A Near-Earth Optical Communications Terminal With a Corevolving Planetary Sun Shield

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The umbra of a planet may serve as a sun shield for a space-based optical communications terminal or for a space-based astronomical observatory. An orbit that keeps the terminal or observatory within the umbra is desirable.

There is a corevolution point behind every planet. A small body stabilized at the planet corevolution point will revolve about the Sun at the same angular velocity as the planet, always keeping the planet between itself and the Sun. This corevolution point is within the umbra of Mars but beyond the end of the umbra for Mercury, Venus, and Earth.

The Mars corevolution point is an ideal location for an astronomical observatory. There Mars obstructs less than 0.00024 percent of the sky at any time, and it shades the observatory completely from the Sun. At the Earth corevolution point, between 51 percent and 84 percent of the solar disk area is blocked, as is up to 92 percent of the sunlight. This provides a reduction from 3 dB to 11 dB in sunlight that could interfere with optical communications if scattered directly into the detectors. The variation is caused by revolution of the Earth about the Earth–Moon barycenter.

Corevolution is also possible closer to the planet than the corevolution point, but only if continuous outward thrust is provided or if a tether is attached to another body located beyond the corevolution point. Corevolution close enough to the Earth to make the Earth disk appear 75 percent larger in diameter than the solar disk (in order to block sunlight at any position of the Earth and Moon) is practically unobtainable, since it would require a tether 900,000 km long, or 5.6 newtons (1.3 pounds) of continuous outward thrust for a 10,000-kg terminal or observatory.

I. Introduction: Natural Sun Shields

Planetary bodies are usually used as sun shields for astronomical observations. On Earth we look at the stars at night. There are four inherent difficulties: the atmosphere of the Earth interferes with visibility; the gravity of the Earth distorts the optics; the bulk of the Earth and the thickness of the atmosphere cut the view to about three-eighths of the heavens at any one time at night; and scattered sunlight during the day obscures the stars. The great advantage of Earth basing is ease of access to the telescope.

An Earth-based optical communications terminal for deep-space data links will operate by day as well as by night. It will filter out most sunlight interference with a narrowband filter. An additional reduction of sunlight interference can be achieved by tuning the communications lasers to a solar Fraunhofer line [1]. Long sunshades, some with internal, venetian-blind slats across the aperture, will permit viewing within a small angle θ of the center of the Sun. The Sun will therefore obstruct an additional $(1 - \cos \theta)/2$ of the sky. For $\theta = 5$ degrees, the solar obstruction is 0.19 percent.

Inaccessibility of five-eighths of the sky at any given time would require placement of a network of three terminals around the Earth for continuous coverage. More terminals would be required in order to provide reasonable assurance of visibility at any given time. Otherwise, the telescope in the terminal suffers the same difficulties as an Earth-based astronomical observatory telescope.

A lunar-based observatory or optical communications terminal would not have to contend with the atmosphere. Observing and optical communication would be possible by day as well as by night, since there would be no sky scattering. Gravitational distortion would be much less severe. Half the heavens would still be obstructed at any one time by the bulk of the Moon. The disk of the Sun has a half-angle of one-quarter degree. The solar obstruction of the sky would be only a little more than 0.0005 percent. However, sunlight scattering within the telescope and heating of the telescope would present more severe difficulties for daytime observing than they would for an Earth-based telescope. Thermal cycling would be gradual, since both the lunar day and the lunar night are over two weeks long. Continuous sky coverage by a lunar optical communications network could be achieved by two terminals on opposite sides of the Moon.

A space-based telescope is troubled neither by atmospherics nor by gravity. In low planetary orbit, somewhat less than half the sky is obscured at any one time by the bulk of the host planet. The obscured portion changes rapidly as the telescope revolves around the planet. Thermal cycling can be as rapid as 45 minutes each of heating and cooling. Higher orbits reduce the amount and rate of movement of the sky obstruction caused by the planet. They lengthen the thermal cycle and increase the ratio of heating to cooling in each cycle. The solar sky obstruction and internal light scattering remain difficulties.

There is an ideal location in space, at the corevolution point within the umbra of a planet, where most of these difficulties are removed. The sky obstruction of the host planet and the Sun may be consolidated into one small obstruction which is the larger of the two. This location will now be defined, described, and analyzed.

