Seismic Analysis of the Large 70-Meter Antenna, Part I: Earthquake Response Spectra Versus Full Transient Analysis

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As a check on structure safety aspects, two approaches in seismic analysis for the large 70-m antennas are presented. The first approach, commonly used by civil engineers, utilizes known recommended design response spectra. The second approach, which is the full transient analysis, is versatile and applicable not only to earthquake loading but also to other dynamic forcing functions. The results obtained at the fundamental structural frequency show that the two approaches are in good agreement with each other and both approaches show a safe design. The results also confirm past 64-m antenna seismic studies done by the Caltech Seismology Staff.

I. Introduction

Since the trial structural-mechanical designs and the upgrading and rehabilitation effort of the present 64-m antenna to a 70-m aperture have been completed (Ref. 1), it is essential to check the candidate design for severe environmental loading conditions to satisfy safety requirements. The purpose of this two part study is to investigate the earthquake response of the large antenna structure, to present the modern methodology and to check the structure safety aspects. The emphasis in the first part is placed on the mathematical description of the method of analysis with detailed design safety features to be presented in the second part.

During the early design phase of the 64-m antenna network, the safety against earthquake loading was one of the important tasks that was taken into consideration. The conclusion of that early seismic analysis, conducted by Prof. G. W. Housner of the California Institute of Technology (Ref. 2), was that the structural design should withstand the horizontal acceleration of about $0.25G$ where G is the acceleration of gravity.

Today, with a wider access to modern digital computers and new numerical techniques in structural mechanics, more subtle dynamic analysis of structures is available. This first part of the study will compare the results of two approaches: the response spectra approach versus the full transient analysis. Cross verification of the results should add to a better confidence in understanding the behavior of the complex antenna structure in response to random earthquake excitation.

II. Methodology Description

The analysis of earthquake-excited structures needs to take into account the nonperiodic form of the external forcing term. Such a problem requires the use of special analytical procedures, as presented in Refs. 3, 4, and 5, which are classified into two broad possibilities:

(1) The frequency response procedure

(2) The modal analysis procedure

Procedure (1) simply determines the natural undamped frequencies of the structure which the designer uses in comparison against the frequencies of the external forces in an effort to avoid resonance. Procedure (2) is more important and widely used in practice and will be described in this study in detail. The differential equation of the structure under dynamic loading is written in the form:

$$[M]\,\ddot{y} + [C]\,\dot{y} + [K]\,y + \{f\} = 0 \tag{1}$$

where y is generalized displacement, $[M]$, $[C]$, $[K]$ are mass, damping, and stiffness matrices, respectively, and $\{f\}$ is an external force vector where $f(t)$ is an arbitrary function of time. Equation (1) is the basic equation describing the dynamic behavior of the structure.

The first step in solving Eq. (1) is the determination of the free undamped response where no damping or forcing terms exist. The special dynamic form of Eq. (1) is reduced to

$$[M]\,\ddot{y} + [K]\,y = 0 \tag{2}$$

The general solution of free responses represented by Eq. (2) is written in the form:

$$y_o = \sum_{i=1}^{n} y_{oi} \exp(j\omega_i t) \tag{3}$$

where ω_i are the eigenvalues (natural frequencies) of the system, y_{oi} are the eigenvectors, n is the degree of freedom and $j = \sqrt{-1}$.

For forced responses represented by Eq. (1) the solution can be written also in a linear combination of modes as

$$y = \sum_{i=1}^{n} y_{oi}\, z_i(t) = [y_o]\ \{z\} \tag{4}$$

where the matrix $[y_o]$ lists all the modes (assumed to be normalized) and $z_i(t)$ are scalar mode participation factors, $i = 1, \ldots, n$, $z_i(t)$ is a function of time and represents the proportion of motion in each mode.

An important advantage of the linear mode superposition in Eq. (4) is that an approximate solution can be obtained by truncating modes and including only part of the total modal contributions. In general, lower modes make the principal contribution to the dynamic response, and good approximation is usually obtained by considering only the first few modes in the analysis.

If Eq. (4) is substituted into Eq. (1) and the result is premultiplied by $[y_o]^T$, then

$$[y_o]^T\,[M]\,[y_o]\,\{\ddot{z}\} + [y_o]^T\,[C]\,[y_o]\,\{\dot{z}\} + [y_o]^T\,[K]\,[y_o]\,\{z\} + [y_o]^T\,\{f\} = 0 \tag{5}$$

By the orthogonality property,

$$\{y_{oi}\}^T\,[M]\,\{y_{oj}\} = \begin{cases} 0 & i \neq j \\ M_i^* & i = j \end{cases} \tag{6}$$

where $i, j = 1, \ldots, n$ and M_i^* is a generalized mass.

