A Method to Dramatically Improve Subcarrier Tracking

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A method is presented for achieving a dramatic improvement in phase tracking of square wave subcarriers or other square waves. The method is to set the amplitude of the phase quadrature reference signal to zero except near the zero crossings of the input signal. Without changing the loop bandwidth, the variance of the phase error can be reduced to approximately $W\sigma_0^2$, where σ_0^2 is the phase error variance without windowing, and W is the fraction of cycle in which the reference signal has a nonzero value. Simulation results confirm the analysis and establish minimum W versus SNR. Typically, the window can be made so narrow as to achieve a phase error variance of $1.5\sigma_0^4$.

I. Introduction

In deep space communications, the loss in data signal-to-noise ratio due to phase tracking error is often more severe for subcarrier tracking than for carrier tracking. This is because the subcarriers are often square waves, and the carriers are sinusoidal. The SNR loss varies approximately as the mean square phase error for sinusoids, but only as the rms phase error for square waves.

Subcarrier tracking loss is most significant in low-rate telemetry systems where the subcarrier loop bandwidth cannot be made narrow enough to reduce the rms phase error to a small enough value. For example, the loss in average symbol SNR for the Pioneer 10 spacecraft at a symbol SNR of 0 dB with the narrowest bandwidth Block III or Block IV subcarrier demodulator assembly is 0.4 dB at 16 bps and 0.6 dB at 8 bps. The actual loss in decoder threshold is even greater, just as radio loss is greater for coded than for uncoded systems.

These losses motivated the analysis and simulation of the improved subcarrier tracking method presented here. The method is capable of reducing the loss in the average symbol SNR (SSNR) to under 0.1 dB for the Pioneer example, without reducing the loop bandwidth.

II. Method and Performance

The improvement in subcarrier tracking is achieved by windowing one of the subcarrier channel reference signals as done in a Digital data Transition Tracking Loop (DTTL) bit synchronizer (Ref. 1). A theoretical basis for this method was presented by Layland (Ref. 2), who concluded that, for a first order phase-locked loop and high loop SNR the optimum reference signals needed to track square waves resemble alternating trains of narrow pulses.

Figure 1 shows the windowed quadrature phase referencing waveform and its relationship to the subcarrier and to the standard reference waveform. Let W be the fraction of each cycle of the reference signal which has nonzero value. The reference signal looks like a square wave, multiplied by zero except for the regions within plus or minus $W/4$ of the zero crossings as illustrated in Fig. 1. The theoretical improvement in loop SNR is approximately a factor of $1/W$, provided that the phase error is small enough that the loop is in the linear region. Based on simulation results, values of W from 1/16 to 1/64 appear practical in cases for which the loop SNR would otherwise be low enough to cause significant symbol SNR loss. This means that the loss can be reduced by a factor of 4 to 8. A 0.4 dB loss can be reduced to 0.05 dB, and a 0.6 dB loss can be reduced to under 0.1 dB. These examples are typical of Pioneer 10 at 0 dB SSNR and data rates of 16 bps and 9 bps, respectively.

The implementation of the windowed reference signal is remarkably simple. The waveform can be generated in a read only memory (ROM) whose input address is the phase. The only change from the full square wave case is to zero the reference signal ROM for the appropriate regions of phase. There is no change in the gain of the subcarrier phase detector due to the windowing.

III. Analysis

In this section we develop the equations that describe the operation of the subcarrier loop, shown in Fig. 2. Using linear analysis we compute analytically the variance of the phase tracking error and validate our results with computer simulations. Throughout this article we neglect quantization errors, nonzero data rise time, filtering distortions, etc. We also assume that perfect symbol synchronization is available and has already been established.

