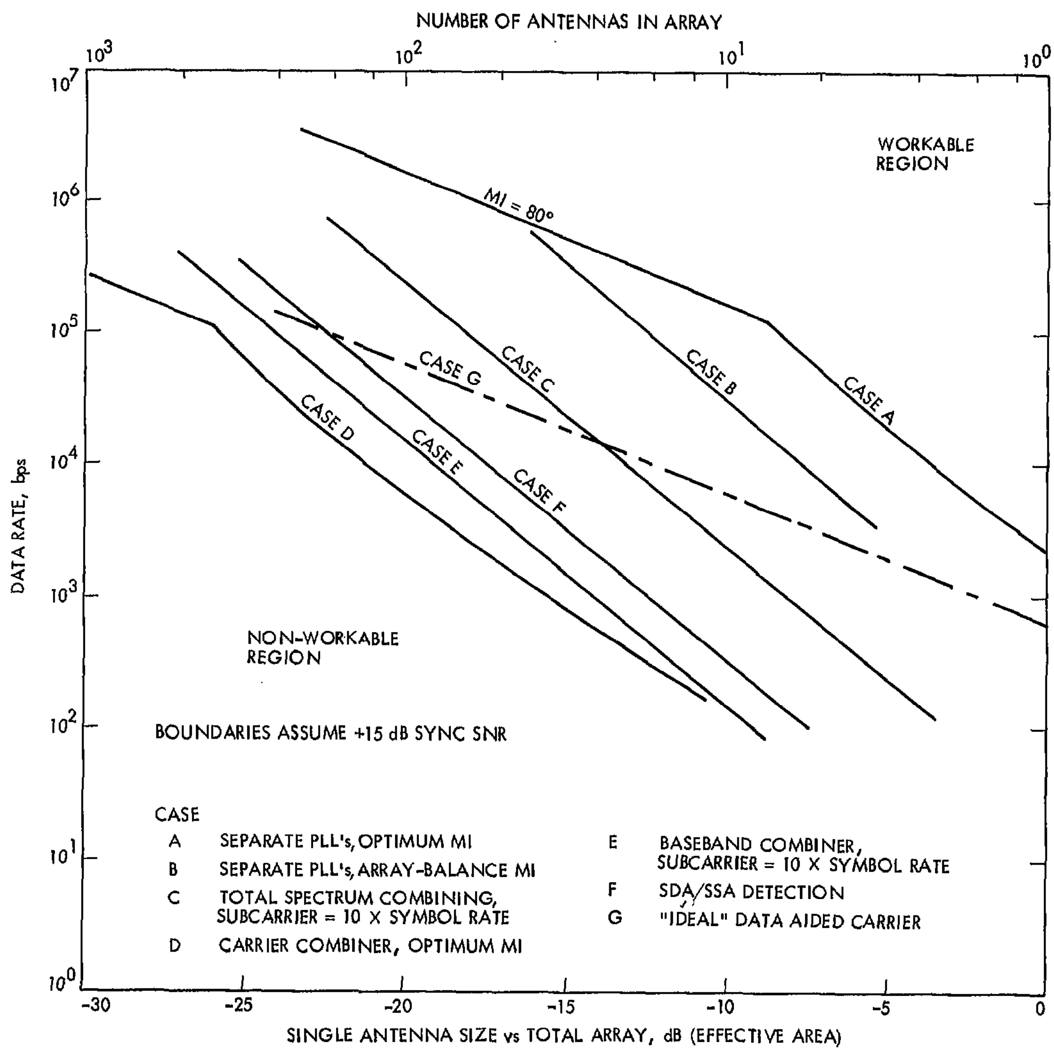

Fig. 3. Arraying limits