Also by definition of the eigenvalue problem,

$$[K]\,\{y_{oi}\} = \omega_i^2\,[M]\,\{y_{oi}\}$$

then

$$\{y_{oi}\}^T\,[K]\,\{y_{oj}\} = \begin{cases} 0 & i \neq j \\ \omega_i^2 M_i^* & i = j \end{cases} \tag{7}$$

The matrix $[C]$ is of such a form that

$$\{y_{oi}\}\,[C]\,\{y_{oj}\} = \begin{cases} 0 & i \neq j \\ 2\,\omega_i\,\xi_i\,M_i^* & i = j \end{cases} \tag{8}$$

where ξ_i is the damping ratio. The natural frequency of a structure affects its response to an earthquake. Most of the energy content of an earthquake is in the 1 to 20 Hz frequency range. The duration of "violent" shaking may last 20 or more seconds. If a structure has a fundamental natural frequency in the 1 to 20 Hz range, then it will have time to build up a resonant response. The level of resonance built up depends on the structural damping. For antenna structures considered here, a damping ratio of 7% for the concrete and 4% for the steel is reasonable. Table 1 lists recommended damping ratio values (Ref. 6). For parameterization, we took damping ratios as 2, 5 and 10% as described later in Section III.

Then, the system of Eq. (5) contains only diagonal terms and forms simply a set of ordinary differential equations which, after the modes are normalized, are written as

$$
\left.\begin{aligned}
\ddot{z}_1 + 2\,\xi_1\,\omega_1\,\dot{z}_1 + \omega_1^2\,z_1 &= -\,\{y_{01}\}^T\,\{f\}\,/\,M_1^* \\
\ddot{z}_2 + 2\,\xi_2\,\omega_2\,\dot{z}_2 + \omega_2^2\,z_2 &= -\,\{y_{02}\}^T\,\{f\}\,/\,M_2^* \\
&\vdots \\
\ddot{z}_n + 2\,\xi_n\,\omega_n\,\dot{z}_n + \omega_n^2\,z_n &= -\,\{y_{0n}\}^T\,\{f\}\,/\,M_n^*
\end{aligned}\right\} \qquad (9)
$$

Each ordinary differential equation in Eq. (9) can be solved in an elementary manner, and the complete solution is obtained by superposition as in Eq. (4). In the earthquake case, forces $\{f(t)\}$, at each node, vary in the same manner with time. When the antenna is subjected to earthquake motions, the foundation is subjected to a certain "forcing" acceleration $a(t)$, which tends to move with the ground. Since the motion is relatively rapid, it causes severe stresses and deformation throughout the antenna structure. If a mechanical-structural component of the antenna is rigid, it will move with the same acceleration motion of its base, and the dynamic forces acting on it will be very nearly equal to those associated with the base acceleration. By superposing and opposing motions on the whole foundation-to-top structure, we can consider the moving foundation as equivalent to a fixed foundation with forces $(-Ma\{r\})$ acting on the nodes as shown in Fig. 1. The vector $\{r\}$ is an influence vector (Ref. 3) which geometrically connects the acceleration at nodes. The $\{r\}$ consists of ones and zeros only.

A typical modal differential equation from Eq. (9) (for $i = 1, \ldots, n$) is written as

$$
\ddot{z}_i' + 2\,\xi_i\,\omega_i\,\dot{z}_i' + \omega_i^2\,z_i' = a(t) \qquad (10)
$$

with

$$
z_i = \alpha_i\,z_i'
$$

and

$$
\alpha_i = \{y_{oi}\}^T\,[M]\,\{r\}\,/\,M_i^* \qquad (11)
$$

The solution of Eq. (10) can be written simply as the Duhamel integral

$$
z_i' = \frac{1}{\omega_i}\int_0^t a(\tau)\exp\left[-\,\xi_i\,\omega_i\,(t-\tau)\right]\,\sin\omega_i(t-\tau)\,d\tau \qquad (12)
$$

In practice, two approaches to solve Eqs. (10) through (12) are followed:

Approach 1: Full transient analysis. The complete transient response analysis is obtained from integration of Eq. (12), carried out numerically. In principle, superpositions in Eq. (4) will result in the full transient responses required. Often a simple calculation is carried out for each mode to determine maximum responses followed by a suitable "addition" of these responses. More details are given in Appendices A and B.