A. Phase Detector Model

The ith sample of the digitized subcarrier signal for the DSN Advanced Receiver is assumed to be of the form (Ref. 3)

$$r_i = \sqrt{P_D}\, d_k \; \mathrm{Sin}(\theta_i) \cos(\phi_{ci}) + n_i \tag{1}$$

where

P_D = average data power (V^2)

d_k = data value of kth binary symbol (±1 equally probable)

$\mathrm{Sin}(x) = \mathrm{sgn}(\sin(x))$

ϕ_{ci} = instantaneous phase carrier estimation error (rad) assumed zero for the rest of the analysis.

n_i = zero mean white Gaussian noise sample with variance σ_n^2

θ_i = instantaneous subcarrier angle (rad)

At this point, it is convenient to introduce the variables

$\hat{\theta}_i$ = closed loop estimate of θ_i (rad)

$\phi_i = \theta_i - \hat{\theta}_i$ = instantaneous subcarrier phase estimation error of the ith sample (rad)

To facilitate the analysis, we assume that the subcarrier period T_{sc} is related to the symbol duration T_{sym} by

$$T_{sym} = nT_{sc} \text{ for some integer } n \tag{2}$$

This circumvents modeling problems associated with "end" effects, which greatly complicate the analysis and lie outside the intended scope of the present discussion. We also postulate that a large number of samples per symbol time are available.

The loop operates as follows (see Fig. 2): The digitized incoming signal r_i is mixed with the reference signals (with no loss of generality we set the multiplier gains equal to one)

$$R_I = \mathrm{Sin}(\hat{\theta}_i) \tag{3}$$

$$R_Q = \widetilde{\mathrm{Cos}}(\hat{\theta}_i) \tag{4}$$

to produce the signals x_i and y_i respectively. These signals are accumulated over the L samples during a symbol interval. Assuming that the instantaneous phase errors of the samples averaged over any particular symbol interval are equal, then the accumulators have responses

$$X_k = d_k L\sqrt{P_D}\,(1 - |u_k|) + n_{xk}, \qquad k = 0, \dots, M \tag{5}$$

$$Y_k = d_k L\sqrt{P_D}\, v_k + n_{yk}, \qquad k = 0, \dots, M \tag{6}$$

where

$$u_k = \frac{2}{\pi}|\phi_k|, \qquad |\phi_k| \leq \pi \tag{7}$$

and

$$v_k = \begin{cases} u_k, & |\phi_k| \leq \pi W/2 \\ \mathrm{sgn}(\phi_k)W, & \pi W/2 \leq |\phi_k| \leq \pi(1 - W/2) \\ 2\,\mathrm{sgn}(\phi_k) - u_k, & \pi(1 - W/2) \leq |\phi_k| \leq \pi \end{cases} \tag{8}$$

These last two equations describe the in-phase and quadrature arms of the phase detector respectively. We show them graphically in Fig. 3.

B. Phase Detector S Curve

The S curve is defined as the mean value of the error control signal conditioned on the phase error. The gain slope at the origin of the S curve and the variance of the error control signal are useful in evaluating the closed loop tracking performance.

In the following section we show that the S curve is given by

$$S(\phi_n) = \frac{\pi}{2}(1 - |U_n|)V_n \tag{9}$$

which for convenience, we have normalized to have unity slope at the origin. This is shown in Fig. 4 for different windows. Notice that for small phase errors, the gain (slope) of the S curve does not change due to the windowing. This has the notable advantage from an implementation point of view of maintaining constant phase detector gain as W is changed. This simplifies implementation in which a wide W is used for acquisition and a narrower W for tracking.

C. Phase Detector Variance

In order to assess the closed loop tracking performance, the variance of the error control signal is needed. Due to the orthogonal nature of the reference signals R_I and R_Q, the noise processes $\{n_{xk}\}$ and $\{n_{yk}\}$ are independent. Strictly speaking, these are cyclostationary processes, but we approximate them by stationary processes. In other words, their first and second order statistics are obtained by time averaging (over just one symbol interval for this case) their ensemble averages. With this in mind, samples of these noise processes have zero mean and variances

$$\mathrm{var}\,(n_{xk}) = L\,\sigma_n^2 \tag{10}$$

$$\mathrm{var}\,(n_{yk}) = WL\,\sigma_n^2 \tag{11}$$

There is also self noise, which is neglected. This self noise is the difference between the actual signal summed over the actual samples, and the mean value which we have used.

