Non-Linearity in Measurement Systems: Evaluation Method and Application to Microwave Radiometers

C. T. Stelzried
Office of Telecommunications and Data Acquisition

A simple method for determination and correction of non-linearity in measurement systems is presented. The technique is applicable to a wide range of measuring systems. The basic concept, an analysis, and a sample application are given. Non-linearity of the Goldstone DSS 13 low noise HEMT 2.3-GHz (S-band) radiometer system results in noise temperature measurement errors. These errors are successfully corrected with this method.

I. Introduction

The Deep Space Network (DSN) requires a large number of measurement systems for engineering and science data. For example, microwave Noise Adding Radiometers (NARs) are used for system noise temperature measurements [1]-[3] with applications for the determination of radio "star" flux density for both antenna efficiency calibrations and scientific value.

Engineering design and test procedures are used to obtain linear system measurement performance. A linear system provides equal changes in output reading for equal changes at the system input, independent of signal level. If a noise diode is turned on and off while the microwave radiometer system is switched to the antenna (cold sky) and then to the ambient load, a linear system will give equal changes in output readings. A non-linear system does not.

Linearity of the DSN NAR, a subassembly of the Precision Power Monitor Assembly, is checked with this technique.[1] If the measured noise temperature of a small noise diode, while the system is sequentially switched to the antenna and then to the ambient load, differs by less than 2 percent, the system is considered to be sufficiently linear for use as an NAR.

The purpose of this article is to evaluate an algorithm [4] useful for a wide variety of measurement systems to determine and correct for this type of error.

System non-linearities are only a single example of many possible measurement error sources. Other sources of error, not discussed in this analysis, include such items as incorrectly calibrated noise standards and nonrepeatable front end switches for microwave radiometer systems. Each separate error source should be systematically analyzed to obtain the required overall measurement accuracy. The following analysis provides a method for including non-linearity errors in this list.

II. Theory

A. Linear Analysis

For a linear system, assume an input unknown, T, to be determined in terms of constants and an output reading R

[1] "Precision Power Monitor Assembly," TM 03115A, JPL internal document, pp. 1-3 and 1-4, June 1, 1986.

$$T = A + BR \tag{1}$$

where

T = system input parameter (for example, temperature, K)

A, B = constants, to be evaluated, defining a particular system

R = system output reading (arbitrary units: K, mW, etc.)

For a microwave receiving system,

$T = T_{op}$ = system operating noise temperature for a receiving system [5], K

$= T_i + T_e$

T_i = source temperature, K

T_e = receiver effective noise temperature, K

Evaluating the constants for Eq. (1)

$$B = \frac{T_4 - T_1}{R_4 - R_1} \tag{2}$$

$$A = \frac{T_1 R_4 - T_4 R_1}{R_4 - R_1} \tag{3}$$

or alternatively

$$A = T_1 - BR_1 \tag{4}$$

where

T_1, T_4 = system input calibration parameters

R_1, R_4 = system output readings with inputs T_1 and T_4

For a microwave radiometer system, the solution for B can be accomplished by switching the receiver input between two matched terminations at known physical temperatures T_{p4} and T_{p1}, and readings R_4 and R_1. The usual assumption is that $R = 0$ when $T = 0$. For this case

$$B = \frac{T_{p4} - T_{p1}}{R_4 - R_1} \tag{5}$$

$$A = 0 \tag{6}$$

A typical DSN microwave radiometer system is presently configured with a single microwave ambient termination at physical temperature T_{p4} so that Eq. (5) is not applicable. Also, Eq. (6) may not be exactly applicable if $R_1 \neq 0$ when $T_1 = 0$. It is possible to simulate T_1 by terminating the reading device input with an ambient termination (at temperature T_{p1}) and either setting $R_1 = 0$ (zero set) or reading R_1 (bias). In this case, T_1 is estimated using $T_1 = T_{p1}/G$, where G is the overall gain between the microwave system input and the reading device input termination. Assuming that T_e (defined from receiver input to the reading device input termination) is known from a laboratory measurement, we have, for this case

$$B = \frac{T_{p4} + T_e - T_1}{R_4 - R_1} \tag{7}$$

$$A = T_1 - BR_1 \tag{8}$$

$$T_1 = \frac{T_{p1}}{G} << 1 \tag{9}$$

assuming G is much greater than 1.

