Symbol Stream Combining in a Convolutionally Coded System

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Symbol stream combining has been proposed as a method for arraying signals received at different antennas. If convolutional coding and Viterbi decoding are used, it is shown that a Viterbi decoder based on the proposed weighted sum of symbol streams yields maximum likelihood decisions.

I. Introduction

The use of symbol stream combining has been proposed for decoding data from deep space missions when the received signal is very weak. Under this scheme, a signal is received at two or more antennas, and each antenna's output is processed separately through carrier and subcarrier demodulation and symbol synchronization. Then the two digital copies (of the same signal plus different noise) are combined before decoding.

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In Ref. 1, this system was studied and found to be superior to baseband combining. The analysis of Ref. 1 determines the optimal weights for adding the outputs of the two symbol synchronizer assemblies (SSAs) before sending the signals to the Viterbi decoder.

The fact that symbol stream combining is superior to baseband combining follows from timing uncertainties (Ref. 1). But why symbol stream combining? Could a maximum-likelihood decision be made on the pair of symbol streams to yield better performance than using the Viterbi (maximum-likelihood) decoder on a weighted sum of the symbol streams? This article shows that symbol stream combining followed by

Viterbi decoding is in fact maximum-likelihood decoding for a pair of symbol streams.

II. Symbol Stream Combining

The digital coding system for a deep space mission can be modeled as shown in Fig. 1 where the output of the convolutional encoder is a stream of bipolar symbols $x_k \epsilon \{-V, V\}$, and the output of the SSA is a stream of real numbers x_k,

$$\hat{x}_k = Ax_k + n_k$$

where $n_1, n_2, \ldots$ are the values of independent, Gaussian, mean zero, variance $N_0/2$ random variables, and A is a positive constant. The Viterbi decoder then makes a maximum-likelihood decision about the symbols $x_1, x_2, \ldots$, and therefore about the data bits, producing the decoded data bits.

Under symbol stream combining, two different symbol synchronizers produce two different output streams:

$$\hat{x}_k = A\, x_k + n_k \tag{1}$$

and

$$\hat{y}_k = \bar{A}\, x_k + \bar{n}_k \tag{2}$$

where $n_1, n_2, \ldots$ and $\bar{n}_1, \bar{n}_2, \ldots$ are samples of two independent stochastic processes, each independent, mean zero, identically distributed, and Gaussian, with variances $N_0/2$ and $\bar{N}_0/2$ respectively, and where A and $\bar{A}$ are positive constants.

As described in Ref. 1, the symbol stream combiner will fit into the system between the two SSAs and one Viterbi decoder, combining the streams $\hat{x}_1, \hat{x}_2, \ldots$ and $\hat{y}_1, \hat{y}_2, \ldots$ to yield one stream

$$z_k = \alpha \hat{x}_k + \beta \hat{y}_k \tag{3}$$

which is sent to the Viterbi decoder, where α and β are "weighting constants" chosen by the symbol stream combiner to maximize the signal-to-noise ratio $(\alpha A + \beta \bar{A})^2 x_k^2/(\alpha^2 N_0 + \beta^2 \bar{N}_0)$ of the combined stream. The ratio of α to β which maximizes signal-to-noise ratio of the combined stream depends on the signal power and noise power of the two streams,

$$\frac{\alpha}{\beta} = \frac{A/N_0}{\bar{A}/\bar{N}_0}$$

Ways to measure the signal-to-noise ratios of the streams, and to determine the weighting constants, are discussed in Refs. 1, 2, and 3.

In this article, we consider whether better use can be made of the two symbol streams than combining them linearly as in Eq. (3). We find that in fact a Viterbi decoder acting on the stream (Eq. [3]) will perform identically to a maximum-likelihood decoder acting on the two streams (Eqs. [1] and [2]) (assuming perfect knowledge of signal and noise power in both cases). We are ignoring quantization, which of course causes some degradation; we assume that the SSAs produce enough quantization levels that this is not a problem.

