# In Search of a 2-dB Coding Gain

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*A recent code search found a (15, 1/5), a (14, 1/6), and a (15, 1/6) convolutional code which, when concatenated with a 10-bit (1023, 959) Reed-Solomon (RS) code, achieves a bit-error rate (BER) of $10^{-6}$ at a big signal-to-noise ratio (SNR) of 0.50 dB, 0.47 dB and 0.42 dB, respectively. All of these three codes outperform the Voyager communication system, our baseline, which achieves a BER of $10^{-6}$ at bit SNR of 2.53 dB, by more than 2 dB. Our 2 dB coding improvement goal has been exceeded.*

## I. Introduction

To enable future planetary missions to provide today's equivalent or better information return, higher communication efficiencies have to be achieved. More efficient communication links will help to lower mission cost.

To achieve reliable communications over a noisy channel with acceptable coding complexity, concatenated coding systems with a convolutional code as the inner code and a Reed-Solomon (RS) code as the outer code are used. NASA's Voyager employs a communication system which uses a (7, 1/2) convolutional code as its inner code and an 8-bit (255, 223) RS code as its outer code. This system, which achieves a bit-error rate (BER) of $10^{-6}$ at a bit signal-to-noise ratio (SNR) of 2.53 dB, is the proposed NASA standard and our baseline. Communication system performance can be improved by many means, including increasing the spacecraft transmitter power, the antenna size, and the coding complexity.

Spacecraft transmitter power is at a premium; larger antennas are also very costly. Fortunately, with the continued progress in VLSI technology, it is possible to implement codes with much higher complexity than our baseline system. Our goal in this search was to find codes which will provide a 2 dB improvement over the baseline system, i.e., the new code needs to achieve a BER of $10^{-6}$ at a bit SNR of 0.53 dB. This 2 dB gain of bit SNR by coding improvement is likely to be a much more cost effective means than increasing spacecraft transmitter power, antenna size, etc.

## II. Code Search Results

Instead of using the criterion of maximum free distance, $d_f$, we searched for good convolutional codes using the criterion of minimizing required bit SNR, $E_b/N_0$, for a given value of desired BER, for the goodness of code. The channel considered is the deep space channel which is the additive white Gaussian noise channel and binary-phase-shift-keying (BPSK) modulation (Ref. 1). Partial results for good convolutional codes of constraint length $K < 13$, and rate $1/N$, $2 < N < 8$, have been reported (Refs. 2 and 3). For ($K$, $1/N$) convolutional codes, the number of all possible codes is $2^{KN}$, which is extremely large for large $K$ and $N$. It is prohibitively time consuming to perform an exhaustive search except for small $K$ and $N$. Using educated guesses combined with the idea that good codes generate good codes (Ref. 2), we performed selective searches for good low rate $1/N$ convolutional codes of constraint length $K$ = 13, 14, and 15. The codes found were first measured by using the free distance and then the transfer function bound (Ref. 3), which is a fast, efficient algorithm developed for our code search effort. Relatively good code resulted. The symbol-error probabilities were determined by computer simulations using 4-bit channel output quantization, and the performance of these relatively good convolutional codes when concatenated with various RS codes was determined.

The convolutional code generators (in Octal) for several relatively good codes of rate 1/4, 1/5, and 1/6 are listed in Table 1, 2, and 3, respectively. The minimum free distances of these codes are also listed. The 10-bit symbol error probabilities, obtained by long computer simulations, are plotted in Fig. 1 for some of these codes (codes marked with an asterisk (*) in Tables 1, 2, and 3). These convolutional codes are concatenated with various 10-bit RS codes. The required $E_b/N_0$ (i.e., outer code SNR) to achieve a BER of $10^{-6}$ is determined (Ref. 5) and shown in Table 4 for concatenated systems with optimal RS code rates.

Similarly, performance results have also been obtained when these convolutional codes are concatenated with various 8-bit RS codes, including the (255, 223) RS code of our baseline system. Results are included in Table 4. It is found that the performance of the 8-bit RS code is only about 0.25 dB less than the 10-bit RS code.

We have found three convolutional codes which, when concatenated with the 10-bit (1023, 959) RS code, exceed our 2 dB coding improvement goal. These are the (15, 1/5), (14, 1/6), and (15, 1/6) convolutional codes whose code generators are given in Tables 2 and 3. The performance of these codes is also illustrated in Fig. 2 along with some historically significant codes used for deep space communications.