For $T_1 = R_1 = 0$

$$B = \frac{T_{p4} + T_e}{R_4} \tag{10}$$

$$A = 0 \tag{11}$$

Eq. (1) is now used to obtain the system temperature on the antenna T_2, from the system output reading R_2, using A and B as calculated above.

$$T_{op} = T_2 = A + BR_2$$

B. Non-Linear Analysis

The linear analysis, Eq. (1), will be in error if the system is non-linear. For small non-linearities, assume a quadratic solution (see Fig. 1). Other models are also possible; a quadratic was chosen for this study since it is the next term in a power series, it is consistent with previous studies [3], and good results are obtained with existing microwave radiometer systems.

$$T = A + BR + CR^2 \tag{12}$$

There are various practical techniques for measuring A, B, and C. For a non-linear system, constants A and B from Eq. (12) are not the same as in Eq. (1). Consider the example of a microwave radiometer system with a manually switched noise diode which adds a temperature T_n to the microwave system input noise temperature when turned on. There are five conditions to consider, with a noise temperature and reading for each. Assume that readings R are taken for each case. Then for the five conditions we find the following input values, T, and output readings, R (see Fig. 1):

(1) calibrated termination, ND off: $T = T_1; R = R_1$

(2) antenna, ND off: $T = T_2; R = R_2$

(3) antenna, ND on: $T = T_3 = T_2 + T_n; R = R_3$

(4) calibrated termination, ND off: $T = T_4; R = R_4$

(5) calibrated termination, ND on: $T = T_5 = T_4 + T_n$; $R = R_5$

Using these conditions with Eq. (12)

$$A + BR_1 + CR_1^2 = T_1 \tag{13}$$

$$A + BR_2 + CR_2^2 = T_2 \tag{14}$$

$$A + BR_3 + CR_3^2 = T_2 + T_n \tag{15}$$

$$A + BR_4 + CR_4^2 = T_4 \tag{16}$$

$$A + BR_5 + CR_5^2 = T_4 + T_n \tag{17}$$

Solving five equations with knowns T_1, T_4, R_1, R_2, R_3, R_4, and R_5 and 5 unknowns, A, B, C, T_2 and T_n:

$$C = \frac{T_4 - T_1}{R_4^2 - R_1^2 - (R_4 - R_1)D} \tag{18}$$

$$B = -CD \tag{19}$$

$$A = T_1 - BR_1 - CR_1^2 \tag{20}$$

$$T_2 = A + BR_2 + CR_2^2 \tag{21}$$

$$T_n = A + BR_3 + CR_3^2 - T_2 \tag{22}$$

where

$$D = \frac{R_5^2 - R_4^2 - R_3^2 + R_2^2}{R_5 - R_4 - R_3 + R_2}$$

T_4 and T_1 are determined for the appropriate configuration, such as a microwave radiometer system, using the same concepts illustrated with Eq. (5).

The A, B, and C constants adjust the non-linear curve to fit the equal input temperature level changes, $T_n = (T_5 - T_4) = (T_3 - T_2)$ for the observed system output readings. This is the central concept of the proposed technique. With a curve defined by the solution of A, B, and C, the "primary" unknown, T_2, is solved with Eq. (21).