Approach 2: Response spectra. Direct earthquake response spectra are obtained without the necessity of carrying out complete transient analysis. For various input earthquake motions, the responses of a single degree of freedom (typical of Eq. [10]) have been evaluated to determine the "envelope" response spectra. These are available in the literature and are known as the earthquake response spectra, as in Fig. 2 and 3. The response spectra are useful to design engineers because they embrace the spectra of many observed earthquakes. A structure designer can safely select these design response spectra as inputs that describe the statistically justified excitation of the ground at a given site in the United States. They included three x, y, z motions (two horizontal and one vertical).

III. Computational Results

Twenty values of natural frequencies and the participation factors from Eq. (11) are made available from the JPL IDEAS program for the 70-m antenna. The two approaches are compared for the natural frequency at the first mode, with three different damping ratios 2, 5 and 10%. Table 2 represents the value of permissible displacement S_d and acceleration S_a taken from spectrum curves proposed by Housner in Refs. 7 and 8. The first row in Table 2 concerns an earthquake of the intensity expected at Goldstone, the second row an earthquake of the 1940 El Centro intensity. The third row represents values from the full transient analysis. The computer program TRANST (see also Appendix A) was used to solve Eq. (10) representing the motion of a single degree of freedom of the system. The program solves optionally the eigenvalue problem before proceeding to the transient response solution. The fundamental frequency $f = 1.59$ Hz was used first for computation. A "simplified" 1940 El Centro earthquake was selected as the acceleration input function $a(t)$ in Eq. (10) (Fig. 4). Transient displacement responses are shown in Fig. 5 for damping ratios 2, 5 and 10%, respectively.

Another numerical method was also used to evaluate the Duhamel integral Eq. (12) representing the transient response solution of Eq. (10). This numerical method uses trigonometric identity and converts the original Duhamel integral into a summation of closed form solutions. A description of this method, as well as a flow chart of the computer program are given in the Appendix B. The transient response for the fundamental frequency f = 1.59 Hz, with damping ratios 2, 5 and 10% are given in Fig. 6. The results obtained from both numerical methods are in excellent agreement.

IV. Conclusions

The development of methods and practices suitable for structural design of the DSN large antennas should include the analysis of the structure for earthquake resistance. In this study two approaches were presented, one which utilizes known recommended design response spectra, and the second which is the full transient analysis, applicable also for a different type of dynamic loading. The comparative results obtained at the fundamental frequency show that the two approaches are in good agreement. The preliminary results agree with past 64-m antenna study done by Caltech Seismology Staff that shows that the center of the mass of the structure should not exceed about 0.25 to 0.35G. In this first part of the study the emphasis was on the mathematical tools to solve the responses and on the cross verification of different approaches. The second part of this study will compute the forces developed in antenna structure components due to seismic excitation and will compare these with seismic design requirements according to building codes.

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References

1. McClure, D. H. and McLaughlin, F. D., "64-Meter to 70-Meter Antenna Extension," *TDA Progress Report 42-79*, Jet Propulsion Laboratory, Pasadena, Calif., pp. 160-164.
2. TDA Technical Staff, The NASA/JPL 64-Meter-Diameter Antenna at Goldstone California: Project Report, *JPL Technical Memorandum 33-671*, Jet Propulsion Laboratory, Pasadena, California, July 15, 1974.
3. Clough, R. W. and Penzien, L., *Dynamics of Structures*. New York: McGraw-Hill, 1975.
4. Wiegel, R. L., *Earthquake Engineering*. New York: Prentice-Hall, Inc., 1970.
5. Zienkiewicz, O. C., *The Finite Element Method in Engineering Science*. New York: McGraw-Hill, 1977.
6. Newmark, N. M., and Hall, W. J., *Procedures and Criteria for Earthquake Resistance Design in Building Practices for Disaster Mitigation*, Building Science Series 46, U.S. Dept. of Commerce, National Bureau of Standards, pp. 209-236, Feb. 1973.
7. Housner, G. W., "Behavior of Structures During Earthquakes," *Proc. ASCE 85* (EM4), 1959, pp. 109-129, Proceedings Paper 2220.
8. U.S. Atomic Energy Commission (1963). *Nuclear Reactors and Earthquakes*, TID-7024, Washington, D.C., Office of Technical Services.