III. Maximum Likelihood Decoding

After the two symbol synchronizer assemblies, the information available for maximum-likelihood decoding is

$$\hat{x}_k = A\, x_k + n_k$$

and

$$\hat{y}_k = \bar{A}\, x_k + \bar{n}_k$$

In fact, assuming that x_k was sent, this pair $(\hat{x}_k, \hat{y}_k)$ appears with probability density

$$\frac{1}{2\pi \sqrt{N_0\, \bar{N}_0}} \exp\, [-(\hat{x}_k - A\, x_k)^2/2N_0]$$

$$\times \exp\, [-(\hat{y}_k - \bar{A}\, x_k)^2/2\bar{N}_0]$$

Given that the stream $\{x_1, x_2, \ldots\}$ was sent, the pair of streams $\{\hat{x}_1, \hat{x}_2, \ldots\}$ and $\{\hat{y}_1, \hat{y}_2, \ldots\}$ will be received with probability density

$$K \exp \left\{ - \sum_k \left[(\hat{x}_k - A\, x_k)^2/2N_0 + (\hat{y}_k - \bar{A}\, x_k)^2/2\bar{N}_0 \right] \right\} \tag{4}$$

where K is a constant.

Thus, a maximum-likelihood decoder acting on the pair of streams $\{\hat{x}_1, \hat{x}_2, \ldots\}$ and $\{\hat{y}_1, \hat{y}_2, \ldots\}$ will find the stream $\{x_1, x_2, \ldots\}$ which maximizes the probability density in Eq. (4) or, equivalently, which maximizes

$$\sum_k \left(\frac{A}{N_0}\, \hat{x}_k x_k + \frac{\bar{A}}{\bar{N}_0}\, \hat{y}_k x_k \right)$$

Now, if instead, we have a maximum likelihood decoder acting on the stream

$$z_k = \frac{A}{N_0}\hat{x}_k + \frac{\bar{A}}{\bar{N}_0}\hat{y}_k$$

it will maximize

$$K \exp\left\{\frac{-\sum_k \left(z_k - \left(\frac{A^2}{N_0} + \frac{\bar{A}^2}{\bar{N}_0}\right) x_k\right)^2}{2\left(\frac{A^2}{N_0} + \frac{\bar{A}^2}{\bar{N}_0}\right)}\right\}$$

which is equivalent to maximizing

$$\sum_k z_k x_k = \sum_k \left(\frac{A}{N_0}\hat{x}_k + \frac{\bar{A}}{\bar{N}_0}\hat{y}_k\right) x_k$$

Thus the two schemes, maximum-likelihood decoding on the pair of streams and Viterbi decoding on the combined stream, will produce identical results.

IV. Determining Signal and Noise Power

In the Deep Space Network, signal and noise power are estimated by the symbol synchronizer assembly, so this information would be available to a symbol stream combiner. If another estimate were desirable, a feed-back system could involve estimates based either on the Viterbi decoder metrics or on channel symbol errors.

References

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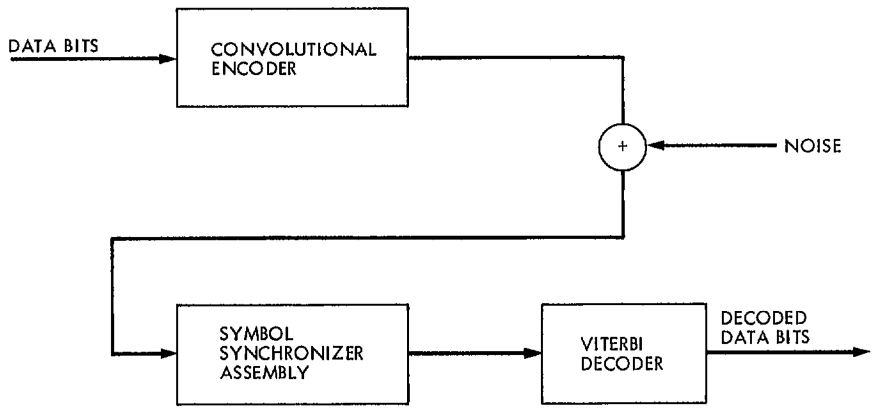

Fig. 1. Digital coding system for a deep space mission