The outputs of the in-phase and quadrature arm summers are multiplied together and accumulated subsequently over M symbols to produce the error control voltage that drives the Costas loop. This signal is

$$\epsilon_n = ML^2 P_D (1 - |U_n|) V_n + N_n \tag{12}$$

with U_n, V_n being the time averages of u_k and v_k over M samples respectively. It can be argued via the central limit theorem that the noise samples N_n are approximately Guassian with zero mean and variance

$$\begin{aligned}\sigma_N^2 = {} & \sigma_n^2 P_D L^3 M V_n^2 \\ & + W\sigma_n^2 P_D L^3 M (1 - |U_n|)^2 \\ & + W\sigma_n^4 L^2 M\end{aligned} \tag{13}$$

D. Linear Tracking in the Presence of Noise

When the loop is in the linear region, U_n and V_n in Eqs. (12) and (13) are close to zero and

$$\begin{aligned}\sigma_N^2 \approx {} & W\sigma_n^2 P_D L^3 M \\ & + W\sigma_n^4 L^2 M\end{aligned} \tag{14}$$

If we assume that the noise samples N_n are stationary, the steady state variance of $\{\phi_n\}$ is given by (Refs. 4 and 5)

$$\sigma_\phi^2 = \frac{1}{2\pi j}\oint_{|z|=1} H(z)H(z^{-1})z^{-1}\frac{R_N(z)}{A^2}\,dz \tag{15}$$

where

$$R_N(z) = Z\{E(N_j N_{j+n})\}$$

is the Z transform of the autocorrelation function of the noise process at the input of the loop filter. The function $H(z)$ denotes the closed loop transfer function and A is the gain of the control voltage signal (without noise) evaluated at zero phase error, which is

$$A = \frac{2}{\pi}L^2 M P_D \tag{17}$$

Since the samples $\{N_n\}$ are uncorrelated, zero mean with variance σ_N^2, then

$$R_N(z) = \sigma_N^2 \tag{18}$$

so

$$\sigma_\phi^2 = \frac{2B_L T\,\sigma_N^2}{A^2} \tag{19}$$

where

$$B_L = \frac{1}{2T}\frac{1}{H^2(1)}\frac{1}{2\pi j}\oint_{|z|=1} H(z)H(z^{-1})\frac{dz}{z} \qquad (20)$$

is the one-sided noise bandwidth of the loop, and T is the filter update time related to the symbol duration by

$$T = MT_{sym} \qquad (21)$$

Substitution of Eqs. (14) and (17) into Eq. (19) results in

$$\sigma_\phi^2 = \frac{2\left(\frac{\pi}{2}\right)^2 W\sigma_n^4\left(1+\frac{P_D L}{\sigma_n^2}\right)B_L T_{sym}}{L^2P_D^2} \qquad (22)$$

If we assume that the received noise samples n_l are obtained by sampling white noise of one-sided spectral density N_o at a rate $1/T_s$, then

$$\sigma_n^2 = \frac{N_o}{2T_s} \qquad (23)$$

Using this result, and the fact that

$$P_D = \frac{E_s}{T_{sym}} \qquad (24)$$

$$T_{sym} = L\,T_s \qquad (25)$$

E_s being the symbol energy, then the variance of the tracking error can be put in the form

$$\sigma_\phi^2 = W\left(\frac{\pi}{2}\right)^2\frac{B_L T_{sym}}{E_s/N_o}\left(1+\frac{1}{2E_s/N_o}\right) \qquad (26)$$

Notice that windowing improves the loop SNR by a factor of $1/W$. At first glance, it might appear (erroneously though) that arbitrarily small windows can be selected to obtain any desired performance. This is not so, since for very small windows, linear theory is not valid, and the actual tracking variance is much larger than that predicted by Eq. (26). This will be quantified more precisely in the following section.

IV. Simulation Results

Computer simulations were performed to validate the analysis and determine the range of usable values for the windows. To perform the simulations, an equivalent PLL type model is found first for the Costas loop. This resembles a standard PLL, except that the sinusoidal nonlinearity is replaced by the normalized phase detector characteristic given by Eq. (9).

In Fig. 4 we summarize the simulation results and also include results dictated by linear analysis for several window sizes. We do this by computing the variance of the phase error using as parameters loop SNR's when $W = 1$. Thus, when $W = 1$ and loop SNR = 14 dB, the phase error variance is approximately 0.04 rad^2. By just narrowing the window, we can lower the variance by a factor of roughly 16.