Table 1. Recommended damping ratios

Stress level	Type and condition of structure	Percentage of critical damping
Working srress, no more than about 1/2 yield point	Vital piping	0.5 to 1.0
	Welded steel, prestressed concrete, well reinforced concrete (only slight cracking)	2
	Reinforced concrete with considerable cracking	3 to 5
	Bolted and/or riveted steel, wood structures with nailed or bolted joints	5 to 7
At or just below yield point	Vital piping	2
	Welded steel, prestressed concrete (without complete loss in prestress)	5
	Prestressed concrete with no prestress left	7
	Reinforced concrete	7 to 10
	Bolted and/or riveted steel, wood structures, with bolted joints	10 to 15
	Wood structures with nailed joints	15 to 20

Table 2. Displacement and acceleration for fundamental frequency

ω = 9.99 rad/s
f = 1.59 Hz
T = 0.629 s

Damping ratio, %	Displacement S_d, in. (cm)	Acceleration, S_a/G
2	1.4 (3.556)	0.362
	2.4 (6.096)	0.620
	3.05 (7.747)	0.826
5	0.9 (2.286)	0.233
	1.56 (3.963)	0.403
	2.76 (7.01)	0.747
10	0.65 (1.651)	0.168
	0.96 (2.438)	0.248
	2.31 (5.867)	0.260

