Soft-Decision Decoding of Some Block Codes

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The performance of certain binary block codes with soft-decision decoding is evaluated by simulation. A construction is proposed to introduce memory on block codes, and simulation results are shown for a trellis code derived from the Nordstrom-Robinson code.

I. Introduction

Recent technological advances make soft-decision decoders, for moderate length block codes, a practical alternative to convolutionally coded systems. In particular, in order to improve the performance of current concatenated coding systems used in deep space communication, it is interesting to find the performance of some block codes, which might be used as inner codes with soft-decision decoders. Soft-decision decoding is mandatory for inner codes to avoid a penalty of approximately 2 dB incurred by hard quantization of channel outputs.

A well-known upper bound on the word error probability P_w of an (n, k) linear binary block code with soft-decision decoder is given by [1]

$$P_w \leqslant \frac{1}{2} \sum_{m=1}^{M-1} \operatorname{erfc}\left(\sqrt{R w_m E_b / N_0}\right) \tag{1}$$

where $M = 2^k$, $R = k/n$, E_b/N_0 is the bit signal-to-noise ratio, and w_m is the weight of the mth code word. This bound is tight for large E_b/N_0, and the coding gain is proportional to the product $Rd_{\min}$, where $d_{\min}$ is the minimum distance of the block code.

Unfortunately, Eq. (1), which is based on the union bound, yields a very loose performance estimate at low E_b/N_0. We therefore resort to simulation techniques to find the true soft-decision, maximum likelihood performance of some selected binary block codes.

II. Soft-Decision Decoding

We assume a channel with no output quantization. Therefore, each binary waveform is obtained by the optimum demodulator (a matched filter followed by a sampler), and a code word is represented by a sequence of n random variables.

Let E denote the energy of one of these n binary waveforms. Then each binary decision variable can be written, up to an irrelevant constant, as

$$r_i = \begin{cases} \sqrt{E} + n_i, & \text{if } i\text{th bit} = 1 \\ -\sqrt{E} + n_i, & \text{if } i\text{th bit} = 0 \end{cases}$$

and $i = 0, 1, \ldots, n-1$, where the variables n_i are samples of additive white Gaussian noise (AWGN) with zero mean and variance $N_0/2$.

From knowledge of the $M = 2^k$ code words, and upon reception of the sequence $r_0, r_1, \ldots, r_{n-1}$, the decoder computes M squared Euclidean distances

$$d_i^2 = \sum_{j=0}^{n-1} (r_j - \sqrt{E}\, x_{ij})^2, \qquad i = 0, 1, \ldots, M-1$$

where $x_{ij} = \pm 1$ corresponds to the jth bit of the ith code word. The decoded code word is the one corresponding to the minimum distance.

III. Simulation Results for Various Block Codes

A few rate 1/2 binary block codes are considered:

(a) The extended Golay (24, 12) code with $d_{min} = 8$.

(b) The first-order Reed-Muller (8, 4) code with $d_{min} = 4$.

(c) The nonlinear (16, 8) Nordstrom-Robinson (N-R) code [2] with $d_{min} = 6$.

The word error probability P_w of these codes, obtained by simulation, is shown in Fig. 1, in the range of interest for use as inner codes. The results are also compared to the performance of the NASA (7, 1/2) convolutional code, assuming a word length of 8 bits. Care must be exercised in comparing the results, due to different word lengths.

Figure 2 shows the performance of the first-order Reed-Muller (16, 5) code and the (7, 1/3) convolutional code with 8-bit words. Figure 3 shows a similar comparison between the first-order Reed-Muller (32, 6) code and the (14, 1/4) and (14, 1/5) convolutional codes with 10-bit words.

The bit error probabilities of two of the codes considered in Fig. 1, the Golay (24, 12) and the NASA (7, 1/2) codes, are compared in Fig. 4.

In Fig. 1 it appears that the NASA (7, 1/2) code is still the best, except at very low E_b/N_0, and that longer block codes, longer than those considered here, are necessary to compare favorably. Nevertheless, the simulation of the Nordstrom-Robinson code gave rise to a new approach.

Instead of pursuing the path of longer codes we devised some simple methods to introduce limited memory on a block code. Given that the Nordstrom-Robinson code is composed of a first-order Reed-Muller (16, 5) code and 7 of its cosets [3], we were able to construct a 4-state trellis code by properly assigning these 8 cosets to the branches of the state diagram in Fig. 5. This construction, described in detail and generalized in [4], yields in this case a 4-state (16, 7) code with free distance $d_f = 8$. Notice that even though the product ($Rd_f = 3.5$) for this code is lower than that for the Golay code ($Rd_f = 4$) and for the NASA (7, 1/2) code ($Rd_f = 5$), the code performs better at low E_b/N_0, and has lower complexity than the Golay code. Figures 6 and 7 show simulation results for the word error probabilities P_w and bit error probabilities P_b respectively, for the three codes just mentioned.

