New Reflective Symmetry Design Capability in the JPL-IDEAS Structure Optimization Program

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The JPL-IDEAS antenna structure analysis and design optimization computer program was modified to process half-structure models of symmetric structures subjected to arbitrary external static loads, synthesize the performance, and optimize the design of the full structure. Significant savings in computation time and cost (more than 50%) were achieved compared to the cost of full-model computer runs.

I. Introduction

The present X-band upgrade of NASA's Deep Space Network (DSN) 64-m diameter antennas to 70 m requires stripping the existing support structure trusswork back to a 34-m diameter, reinforcing the remaining interior structure, and adding new trusswork to provide the 70-m aperture. To achieve these extensive structural modifications while maintaining very close tolerances on structural deformation, it was decided to use the IDEAS program (Ref. 1, 2, 3) to size the truss members used in the new construction and to determine the amount of reinforcement needed for the existing structure. The IDEAS program is a special purpose finite element structural analysis computer program with unique features for the analysis and optimal design of microwave antenna structures. Unique features include the ability to automatically analyze and optimize the antenna reflector structure for surface accuracy and/or boresight errors due to various gravity, wind, thermal, and other environmental loads. An optimality criteria algorithm is used for the iterative optimization (redesign) of structural elements to achieve the

displacement constraints. Stress and buckling limits are automatically included side constraints for all optimization design constraints.

At the beginning of the project it was realized that the IDEAS program, operating on a UNIVAC 1100/81 computer, could not accommodate the large number of degrees of freedom (over 12,000) needed to represent the mathematical model of the entire 70-m diameter structure. The structure has, though, one plane of structural symmetry, and the capacity of the program was adequate to analyze a half-structure model.

It is known that an arbitrary loading condition for a geometrically symmetrical structure can be decomposed into symmetric and anti-symmetric components. After separate load-displacement solutions, the responses (displacements, stress resultants, etc.) of the two half-structure models can be combined to recover the responses for the full structure. While the IDEAS program was organized to accomplish analysis and redesign of a geometrically symmetrical structure based on

the modeling of one half, only symmetrical loading conditions (e.g., gravity or zero-azimuth wind load) would have physical significance. Independent optimization of half-structure models could not consider effects of general wind or thermal loading conditions. This would have been inconsistent with a comprehensive design.

The large size of the new 70-m antenna structure model prevented automated optimization of the full-structure. It was necessary to reorganize the IDEAS program to automate multiple half-structure analyses and full structure response synthesis. In this way, the IDEAS multiple-constraint design optimization scheme could be applied for any general loading condition with the additional benefit of reduced time and cost. The automation of the optimal design of symmetrical structures based on half-structure model analyses is, we believe, unique to the IDEAS program.

II. Discussion

The 2-axis steerable microwave antennas which are treated by the IDEAS program are represented in a particular right-handed Cartesian coordinate system fixed to the tilting reflector. The Z-axis is the boresight axis, the X-axis is parallel to the elevation axis and the Y-axis is positive upwards when the antenna points to the horizon (Figs. 1 and 2). The Y-Z plane is assumed to be the plane of geometrical symmetry. The right (positive X coordinate) half of symmetric structures is that represented by our half-structure models. This convention was incorporated into the program to facilitate program design and input data preparation. The remaining discussions are based on this convention.

An arbitrary load on a structure is modeled as discrete loads applied to discrete points of a finite element model. A geometrically symmetric structure has pairs of symmetric discrete points, one point on each side of the plane of symmetry. The discussion of discrete loads applied to one of these pairs of points can be generalized to the remaining pairs. The general load applied to the pair of points can be represented by their discrete components applied to the right side point (R) and left side point (L). These R and L components must be further broken down into their components parallel to and normal to the plane of symmetry (subscripts p and n). The symmetric and anti-symmetric components $(S$ and $A)$ of the original loading case can be calculated as follows: For loads normal to the plane of symmetry,

$$S_n = \frac{R_n - L_n}{2}$$

$$A_n = \frac{R_n + L_n}{2}$$

and for loads parallel to the plane of symmetry,

$$S_p = \frac{R_p + L_p}{2}$$

$$A_p = \frac{R_p - L_p}{2}$$

Figures 3 and 4 illustrate the decomposition of a general load and the method of recovery of the general displacement condition. For clarity, only loads parallel to the structures plane of symmetry are shown.