The most striking piece of information contained in Fig. 4 is that, given an initial tracking variance, say σ_0^2, this variance can be reduced conservatively to

$$(\sigma_1^2)_{min} = 1.5\,\sigma_0^4 \qquad (27)$$

by selecting the optimum window size whose value is approximately

$$W = 0.5 \text{ to } 1 \times \sigma_0^2 \qquad (28)$$

V. SNR Loss Due to Phase Error and Design Example

First we determine the average loss in symbol SNR (SSNR) due to subcarrier phase error. Equation (5) represents the output of the decision arm in the Costas loop. It is observed that a subcarrier phase estimation error causes the signal voltage term to be degraded by

$$D = 1 - \frac{2}{\pi}|\phi| \qquad (29)$$

The symbol signal to noise ratio (SSNR) is then degraded on the average by the statistical expectation of the square of the above term. If the loop SNR is high, then it is reasonable to assume a Gaussian density function for the phase error. Carrying out the details of the expectation leads to

$$E\{D^2\} = 1 - 2\left(\frac{2}{\pi}\right)^{3/2}\sigma_\phi \qquad (30)$$

where only first order terms were retained.

A design example is considered next: The average loss in SSNR for the Pioneer 10 spacecraft at a coded symbol SNR of 0 dB with a two-sided design point bandwidth of 0.03 Hz and a symbol rate of 33-1/3 sps is about 0.45 dB (see footnote 1). If a narrower window is employed, this degradation can be significantly reduced.

For the parameters previously mentioned, the initial ($W = 1$) loop SNR is 27.8 dB. From the simulation results, it appears that a window of size $W = 1/256$ is feasible. If this smaller time window is used, with the other parameters kept constant, the average loss in SSNR can be reduced to 0.01 dB.

[1] "Deep Space Network/Flight Project Interface Design Handbook," JPL internal document 810-5, Rev. D, Vol. I, Module TLM-10, p. 67, Jet Propulsion Laboratory, Pasadena, Calif.

VI. Conclusions

A subcarrier Costas loop capable of tracking square waves with less phase error has been described. By setting the quadrature reference signal to zero at the appropriate phases, the variance of the tracking phase error is reduced by a factor of $1/W$ under linearized conditions. Computer simulations validate the previous statement, and give practical values for W when the loop is not adequately described by linear theory.

References

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5. Lindsey, W. C., and Chie, C. M., "A Survey of Digital Phase-Locked Loops," *Proceedings of the IEEE 69*, No. 4, pp. 410–431, April 1981.