References

[1] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1983.

[2] A. W. Nordstrom and J. P. Robinson, "An optimum nonlinear code," *Inform. and Control*, vol. 11, pp. 613–616, 1968.

[3] F. J. MacWilliams and N. J. Sloane, *The Theory of Error-Correcting Codes*. New York: North-Holland, 1977.

[4] F. Pollara, R. J. McEliece, and K. Abdel-Ghaffar, "Construction for finite-state codes," *TDA Progress Report 42-90*, vol. April–June 1987 (this issue), Jet Propulsion Laboratory, Pasadena, Calif., August 15, 1987.

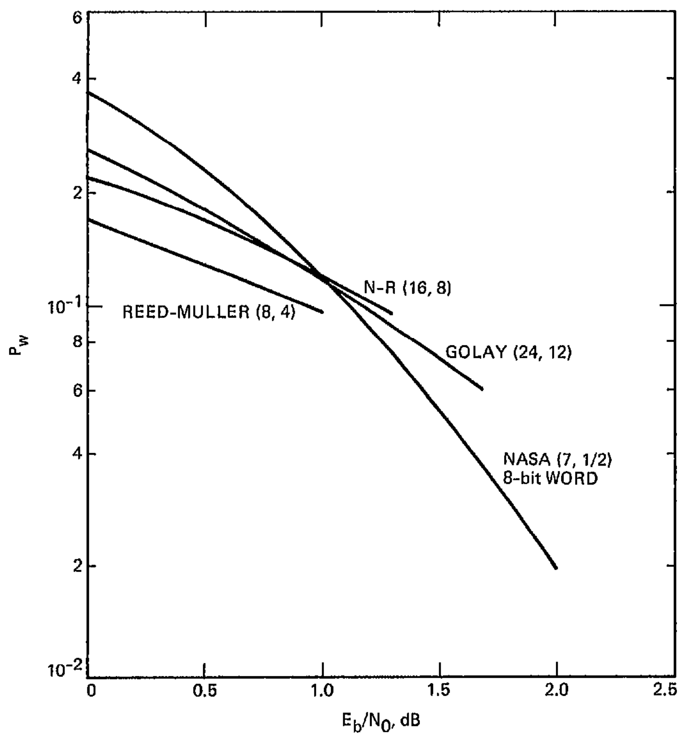

Fig. 1. Soft-decoding performance of three rate 1/2 block codes

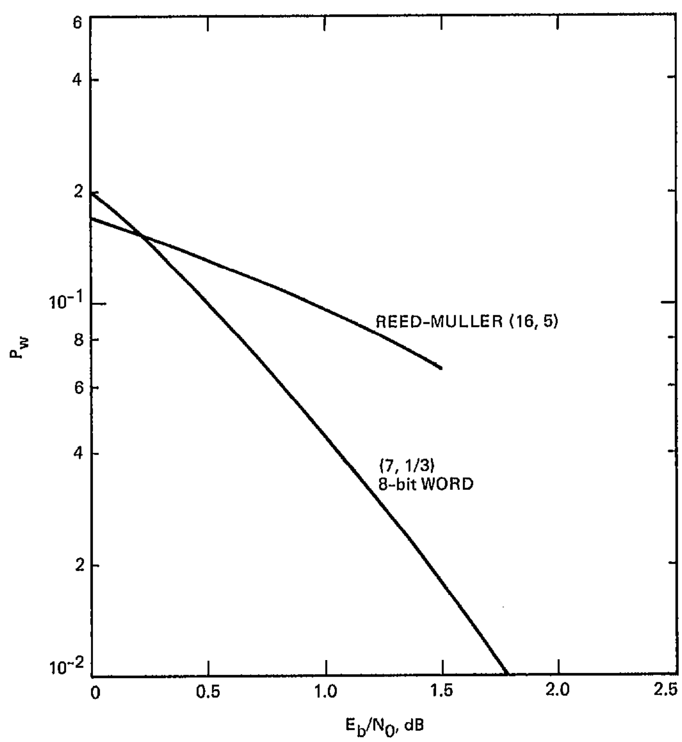

Fig. 2. Soft-decoding performance of first-order Reed–Muller (16, 5) code

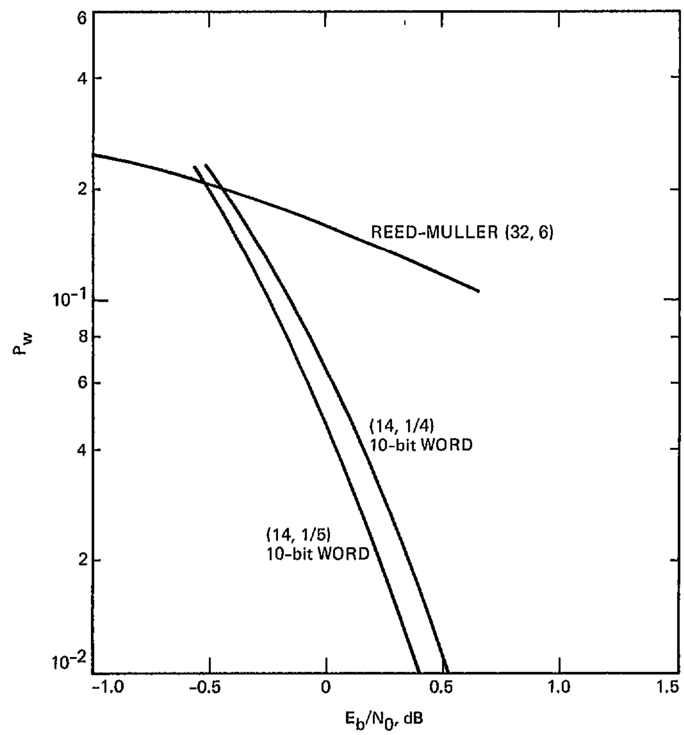

Fig. 3. Soft-decoding performance of first-order Reed–Muller (32, 6) code