When applied to a half-structure model, these loads produce decomposed components of the response of the full structure. Appropriate boundary constraints (supports) must be imposed on the half-structure model to insure compatible deflections at the plane of symmetry (Fig. 3). For symmetrical loads (thus symmetrical deformations) the plane of symmetry cannot translate normal to the plane of symmetry or rotate about an axis in the plane of symmetry. Thus for anti-symmetric deformations, the plane of symmetry cannot translate parallel to the plane of symmetry or rotate about an axis normal to the plane of symmetry.

The application of these decomposed components of the loads permits the calculation of decomposed response components which are then superimposed to obtain the response of the full structure. Since the models are linear elastic, response components (deflections, stress resultants, etc.) are proportional to load components. Hence, for simplicity, components of load and the responses to that load will be expressed using the same notation. Thus, symmetric (S) and antisymmetric (A) components of load will cause symmetric (S) and anti-symmetric (A) components of deflection. The right (R) and left (L) side components of load or deflection can then be recovered from the symmetric (S) and antisymmetric (A) components of the right side model according to the formulas:

$$R_p = S_p + A_p$$

$$L_p = S_p - A_p$$

for deflections normal to the plane of symmetry, and

$$R_n = S_n + A_n$$

$$L_n = A_n - S_n$$

for deflections parallel to the plane of symmetry.

III. Program Organization

Data input to IDEAS to describe the structure and loading conditions is similar to that used by the NASTRAN structure analysis program. With minor modifications, the same bulk data deck can be used to make a NASTRAN analysis run. Other input is accomplished via FORTRAN V NAMELIST for parameter input and NASTRAN-like cards for bulk data input.

The modifications to implement reflective symmetry, which are described below, were made in view of the organization of the existing IDEAS program. The program performs the following steps:

(1) Construct data base

    (a) Input data

    (b) Collate data

(2) Determine external and inertial loading vectors,

(3) Decompose stiffness matrix,

(4) Analyze natural frequency and, as an option for the frequency design constraints, calculate (equivalent) real and virtual stress resultants,

(5) Compute displacement and solve for reactions and elemental stress resultants,

(6) Calculate antenna performance (path length and bore-sight error analysis) and calculate virtual loads and virtual stress resultants,

(7) Calculate virtual work from real and virtual stress resultants,

(8) Apply the re-design algorithm for resizing rod areas and plate thicknesses.

Note: If displacement performance is not acceptable and/or structure weight change from the preceding cycle is not within convergence criteria, analysis design iterations resume starting at step (2).

IV. Program Modifications for Reflective Symmetry Design

Much of the effort of program reorganization for reflective symmetry was within step (1b). Additional input data is required for reflective symmetry analysis and design to identify:

(1) Which boundary constraint sets and loading cases are symmetric and which are antisymmetric,

(2) How these loads are to be synthesized to obtain full-structure response,

(3) Which loads are to be considered as the design loads.

Extensive data checks were implemented to ensure that input data errors and inconsistencies were identified during the data input and collation phases of the program.

The program was reorganized to repeat the sequence of step (2) through step (5) first for the symmetric and then for antisymmetric configurations. The reactions and displacements from each sequence are saved. After completion of both the symmetric and anti-symmetric sequences, the responses are combined to form the response of the full structure.

The antenna performance algorithms in step (6) were rewritten to process both half- and/or full-structure displacement data. Generating the virtual stress resultants for virtual work coefficient calculations was rather involved. Virtual displacements for the full structure were recovered from the symmetric and antisymmetric components of the response to decomposed full-structure virtual loads.

For the redesign algorithm, steps (7) and (8), individual finite elements were combined into user-specified design variable linking groups. For a structure assumed symmetrical, the full- and half-structure models will have the same number of design variable linking groups — an element on the right and its opposite on the left will be in the same group. Virtual work sums for design variables include contributions from pairs of modeled (right side) and inferred (left side) elements. As an illustration using one right-left pair of 1-dimensional rods,

$$C_{ij} = (F_{ij} + F_{ij}) \frac{1_i}{a_i E_i}$$

in which the product of the real and virtual stress resultants (F) for the left side is as follows where P is the internal bar force:

$$F_{ij} = P_{ir} P_{id}$$

$$= (P_{ir} - P_{ir})(P_{id} - P_{id})$$

and for the right side is

$$F_{ij} = P_{ir} P_{id}$$

$$= (P_{ir} - P_{ir})(P_{id} - P_{id})$$

This simplifies to

$$C_{ij} = 2(P_{ir} P_{id} + P_{ir} P_{id}) \frac{1_i}{a^i E^i}$$

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Once the virtual work for the half and/or full structures is calculated, the redesign algorithm is invoked. The redesign algorithm only operates on design variable groups so that it does not have to be changed for reflective symmetry.