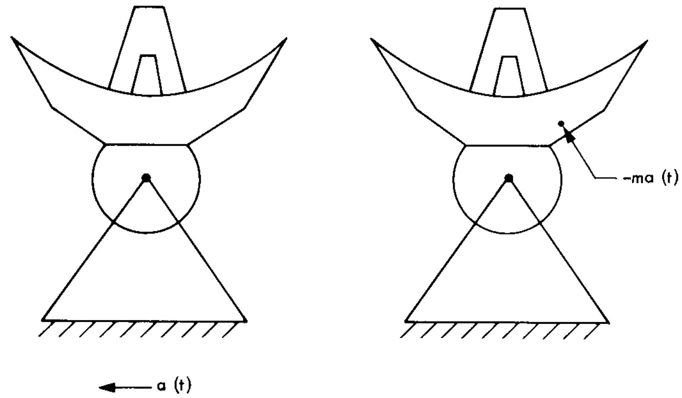

Fig. 1. Foundation motion as equivalent force

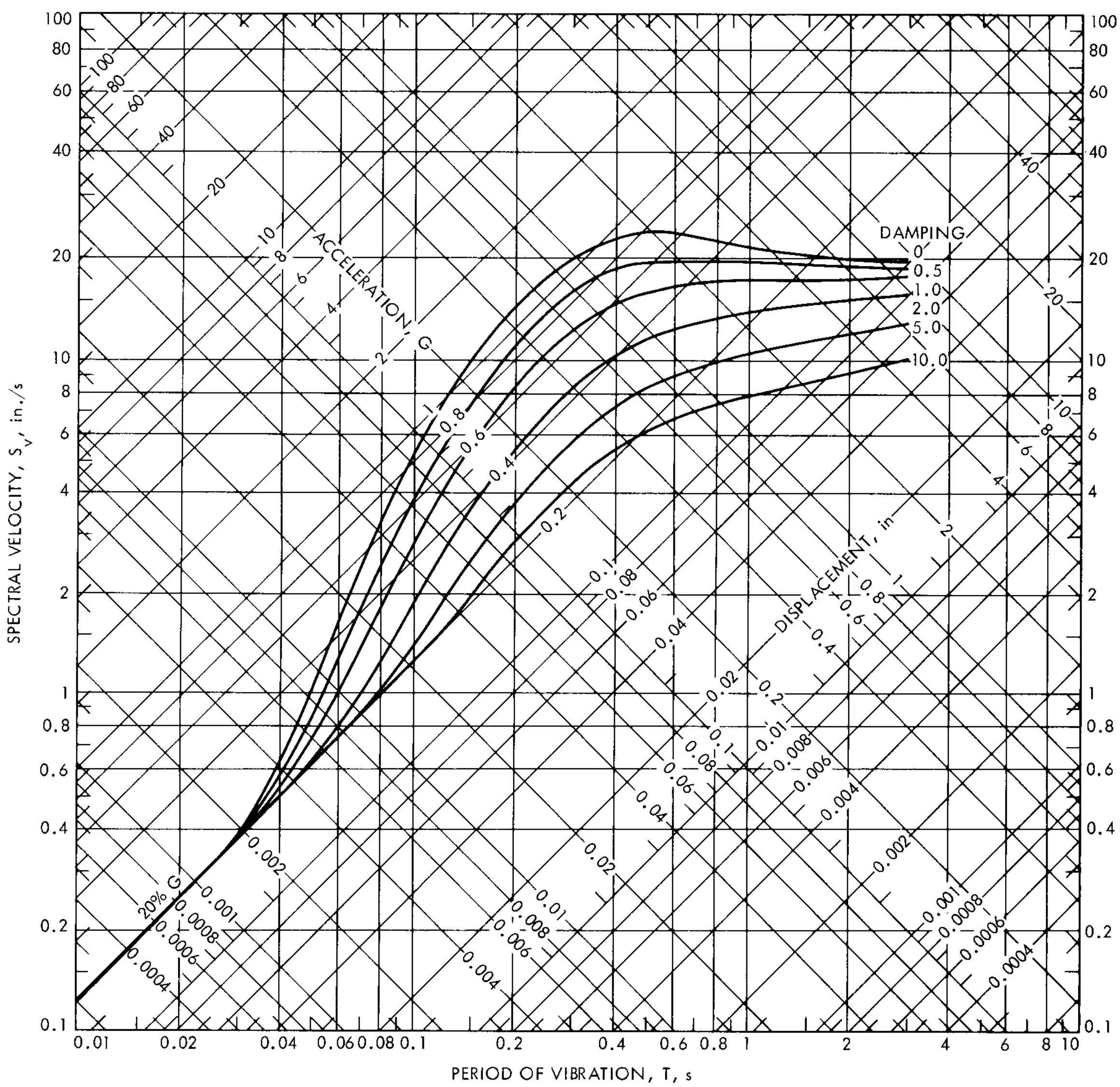

Fig. 2. Combined earthquake response spectra (adapted from Ref. 3)

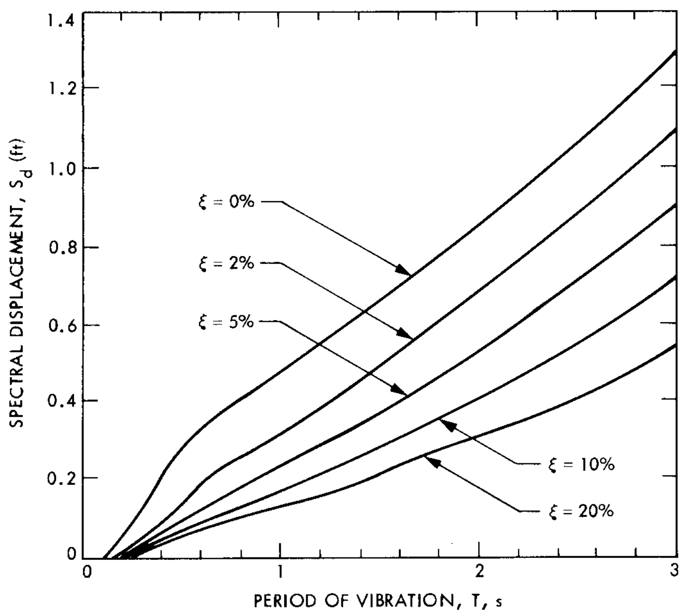

Fig. 3. Average displacement response spectrum of 1940 El Centro intensity (adapted from Ref. 3)