Our 2 dB coding gain in SNR comes from increased decoder complexity and expanded bandwidth. Since implementation complexity may be an issue, let us examine the complexity of these codes. The encoders are relatively simple. For example, the best convolutional code (15, 1/6) that we have found is only slightly more complicated that the current Voyager baseline (7, 1/2) convolutional code as depicted in Fig. 3. They are indeed simple logic circuits. An assessment of the increased decoder complexity and expanded bandwidth of the two most powerful codes that we have found is summarized in Table 5 by comparing them to our baseline system. Replacing the 8-bit RS code in our baseline system by the 10-bit RS code contributes only about 0.25 dB in coding gain, but the increase in decoder complexity is rather large. Since most of the coding gain comes from the long constraint length, low rate convolutional codes, we should concentrate on their implementation. We feel that the realization of our newly found good convolutional codes is within reach by VLSI technology.

## III. Conclusion

Our selected search of the astronomical number of possible convolutional codes of constraint length K = 13, 14, and 15, and rate 1/2 to 1/6 was guided by educated guesses and the idea that good codes generate good codes. We used the practical criterion of minimizing the required bit SNR for a given BER for the goodness of code. By using a specially developed fast efficient transfer function bound algorithm and computer simulation programs, we obtained many good convolutional codes. In particular, we found a (15, 1/5), a (14, 1/6), and a (15, 1/6) convolutional code. Each, when concatenated with a 10 bit (1023, 959) RS code, outperforms our baseline system by 2.03 dB, 2.06 dB, and 2.11 dB respectively. They exceed the 2 dB coding improvement goal. Considerations have been given to the decoding complexity. We feel that VLSI technology will enable the realization of our newly found convolutional codes.

## References

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**Table 1. Rate 1/4 convolutional codes**

| $K$ | Generators (Octal) | | | | $d_{free}$ |
|---|---|---|---|---|---|
| 13 | 11145 | 12477 | 15573 | 16727 | 33*† |
| 14 | 20553 | 25271 | 33447 | 37515 | 34 |
| | 21113 | 23175 | 35557 | 36527 | 36 |
| 15 | 46321 | 51271 | 63667 | 73257 | 37 |
| | 46321 | 51271 | 63667 | 73277 | 38 |

†This code was found by Larsen (Ref. 4).

**Table 2. Rate 1/5 convolutional codes**

| $K$ | Generators (Octal) | | | | | $d_{free}$ |
|---|---|---|---|---|---|---|
| 13 | 10661 | 11145 | 12477 | 15573 | 16727 | 41* |
| | 10671 | 11145 | 12477 | 15573 | 16727 | 42 |
| 14 | 21113 | 23175 | 27621 | 35557 | 36527 | 44* |
| 15 | 46321 | 51271 | 63667 | 70535 | 73277 | 47 |
| | 46321 | 51271 | 63667 | 70565 | 73277 | 47* |

**Table 3. Rate 1/6 convolutional codes**

| $K$ | Generators (Octal) | | | | | | $d_{free}$ |
|---|---|---|---|---|---|---|---|
| 14 | 21113 | 23175 | 27621 | 33465 | 35557 | 36527 | 53* |
| | 21113 | 23175 | 27631 | 33465 | 35557 | 36527 | 54 |
| 15 | 46321 | 51271 | 70535 | 63667 | 73277 | 76513 | 56* |

Table 4. Required $E_b/N_0$ to achieve a BER of $10^{-6}$

| Inner Convolutional Codes | Outer RS Codes | Required $E_b/N_0$ for $P_b = 10^{-6}$, dB | Improvement Over Baseline System, dB |
|---|---|---|---|
| (13, 1/4) | (1023, 895) | 0.84 | 1.69 |
| (13, 1/5) | (1023, 927) | 0.68 | 1.85 |
| (14, 1/5) | (1023, 927) | 0.57 | 1.96 |
| (15, 1/5) | (1023, 959) | 0.50 | 2.03 |
| (14, 1/6) | (1023, 959) | 0.47 | 2.06 |
| (15, 1/6) | (1023, 959) | 0.42 | 2.11 |
| (13, 1/5) | ( 255, 223) | 0.93 | 1.60 |
| | ( 255, 229) | 0.91 | 1.62 |
| (14, 1/5) | ( 255, 223) | 0.82 | 1.71 |
| | ( 255, 231) | 0.79 | 1.74 |
| (14,1/6) | ( 255, 223) | 0.71 | 1.82 |
| | ( 255, 233) | 0.63 | 1.90 |