V. New Program Testing and Verification

Test models verified the new program. Several half-structure computer runs produced iterative design results identical to those of full-structure models. All natural frequency modes present in the full structure were found in either the symmetric or the anti-symmetric modes of the half-structure.

Some of the verifications were done on full- and half-structure models of a medium sized antenna backup structure (2,048 nodes in the full model, 1,058 in the half), as seen in Table 1. The computer costs of the two executions showed them to be in the ratio of about 2.5:1. Actual run times were about 1/2 hour for the half and 2 hours for the full model.

The projected cost for one full-structure 70-m antenna model containing about 4020 nodes would be about $520. The projected cost for the two-half-structure model cycle containing 2,050 nodes is $200. On the other hand, very small models suffered slightly in cost comparisons because of the overhead associated with two solution sequences; this difference might be outweighed by the fact that half models are considerably easier to generate and verify.

VI. Summary

The addition of the new reflective symmetry analysis-design capabilities to the IDEAS program allows processing of structure models whose size would otherwise prevent automated design optimization. The new program produced synthesized full-model iterative design results identical to those of actual full-model program executions at substantially reduced cost, time, and computer storage.

Nomenclature for Variables and Subscripts

R, L Right and left components of a generalized load, displacement, etc., condition

S, A Symmetric or antisymmetric components of a generalized load, displacement, etc., condition

a Group rod area (design variable)

l Sum of group rod lengths

E Elastic modulus

F Sensitivity coefficient

P Load

u Displacement

p, n Indicates load or displacement components parallel or normal, respectively, to the structure's plane of symmetry

i Design variable index

j Design constraint index
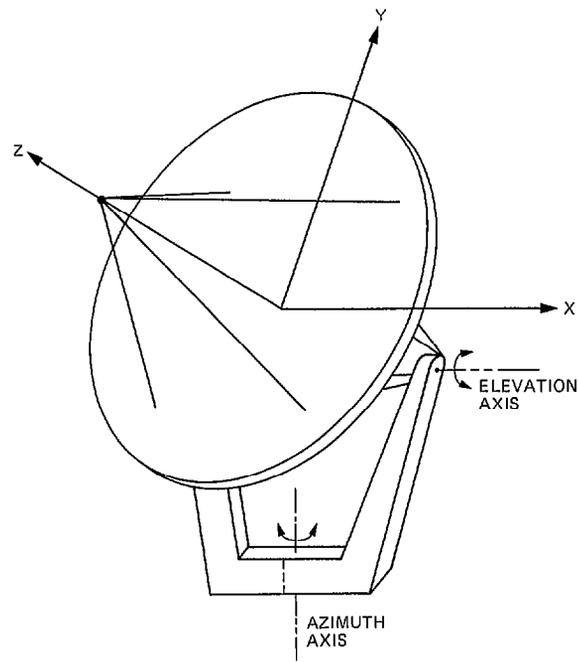
r Real load index

d Virtual load index

References

1. Levy, R., and Strain, D., "Design Optimization of 70-m Antenna with Reflective Symmetry," Proc. 21st Annual Meeting, Society of Engineering Science, Virginia Polytechnic Institute and State University, Blacksburg, Va., Oct. 15–17, 1985, pp. 453–454.

2. Levy, R., and Melosh, R. J., "Computer Design of Antenna Reflectors," *Journal of the Structural Division*, Proc. ASCE 99(ST–11), Proc. Paper 10178, pp. 2269–2285, Nov. 1973.

3. Levy, R., "Computer-Aided Design of Antenna Structures and Components," *Computers and Structures*, Vol. 6, pp. 419–428, 1976.

Table 1. Comparison of 40-m verification model characteristics*

Full Structure	Half Structure	Characteristics
2048	1056	Nodes
6132	3098 3034	Unconstrained D.O.F.
201	102	Matrix wavefront, D.O.F.
9.8 min	1.4 min	Matrix decomposition CPU time
$50 (for 1)	$12 (for 2)	Decomposition cost
$475	$150	Cost of analysis/design sequence
2 hr	0.5 hr	Design sequence through-put time
104 K	54 K	Core size, words

*The half-structure column represents two half-structure finite element models equivalent to the full-structure model represented in the other column. The design sequence was five iterations of static analysis/optimum redesign. The intermediate and final results of both runs were identical.
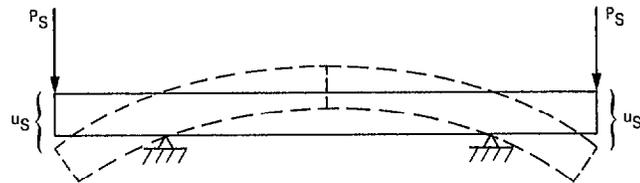
Fig. 1. The IDEAS program antenna coordinate system