These results are general, and can be used either to provide an estimate of system non-linearity or to apply a correction. It is useful to determine the difference between T_2 provided by the linear and non-linear analyses, Eqs. (1) and (12). An assumption that the non-linear analysis corrects for non-linearity and the linear analysis does not can be used to obtain an estimate of the non-linearity error. Defining a linearity factor:

$$\begin{aligned} LF &= T\text{ (linear analysis)}/T\text{ (non-linear analysis)} \\ &= T\text{ (computed from Eq. [1])}/T\text{ (computed from Eq. [12])} \end{aligned} \tag{23}$$

Typically, LF is greater than 1 for an NAR system and less than 1 for a total power system. The results section has further discussion of the linearity factor.

It is also useful to consider a linearity correction factor (CF) for difference measurements based on Eq. (12). This is useful for radio source noise temperature measurements. Assume that the source measurement corrected with Eq. (12) is given by

$$T_{sc} = T_{su} CF \tag{24}$$

where

$CF = B + 2CR_{\text{off}} + CT_{su}$ (CF is a ratio obtained from Eq. [12])

$T_{su} = R_{\text{on}} - R_{\text{off}}$ = uncorrected source measurement, K

R_{off} = uncorrected off source measurement

R_{on} = uncorrected on source measurement

The best strategy for correction of source noise temperature difference measurements depends on the requirements and instrumentation. In some cases the correction should be made in near real time, during the sequence of measurements. In other cases, particularly with small corrections, the correction could be made after the sequence of measurements. Consider an example with $B = 1.02$, $C = 0.0001$, $T_{su} = 1$ K, and R_{off} values of 20 K to 40 K with an elevation angle over the measurement sequence. The corrected source noise temperature difference value computed with the averaged $R_{off} = 30$ K is $T_{sc} = 1.026$ K. Individual corrected values would range from 1.024 K to 1.028 K. The peak "error" in the single correction relative to the full range of uncorrected measurements is only about 0.01 dB for this example. This indicates that a simple single correction may be satisfactory for many applications.

III. Results

The above equations have been programmed for an IBM compatible personal computer (PC) SuperCalc computer program, assuming a single temperature-calibrated ambient microwave termination and a known receiver noise temperature, T_e. Noise temperature measurements were made to verify the procedures using the Goldstone DSS 13 antenna at 2.3 GHz (S-band). A typical data set taken July, 1987, is shown in Fig. 2. Data sets 101, the NAR case, and 102, the total power case, are time meshed together in order to minimize other than nonlinearity effects. For this data, $T_1 \approx 0$, since there is high amplifier gain between the antenna and the square law detector. The computer program allows finite values for T_1 and R_1 ("bias") if desired.

The NAR case used an IBM compatible PC configuration. An S-band HEMT low noise amplifier was followed by a mixer, an IF amplifier, a square law diode detector assembly, and an analog voltage to digital interface to the PC. The PC controlled the periodic on-off switching of the NAR noise diode at the HEMT amplifier input. The noise diode required for non-linearity calibration was turned on and off manually as required. The total power case used a voltmeter connected directly to the analog voltage output of the square law detector.

Eqs. (1) and (12) are used for the linear and non-linear solutions tabulated in Table 1. One-sigma statistical measurement errors are in parentheses. The A, B, and C constants are quite different for the linear and non-linear analyses. This indicates significant non-linearity for these configurations. For the non-linear analysis system temperature on the antenna T_2, results for both the NAR and total power cases are in good agreement.

The linear and non-linear analysis difference for the NAR case, data set 101, is shown plotted in Fig. 3 as the non-linearity error. The error goes to zero on the ambient load since this is a common reference for both the linear and non-linear analysis. The linearity factor, LF (Eq. [23]), is shown plotted in Fig. 4. A ratio of 1.03, obtained on the antenna, indicates about 3 percent non-linearity. This linearity factor is included in Table 1 for both the NAR and total power cases, indicating an approximate equal magnitude for each. Figures 3 and 4 include the plus and minus 1-sigma statistical measurement error obtained from the 6 independent data measurements. Most of the non-linearity of this configuration is due to the square law diode detector. This fact was determined by reducing the power level to the detector and repeating the data set and analysis. Reducing the power level to the square law diode reduces the non-linearity error at the expense of increased zero drift in the dc amplifiers following the square law diode, and less resolution in the analog to digital converters.