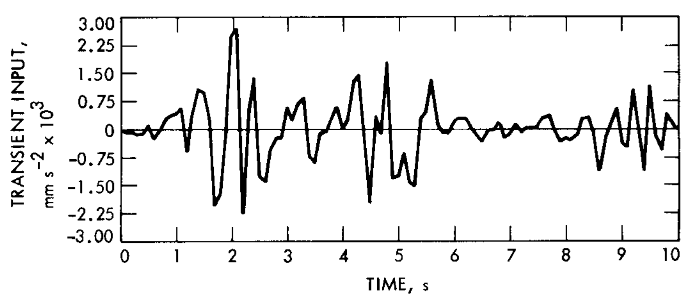

Fig. 4. Earthquake acceleration input function

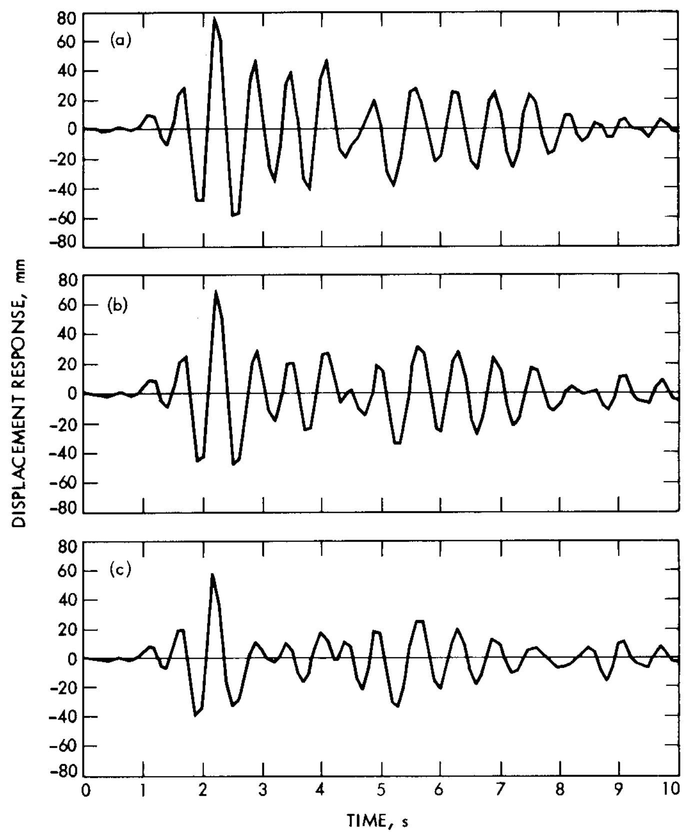

Fig. 5. Transient displacement response damping ratio: (a) 2%, (b) 5%, and (c) 10%

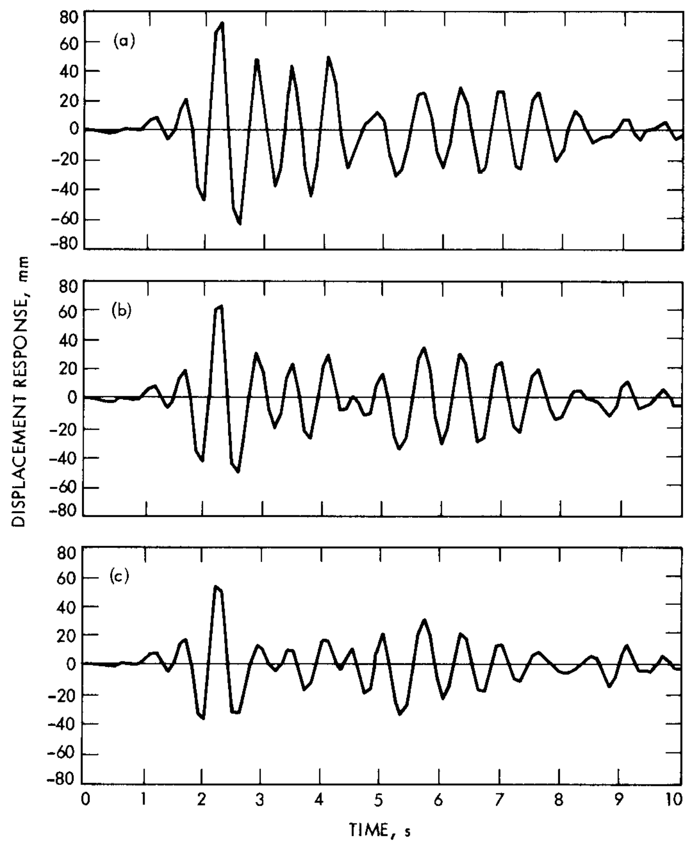

Fig. 6. Numerical evaluation of Duhamel integral for damping ratio: (a) 2%, (b) 5%, (c) 10%

Appendix A
Adams-Moulton Method

The first numerical solution of the antenna transient response follows the Adams-Moulton method in solving a system of linear differential equation of motion, written in the matrix form as:

$$[M]\ \{\ddot{Z}\} + [C]\ \{\dot{Z}\} + [K]\ \{Z\} = \{a(t)\} \qquad \text{(A-1)}$$

where the matrices $[M]$, $[C]$, and $[K]$ are of the order $N \times N$.

Equation (A-1) can be written as

$$\{\ddot{Z}\} + [M]^{-1}\ [C]\ \{\dot{z}\} + [M]^{-1}\ [K]\ \{Z\} = [M]^{-1}\ \{a(t)\} \qquad \text{(A-2)}$$

assuming $[M]^{-1}$ exists.

Equation (A-2) can be written as

$$\{\ddot{Z}\} = - [M]^{-1}\ [C]\ \{\dot{Z}\} - [M]^{-1}\ [K]\ \{Z\} + [M]^{-1}\ \{a(t)\} \qquad \text{(A-3)}$$

Equation (A-3) can be written in the form of

$$\{\dot{Y}\} = [A]\ \{Y\} + \{B\} \qquad \text{(A-4)}$$

if we let

$$\{Y\} = \begin{Bmatrix} Z \\ \dot{Z} \end{Bmatrix} \qquad \text{(A-5)}$$

$$\{\dot{Y}\} = \begin{Bmatrix} \dot{Z} \\ \ddot{Z} \end{Bmatrix} \qquad \text{(A-6)}$$

$$[A] = \begin{bmatrix} [O] & [I] \\ -[M]^{-1}\ [K] & -[M]^{-1}\ [C] \end{bmatrix} \qquad \text{(A-7)}$$

and

$$\{B\} = \begin{Bmatrix} \{O\} \\ [M]^{-1}\ \{a(t)\} \end{Bmatrix} \qquad \text{(A-8)}$$

where $[A]$ is the coefficient matrix of the order $2N \times 2N$, $\{B\}$ is a vector of dimension $2N$, and $[O]$ and $[I]$ are the $N \times N$ null and identity matrices, respectively.