(a) LOOKING INTO DISH
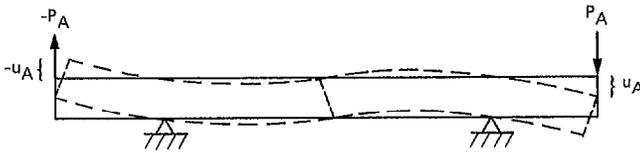
(b) HORIZON LOOK

(c) ZENITH LOOK
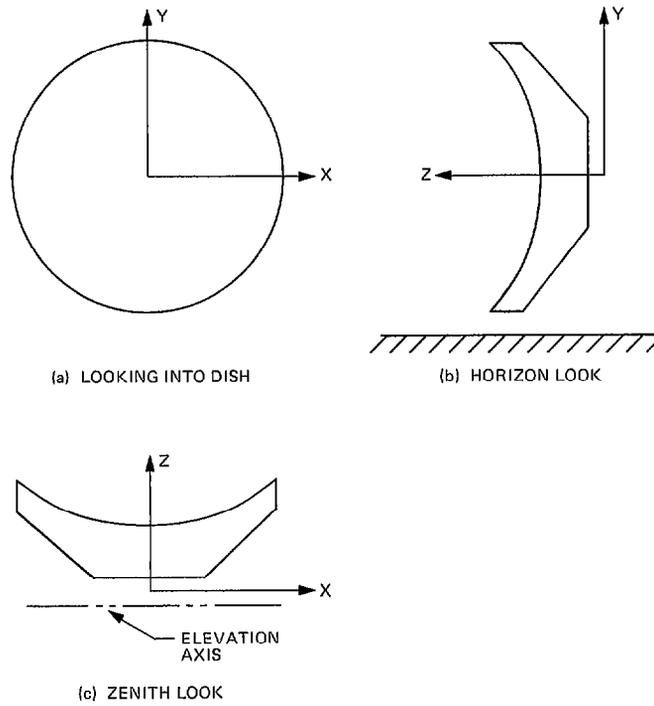
Fig. 2. Details of the IDEAS program antenna coordinate system
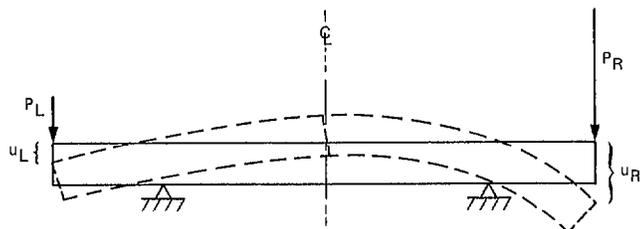
(a) GENERAL LOAD AND DEFLECTION

(b) SYMMETRIC COMPONENTS

(c) ANTISYMMETRIC COMPONENTS
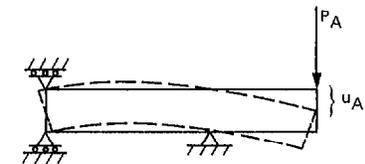
LOADS	SYMMETRIC COMPONENTS	ANTISYMMETRIC COMPONENTS	
		RIGHT SIDE	LEFT SIDE
$P_L = 1$	$P_S = \dfrac{P_L + P_R}{2}$	$P_A = \dfrac{P_R - P_L}{2}$	$P_A = \dfrac{P_L - P_R}{2}$
$P_R = 3$	$= \dfrac{1+3}{2}$	$= \dfrac{3-1}{2}$	$= \dfrac{1-3}{2}$
	$= 2$	$= 1$	$= -1$
			$(= -P_A$ RIGHT SIDE)

Fig. 3. Decomposition of general load into symmetric and anti-symmetric components (load components parallel to plane of symmetry only)
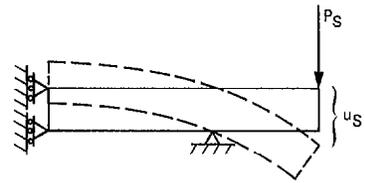
(a) SYMMETRIC COMPONENTS

(b) ANTISYMMETRIC COMPONENTS

SINCE $P_R = P_S + P_A$ THEN $u_R = u_S + u_A$
AND
SINCE $P_L = P_S - P_A$ THEN $u_L = u_S - u_A$

Fig. 4. Recovery of full-structure response from the application of half-structure symmetric and antisymmetric load components