Figure 5 shows a plot of the differences between the linear analysis, Eq. (1), and non-linear analysis, Eq. (12), vs. T_{op} for both the NAR case, data 101, and total power case, data 102. The small differences between the two different cases after correction for non-linearity indicate successful removal of the non-linear effects between two separate radiometer operating modes.

The zenith system noise temperature of the DSS 13 S-band receiving system with a HEMT low noise amplifier was remeasured by station personnel on August 28, 1987, with three different time-interleaved data types. The data cases were: (1) IBM PC NAR with a square law detector and A/D converter; (2) total power with a square law detector and digital voltmeter; and (3) total power with a digital power meter. The results are tabulated in Table 2 and plotted in Fig. 6. This shows poor agreement between the data types using the linear analysis, Eq. (1). The non-linear analysis, Eq. (12), corrects for system non-linearity and provides good agreement between the three data types, increasing confidence in the technique. It is assumed that the difference between the corrected values for the July 2 and August 28, 1987, data is partly due to different atmospheric conditions and partly to other measurement bias errors.

IV. Conclusion

A technique for the determination and removal of system non-linearity has been analyzed and demonstrated. This includes the concept, equations, and results applicable for DSN microwave noise temperature measurements. Comparison corrections for separate radiometer modes of operation resulted in very nearly equal system temperature after correction for the system non-linearity. The primary source of radiometer system non-linearity was shown to be in the square law detector. The best remedy for system non-linearity is proper equipment design and system power level settings. Unfortunately, this is not always achieved. With a given implemented system and operating levels, the above techniques provide a useful tool for estimating the system instrumentation non-linearity and correcting for the non-linearity if necessary. A correction may not be advisable if it is of the same magnitude as the correction measurement statistical error, or if it is small when compared to other error sources.

Acknowledgment

R. Riggs provided stimulating discussions and unpublished computer simulations for the receiver and square law detector model. Many data sets were taken by DSS 13 personnel (J. Garnica, *et al.*) as well as by some DSS 15 personnel (R. Caswell, *et al.*). L. Skjerve provided the IBM compatible PC NAR configuration used for the DSS 13 data.

References

[1] P. D. Batelaan, R. M. Goldstein, and C. T. Stelzried, "A Noise Adding Radiometer for Use in the DSN," *JPL Space Programs Summary 37-65*, vol. 2, Jet Propulsion Laboratory, Pasadena, California, pp. 66–69, September 30, 1970.

[2] C. T. Stelzried, *The Deep Space Network–Noise Temperature Concepts, Measurements, and Performance*, JPL Publication 82-33, Jet Propulsion Laboratory, Pasadena, California, September 15, 1982.

[3] A. J. Freiley, J. E. Ohlson, and B. L. Seidel, "Absolute Flux Density Calibrations: Receiver Saturation Effects," *DSN Progress Report 42-46*, vol. May–June 1978, Jet Propulsion Laboratory, Pasadena, California, pp. 123–129, August 15, 1978.

[4] C. T. Stelzried, "Noise Adding Radiometer Performance Analysis," *TDA Progress Report 42-59*, vol. July–August 1980, Jet Propulsion Laboratory, Pasadena, California, pp. 98–106, October 15, 1980.

[5] "IRE Standards on Electron Tubes: Definitions of Terms, 1962 (62 IRE 7.S2)," *Proc. IEEE*, pp. 434–442, March 1963.