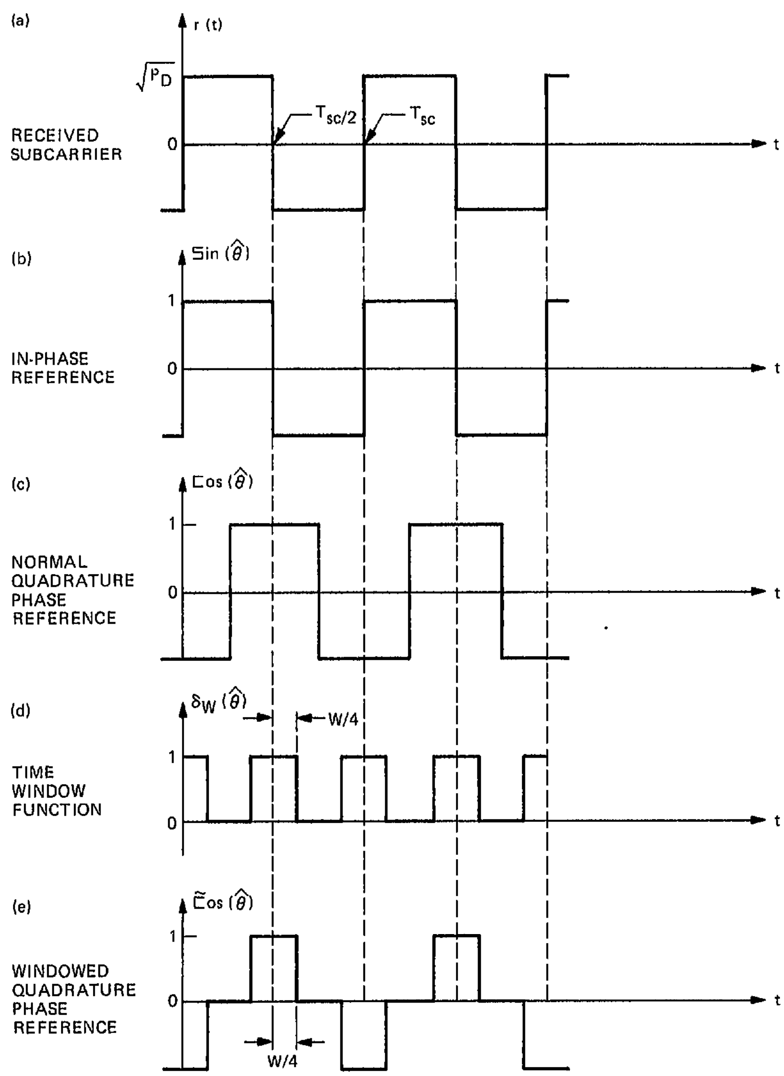

Fig. 1. Loop waveforms in the absence of noise. Continuous and perfect subcarrier synchronization is assumed for pictorial representation.

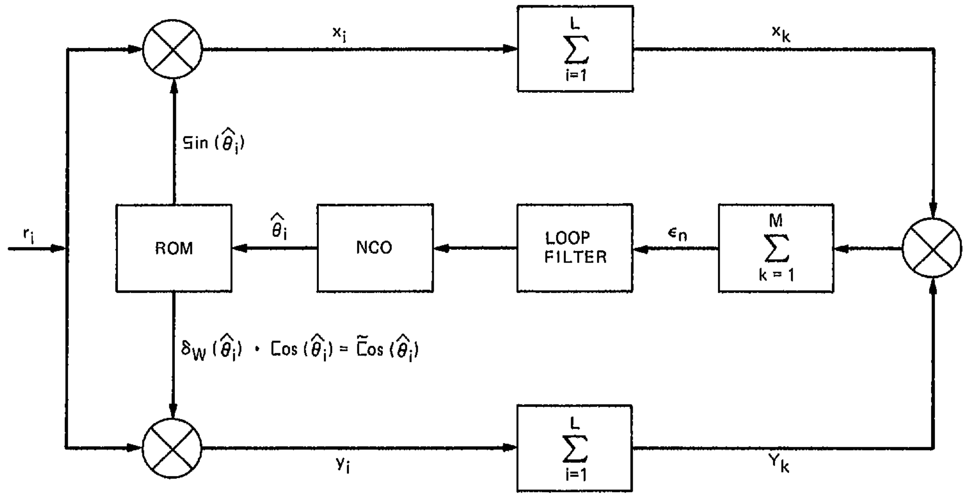

Fig. 2. Block diagram of the all-digital subcarrier Costas loop incorporating a window function in the quadrature reference signal

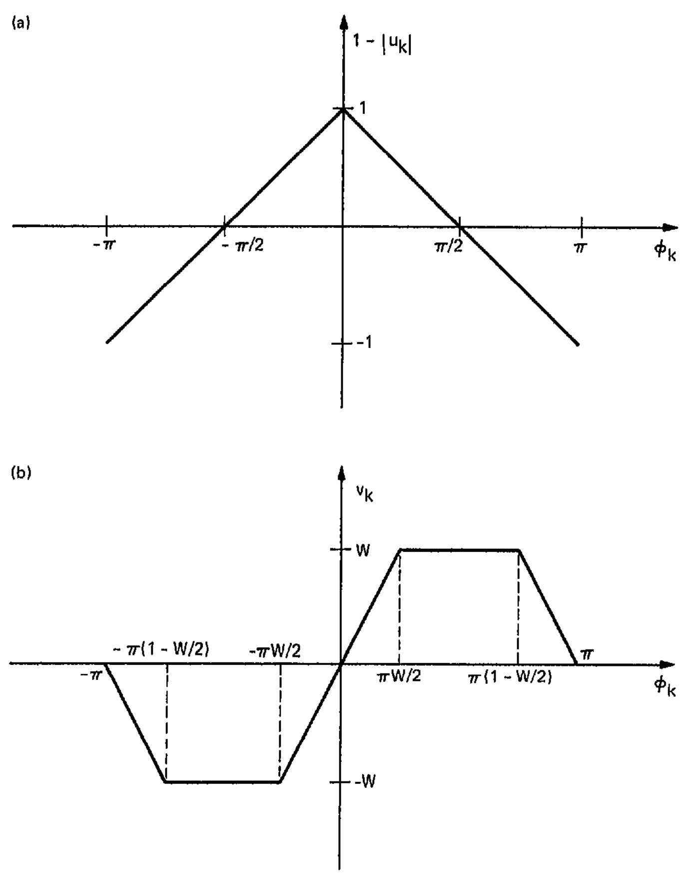

Fig. 3. Phase detector characteristics of in-phase and quadrature arms: (a) in-phase and (b) in-quadrature