**Table 5. Decoder complexity considerations**

| Performance Parameter | Voyager's Code (7, 1/2) + (1023, 959) RS | New Code No. 1 | | New Code No. 2 | | New Code No. 3 | |
|---|---|---|---|---|---|---|---|
| | | (15, 1/5) + (1023, 959) RS | Adv./Disadv. Relative to Voyager's Code | (14, 1/6) + (1023, 959) RS | Adv./Disadv. Relative to Voyager's Code | (15, 1/6) + (1023, 959) RS | Adv./Disadv. Relative to Voyager's Code |
| SNR Required for BER = $10^{-6}$ | 2.53 dB | 0.50 dB | 2.03 dB Improvement | 0.47 dB | 2.06 dB Improvement | 0.42 dB | 2.11 dB Improvement |
| Inner Code | | | | | | | |
| Memory | 64 States | 16384 States | 256 Times | 8192 States | 128 Times | 16384 States | 256 Times |
| Computations | 1 (Normalized) | 640 (Normalized) | 640 Times | 384 (Normalized) | 384 Times | 768 (Normalized) | 768 Times |
| Outer Code | | | | | | | |
| Memory | 255 X 8 | 1023 X 10 | 5 Times | 1023 X 10 | 5 Times | 1023 X 10 | 5 Times |
| Computations | 1 (Normalized) | 20 (Normalized) | 20 Times | 20 (Normalized) | 20 Times | 20 (Normalized) | 20 Times |
| Overall Bandwidth Expansion | 2.2 | 5.3 | 2.5 Times | 6.4 | 2.9 Times | 6.4 | 2.9 Times |

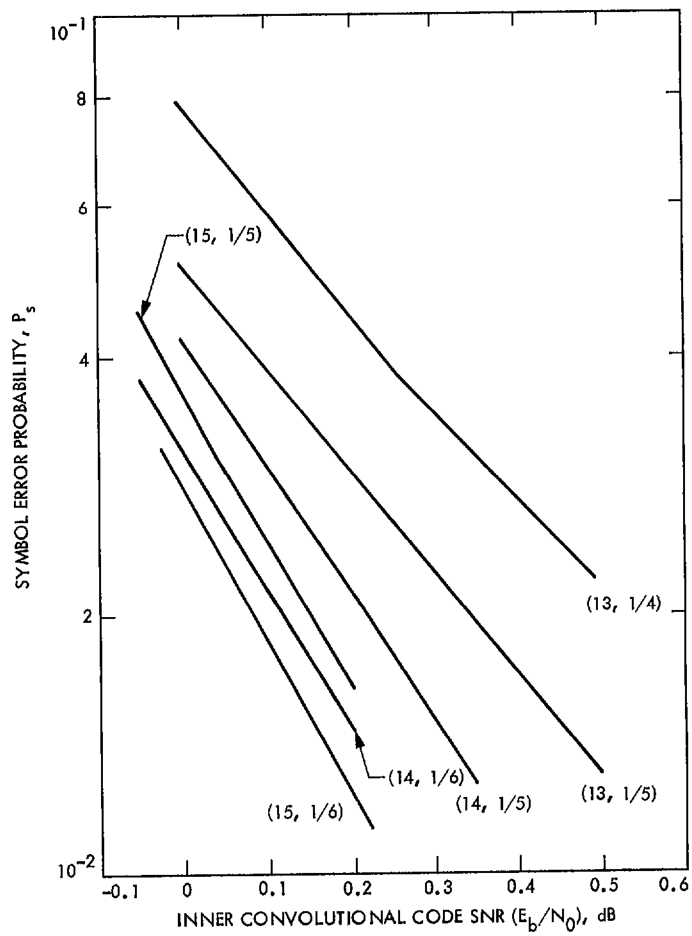
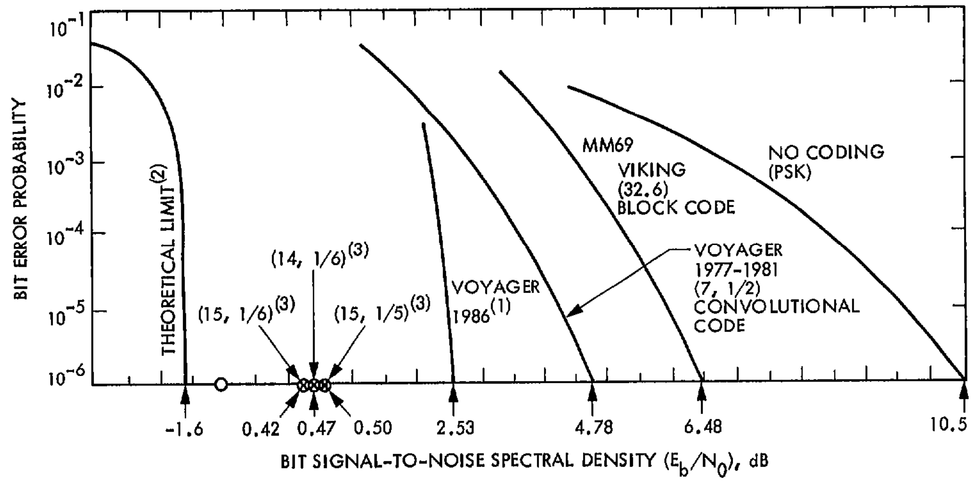