Table 1. DSS 13, July 2, 1987, S-band, zenith, noise temperature measurement analysis results (1-sigma accuracy shown in parentheses)

Radiometer case	Parameter	Analysis method	
		Linear, Eq. (1)	Non-linear, Eq. (12)
NAR (data set 101)	A	0	0
	B	1.0034 (0.0004)	1.0413 (0.0010)
	C	-	-0.00012 (0.000002)
	Noise diode, T_n, K (on antenna)	56.4 (0.02)	57.7 (0.04)
	Noise diode, T_n, K (on ambient load)	60.4 (0.09)	57.7 (0.04)
	System noise temperature on antenna, T_2, K	31.3 (0.02)	32.3 (0.03)
	Linearity factor on antenna, LF (ratio)		1.034 (0.001)
Total power with digital voltmeter (data set 102)	A	0	0
	B	281.23 (0.54)	268.36 (0.59)
	C	-	11.82 (0.20)
	Noise diode, T_n, K (on antenna)	59.3 (0.07)	57.7 (0.05)
	Noise diode, T_n, K (on ambient load)	54.8 (0.04)	57.7 (0.05)
	System noise temperature on antenna, T_2, K	33.7 (0.04)	32.4 (0.05)
	Linearity factor on antenna, LF (ratio)		0.959 (0.001)

Table 2. DSS 13, August 8, 1987, S-band, zenith, noise temperature measurement analysis results (1-sigma accuracy shown in parentheses)

Radiometer case	Parameter	Analysis method	
		Linear, Eq. (1)	Non-linear, Eq. (12)
NAR (data set 101)	System noise temperature on antenna, T_2, K	30.8 (0.02)	31.9 (0.10)
	Linearity factor on antenna, LF (ratio)		1.034 (0.003)
Total power with digital voltmeter (data set 102)	System noise temperature on antenna, T_2, K	32.9 (0.13)	31.9 (0.19)
	Linearity factor on antenna, LF (ratio)		0.969 (0.003)
Total power with power meter	System noise temperature on antenna, T_2, K	31.9 (0.04)	31.8 (0.10)
	Linearity factor on antenna, LF (ratio)		0.997 (0.002)

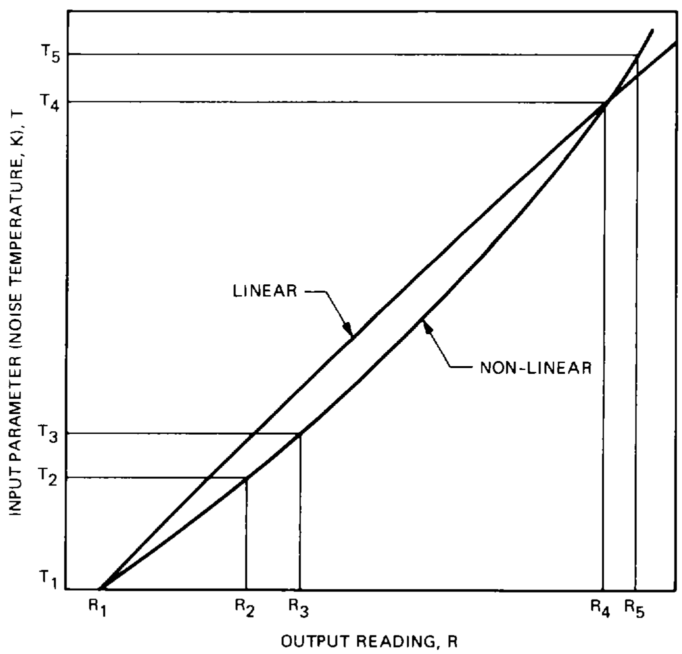

Fig. 1. Representative diagram of non-linear system input parameter vs. output reading, assuming that $T_n = (T_5 - T_4) = (T_3 - T_2)$

LINEARITY CALIBRATION DATA SHEET (CTS LIN2, 4-30-87)