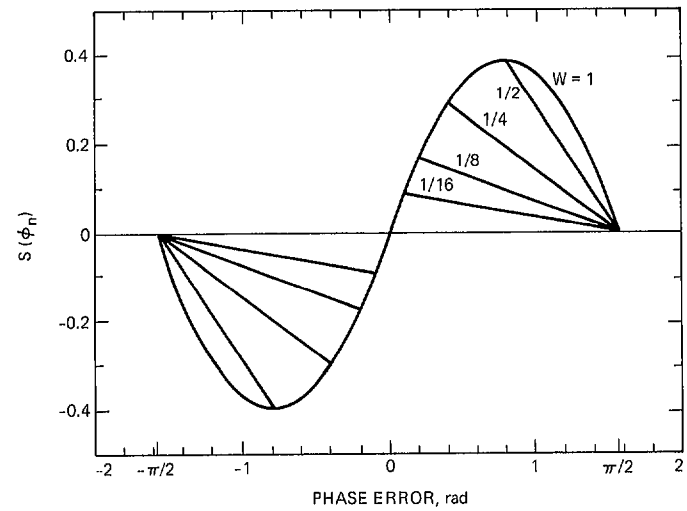

Fig. 4. Normalized loop phase detector characteristics for different window sizes

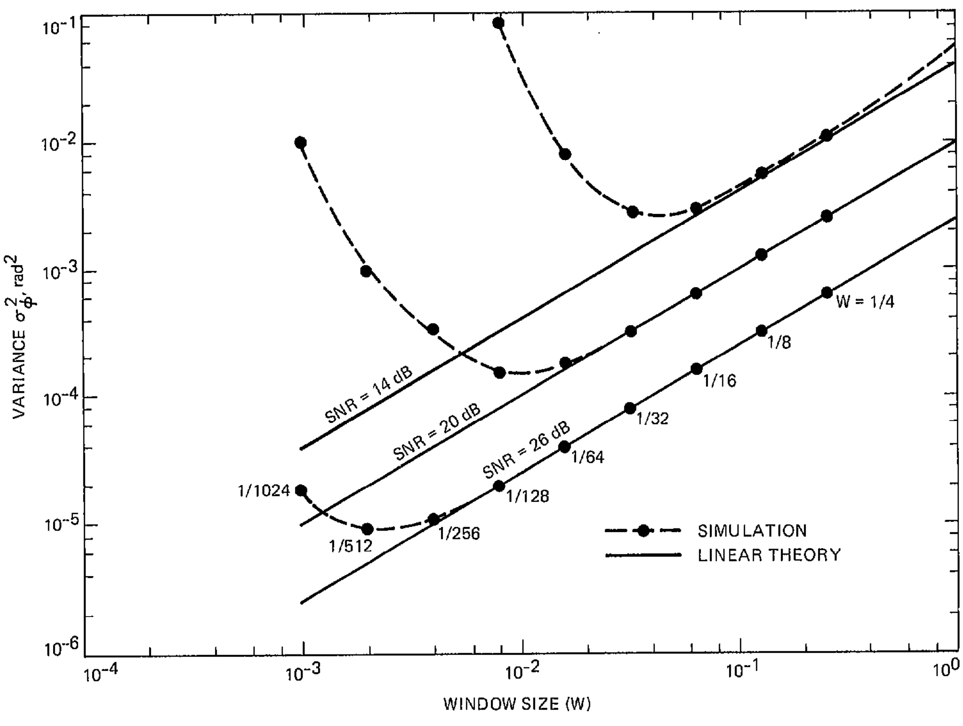

Fig. 5. Variance of the subcarrier phase estimation error as a function of window size with loop SNR as a parameter