**Fig. 1. Simulated 10-bit symbol error probability**




(1) (7, 1/2) CONVOLUTIONAL CODE (VITERBI DECODING) CONCATENATED WITH A (255, 223) REED-SOLOMON OUTER CODE

(2) INFINITE BANDWIDTH EXPANSION

(3) CONCATENATED WITH A (1023, 959) REED-SOLOMON CODE

O THEORETICAL LIMIT FOR RATE 1/5 CODES IS -1.01 dB AND FOR RATE 1/6 CODES IS -1.10 dB

**Fig. 2. Deep space telemetry performance**

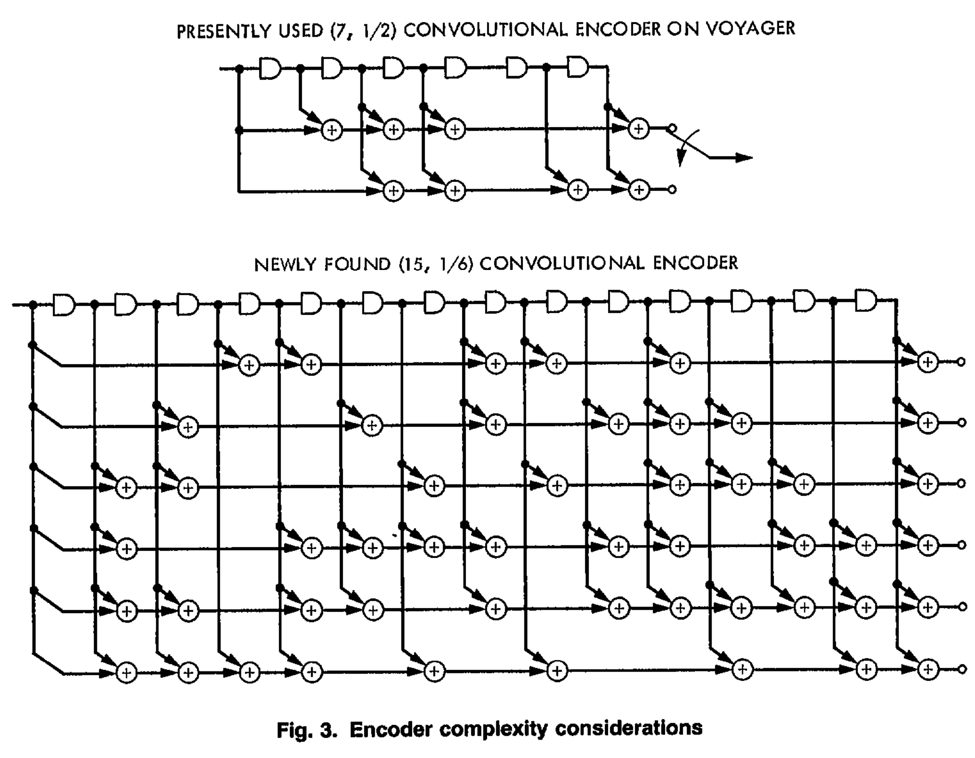



**Fig. 3. Encoder complexity considerations**