OPERATOR C. GOODSON — FREQUENCY, MHz 2295 — DATE 7/2/87

CONFIGURATION 1.8 V — DETECTOR: SN 536

T_f, K 0.471 — T_m (MASER OR LNA), K 10 — INTEGRATION TIME, SEC 40

T_N (IF USING NAR), K 76.6 — WEATHER CLEAR

1.	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00
R_1 (DETECTOR OR POWER METER IN Z_0)						
2.	0.121	0.120	0.120	0.120	0.120	0.119 → TOTAL POWER CASE, DATA 102 (TYP)
	31.136	31.108	31.119	31.173	31.193	31.281 → IBM NAR CASE, DATA 101 (TYP)
R_2 (ANTENNA)						
3.	0.333	0.332	0.332	0.330	0.330	0.329
	87.245	87.295	87.275	87.353	87.391	87.478
R_3 (ANTENNA AND NOISE DIODE)						
4.	1.095	1.093	1.089	1.089	1.084	1.083
	304.887	304.858	304.576	305.718	305.325	305.551
R_4 (AMBIENT TERMINATION)						
5.	1.291	1.288	1.284	1.284	1.278	1.277
	365.041	365.110	364.961	365.632	365.374	365.916
R_5 (AMBIENT TERMINATION AND NOISE DIODE)						
6. R_6 (T_p, C)	22.23	22.40	22.50	22.64	22.77	22.88
7. (TIME, UT)	1734	1740	1745	1753	1759	1806

Fig. 2. Typical radiometer noise temperature data set (time meshed NAR and total power cases) taken July 2, 1987, at DSS 13, with 2.3-GHz HEMT low noise amplifier

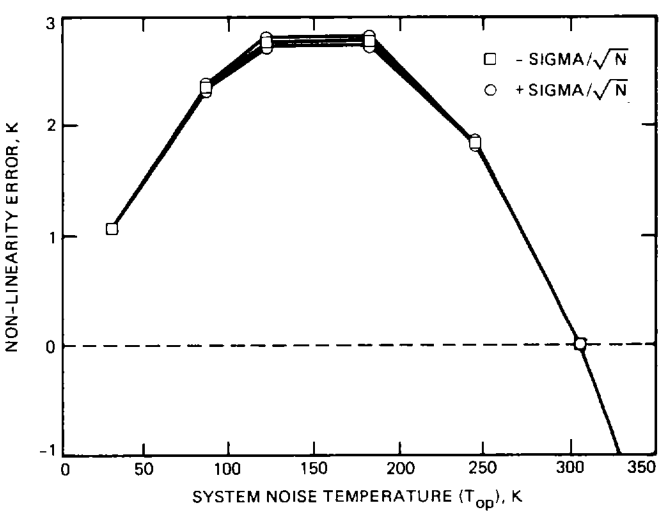

Fig. 3. Plot of the difference noise temperature for the NAR case, data set 101, between the non-linear and linear analyses as a function of system noise temperature

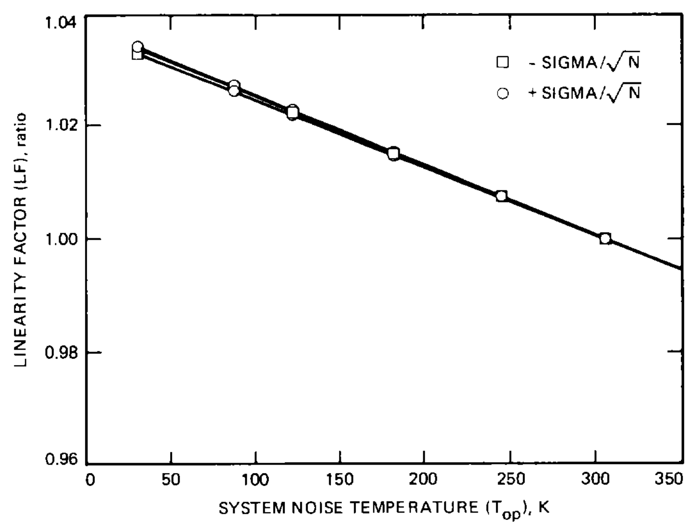

Fig. 4. Plot of the system linearity factor, LF (Eq. [23]), for the NAR case, data set 101, as a function of system noise temperature