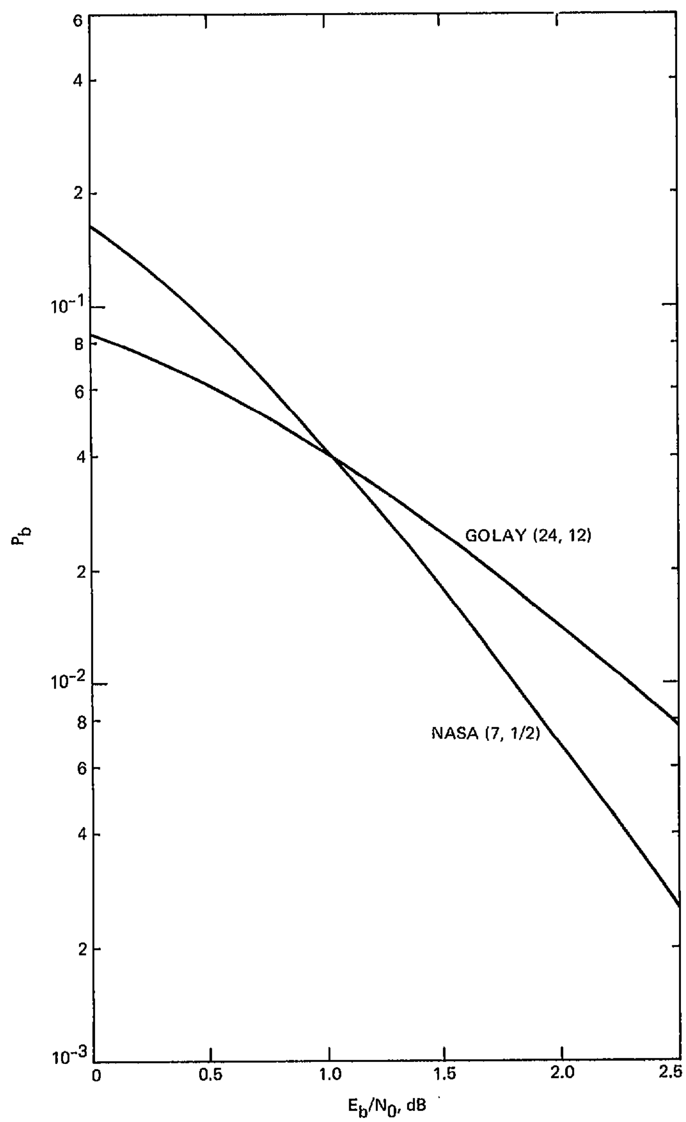

Fig. 4. Bit error probability of Golay code

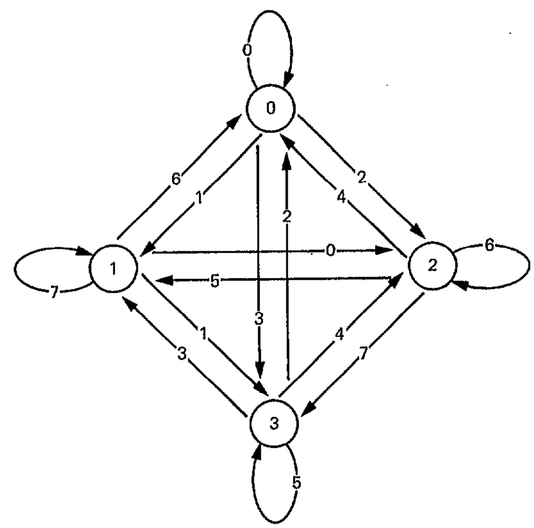

Fig. 5. Diagram of a 4-state trellis

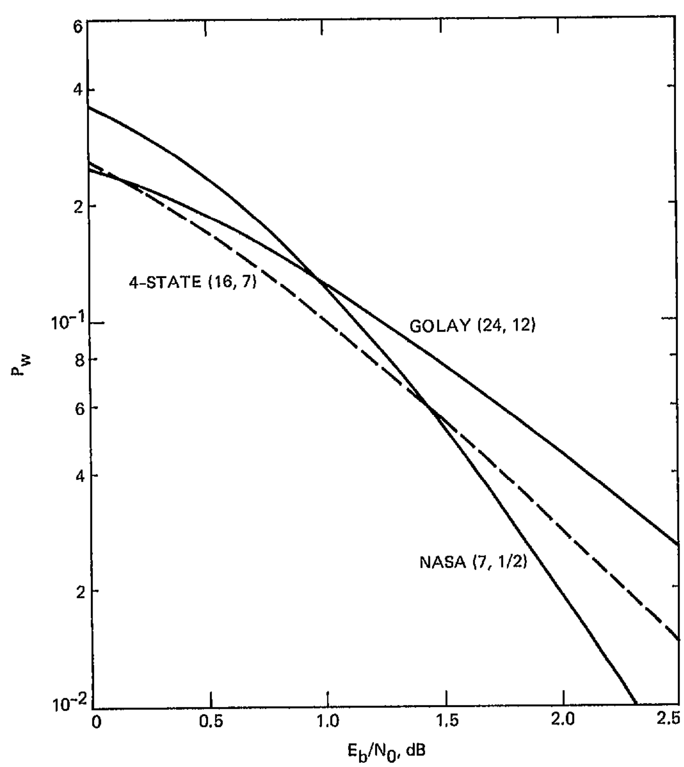

Fig. 6. The P_w of new code compared to Golay and (7, 1/2) codes

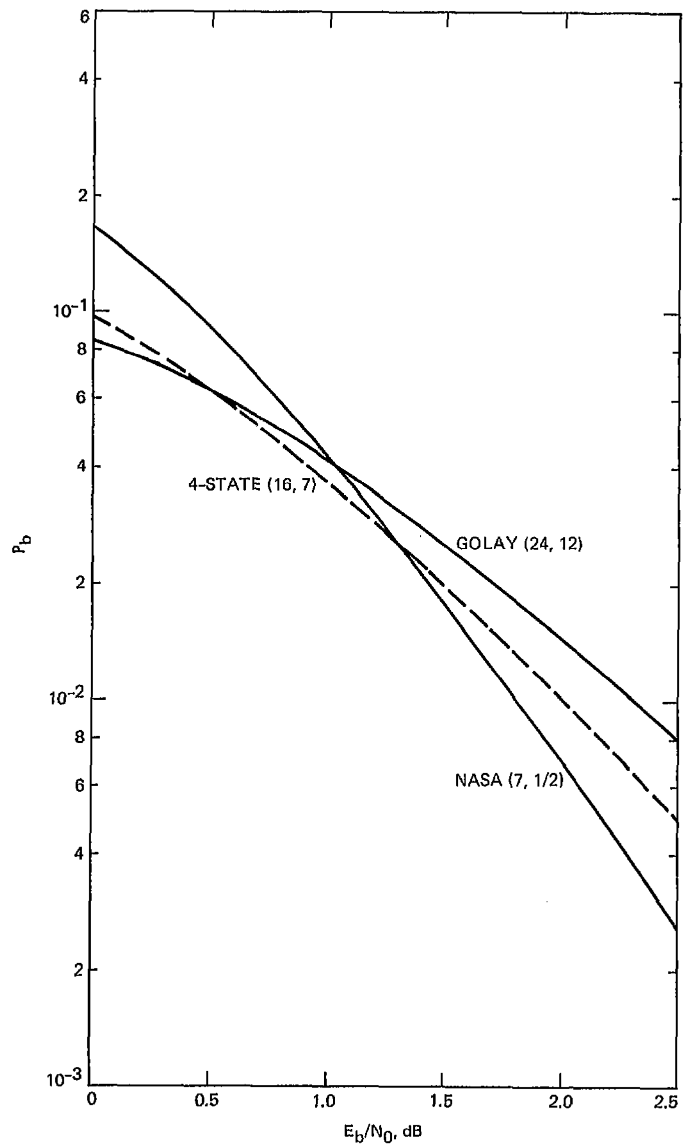

Fig. 7. The P_b of new code compared to Golay and (7, 1/2) codes