Equation (A-4) is a set of $2N$ simultaneous first-order differential equations which are solved by using the Adams-Moulton numerical technique (JPL computer library subroutine SVDQ). The technique uses linear multistep predictor-corrector formulas. Such a technique has the advantage that from successive approximations to each value, an estimate of the truncation error is made.

The fourth-order Runge-Kutta method is used to generate the approximate values of the first four points ($n - 3$, $n - 2$, $n - 1$, n), since the local truncation error is of order h^5. Values at these previous four points are needed to predict or correct the value at the point ($n + 1$). The integration order is selected in such a way as to maximize the step size and to reduce the computation time, consistent with meeting the requested user accuracy.

In the first-order equation

$$\frac{dy}{dx} = f(x, y) \qquad \text{(A-9)}$$

integrating between x_n and x_{n+1}:

$$\int_{x_n}^{x_{n+1}} \frac{dy}{dx}\ dx = \int_{x_n}^{x_{n+1}} f(x, y) \qquad \text{(A-10)}$$

The Adams-Moulton method, like all predictor-corrector methods, starts by predicting y_{n+1} from an initial value of y_n, and then provides successive improvements of y_{n+1}, or else corrects y_{n+1} before calculating the next step.

The Adams-Moulton method uses the following predictor:

$$y_{n+1} = y_n + \frac{h}{24}(55 f_n - 59 f_{n-1} + 37 f_{n-2} - 9 f_{n-3})$$

$$R = O(h^5) \qquad \text{(A-11)}$$

and the following corrector

$$y_{n+1} = y_n + \frac{h}{24}(9 f_{n+1} + 19 f_n - 5 f_{n-1} + f_{n-2})$$

$$R = O(h^5) \qquad \text{(A-12)}$$

where h is the step size and R is the truncation error.

By using the Eq. (A-11) as a predictor and Eq. (A-12) as a corrector, the function $y = y(x)$ is obtained. Further details on the method can be found in Ref. A-1.

Reference

A-1. Carnahan, B., Luther, H., and Wilkes, J., *Applied Numerical Methods*. New York: John Wiley, 1969.

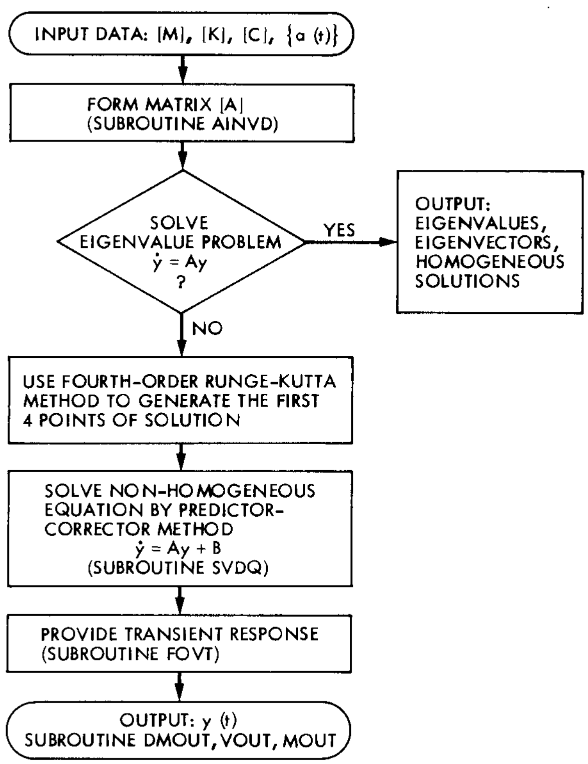

Fig. A-1. Flow chart of TRANST program (using Adams-Moulton method)

Appendix B
Numerical Evaluation of the Duhamel Integral

The Duhamel integral

$$Z_i'(t) = \left(\frac{1}{\omega_i}\right) \int_0^t a(\tau) \exp\left[-\xi_i \omega_i (t-\tau)\right] \sin \omega_i (t-\tau)\, d\tau \quad \text{(B-1)}$$

representing the transient response solution of the equation of motion (Eq. [10]) can be evaluated numerically. Since the acceleration input $a(\tau)$ is given in a tabular form, the integrand can be divided into a number of band functions added together in such a manner as to form the original $a(\tau)$ function.