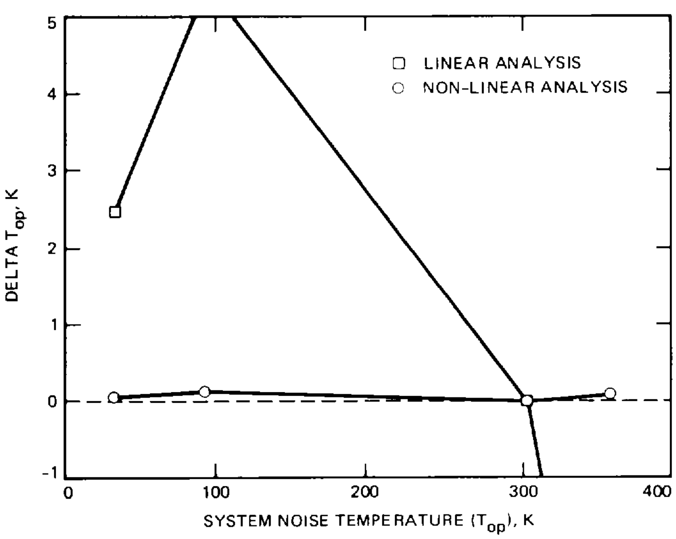

Fig. 5. Plot of the differences, ΔT_{op}, between the NAR and total power cases for both linear and non-linear analyses as a function of system noise temperature (see Table 1)

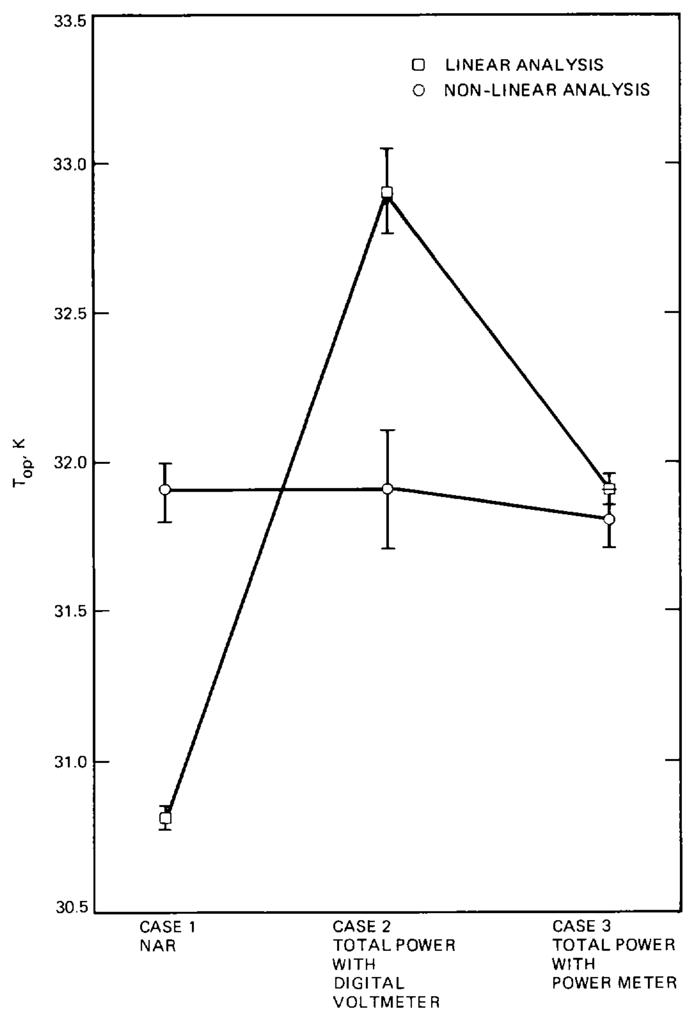

Fig. 6. Plot of the August 28, 1987, zenith, 2.3-GHz noise temperature measurements using the NAR and total power cases showing linear and non-linear analysis results (see Table 2)