Equation (B-1) is converted into the sum of exact solutions; each resembles Eq. (B-1) but with a unit forcing function $a(t)$. The sum solution can be used when the actual input is approximated by band segments.

The approach is explained as follows: Using the trigonometric relation

$$\sin \omega_i (t-\tau) = \sin \omega_i t \cos \omega_i \tau - \cos \omega_i t \sin \omega_i \tau \quad \text{(B-2)}$$

substitute Eq. (B-2) into Eq. (B-1). The Duhamel Integral becomes

$$Z_i'(t) = \frac{\exp(-\xi_i \omega_i t) \sin \omega_i t}{\omega_i} \int_0^t a(\tau) \exp(\xi_i \omega_i \tau) \cos \omega_i \tau \, d\tau - \frac{\exp(-\xi_i \omega_i t) \cos \omega_i t}{\omega_i} \int_0^t a(\tau) \exp(\xi_i \omega_i \tau) \sin \omega_i \tau \, d\tau \quad \text{(B-3)}$$

Using the relationships

$$\int \exp(\alpha x) \cos \beta x \, dx = \frac{\exp(\alpha x)(\alpha \cos \beta x + \beta \sin \beta x)}{\alpha^2 + \beta^2} \quad \text{(B-4)}$$

and

$$\int \exp(\alpha x) \sin \beta x \, dx = \frac{\exp(\alpha x)(\alpha \sin \beta x - \beta \cos \beta x)}{\alpha^2 + \beta^2} \quad \text{(B-5)}$$

let

$$\alpha = \xi_i \omega_i, \qquad \beta = \omega_i \quad \text{(B-6)}$$

and assume that the acceleration $a(\tau)$ is constant ($\bar{a}$) between $\tau = n\Delta\tau$ and $\tau = (n+1)\Delta\tau$, where $\Delta\tau$ is the step of summation. Equation (B-3) then becomes

$$Z_i'(t) = \frac{\exp(-\xi_i \omega_i t) \sin \omega_i t}{\omega_i^3 (1 + \xi_i^2)} \times \sum_{\text{all } \Delta\tau} \bar{a} \left[\exp(\xi_i \omega_i \tau)(\xi_i \omega_i \cos \omega_i \tau + \omega_i \sin \omega_i \tau)\right]_{\tau = n\Delta\tau}^{\tau = (n+1)\Delta\tau}$$
$$- \frac{\exp(-\xi_i \omega_i t) \cos \omega_i t}{\omega_i^3 (1 + \xi_i^2)} \times \sum_{\text{all } \Delta\tau} \bar{a} \left[\exp(\xi_i \omega_i \tau)(\xi_i \omega_i \sin \omega_i \tau - \omega_i \cos \omega_i \tau)\right]_{\tau = n\Delta\tau}^{\tau = (n+1)\Delta\tau} \quad \text{(B-7)}$$

for integration step between $\tau = n\Delta\tau$ and $\tau = (n+1)\Delta\tau$ where n is an integer.

The integral in Eq. (B-1) is then transformed into the summation of the individual contributions from $\tau = 0$ to $\tau = t$. A flow chart of the FORTRAN program used to evaluate Eq. (B-7) is shown in Fig. B-1.

INPUT

NT = TOTAL TIME

DT = TIME INCREMENT

OMEGA = FREQUENCY (ω_i)

CI = DAMPING RATIO (ξ_i)

↓

READ

T (I) = TIME (τ)

ACCEL (I) = INPUT FORCING FUNCTION ($\alpha(\tau)$)

↓

EVALUATE

$$A_1 = [(\exp - \xi_i \omega_i t) \sin \omega_i t]/[\omega_i^2 (1 + \xi_i^2)]$$

$$B_1 = [(\exp - \xi_i \omega_i t) \cos \omega_i t]/[\omega_i^2 (1 + \xi_i^2)]$$

$$A_2 = \alpha(\tau) (\exp \xi_i \omega_i \tau) (\xi_i \omega_i \cos \omega_i \tau + \omega_i \sin \omega_i \tau)$$

$$B_2 = \alpha(\tau) (\exp \xi_i \omega_i \tau) (\xi_i \omega_i \sin \omega_i \tau - \omega_i \cos \omega_i \tau)$$

↓

SUM

$$Z(t) = (A_1) \sum_{i=1}^{N} (A_2) - (B_1) \sum_{i=1}^{N} (B_2)$$

↓

TRANSIENT RESPONSE TIME HISTORY

Fig. B-1. Flow chart of numerical evaluation of Duhamel integral