II. Definitions and Descriptions

A. Planetary Umbral Points

As a planet p revolves around the Sun, its umbra is a cone-shaped region coming to a point a fixed distance directly behind the planet from the Sun, as shown in Fig. 1. At this point, called the umbral point, the angular subtense of the Sun and that of the planet are exactly equal. The distance U_p from the planet center to the umbral point is set by the radii of the planet r_p and of the Sun $r_\odot$, and by the distance R of the planet from the Sun:

$$U_p = R \frac{r_p}{r_\odot - r_p}$$

At the umbral point, the Sun and planet disks are the same size. Exactly 100 percent of the area of the solar disk is blocked by the planet, and no more of the sky is obstructed by the planet than the amount that would be obstructed by the Sun. At any other distance D from the planet center along the line pointing directly away from the Sun, the percentage of the solar disk area obscured by the planet is

$$\left[\left(\frac{r_p}{r_\odot} \right) \left(\frac{D + R}{D} \right) \right]^2 \times 100\%$$

B. The Corevolution Point

The amount of centripetal acceleration required to keep a body moving in a circular path around a central object is v^2/R, where v is the tangential velocity of the body. Solar gravitation, which supplies the available centripetal acceleration, becomes weaker inversely as the square of R, so the outer planets must move more slowly than the inner planets in order to stay in circular orbits.

On a line directly behind a planet from the Sun, the available centripetal acceleration is augmented by the gravitation

of the planet. There is a point on this line, called the corevolution point, at which the total gravitational pull is just enough to keep a small body moving around the Sun at the same angular velocity as the planet. Closer to the planet the pull is too strong, so the small body will accelerate inward; farther from the planet the pull is weaker, so the body will accelerate outward. In either case, the body no longer revolves about the Sun at the same angular rate as the planet, and the orbit is no longer circular.

If the small body stays at the corevolution radius from the Sun but gets ahead of or behind the planet, then the gravitational forces are no longer colinear, and the body will decelerate or accelerate appropriately. Likewise, if the body moves above or below the planetary orbital plane without changing its distance from the Sun, the noncolinear forces will bring it back. In short, the corevolution position is a saddle point of equilibrium for the body. It is unstable in the radial direction but stable in the two angular directions.

The distance C_p of the corevolution point from the planet center depends on the masses of the planet and of the Sun and on the distance from the planet to the Sun. The distance is calculated in the theory section of this article. The stability is analyzed in [2].

C. Relationship Between the Umbral and Corevolution Points

Both the umbral and corevolution points are located on the line directly behind the planet from the Sun, but their relative distances depend on the radius and mass of the planet, respectively. A very dense planet will have its corevolution point beyond the umbra, while a light, fluffy planet will have an umbra reaching beyond the corevolution point. This is illustrated in Fig. 1 for Earth, with a density of 5588 kg/m^3, and for Mars, with a density of 3968 kg/m^3. As a consequence, the Sun as seen from the Earth corevolution point is a thin ring (sometimes complete and sometimes broken as a result of the revolution of the Earth and the Moon about their barycenter), while Mars completely blocks the view of the Sun from its corevolution point.

III. Uses of Corevolution Points

A corevolution point is perpetually shaded by the host planet from most or all sunlight. Furthermore. the two sky obstructions caused by the planet and the Sun are consolidated into the larger of the two, with the rate of movement reduced to the planetary revolution rate. There are two important applications.

A. An Ideal Astronomical Observatory Location

An astronomical observatory would ideally be located at a corevolution point that happened to be just a little closer to the planet than the umbral point. Of the inner planets, only Mars provides an ideal location. The corevolution point is 1.08 million kilometers beyond the center of Mars. At that distance, the disk of Mars blocks out the Sun and only 5 percent more area of the sky. The excess diameter of Mars should be sufficient to compensate for any fluctuations due to the ellipticity of the Mars orbit and for refraction through the thin Martian atmosphere. The observatory would require a radio-isotope thermoelectric generator or another electrical power source, since no sunlight is available at the location for solar power.

B. A Good Location for a Near-Earth Optical Communications Terminal

The corevolution point for the Earth lies 1.5 million kilometers beyond the Earth. The umbral point is closer to the Earth, so the terrestrial disk area is only 84 percent of the solar disk area. This location is useful for optical communications. There is sufficient sunlight to provide electrical power.

The Earth is the only planet with a natural satellite of significant mass relative to its own mass. The lunar mass is 1.2 percent of the terrestrial mass. The Earth and the Moon revolve about their own barycenter, and thus the Earth, as seen from a point directly behind it from the Sun, oscillates in the plane of the lunar orbit with an amplitude of 4671 km. As seen from the corevolution point, the Earth blocks 51 percent of the solar disk at the extremes of its motion and 84 percent as it passes through the center. The variation in heating and cooling is not sinusoidal, so the average value differs somewhat from the median value of 67.5 percent reduction in heating with swings of ± 16.5 percent. The thermal period is half the lunar revolution period—a little over two weeks.

Solar interference with communications is also reduced geometrically by 3 dB to 8.2 dB when light is scattered directly into the telescope detectors from the sun shades or optics. A further reduction is provided by the limb-darkening effect, in which the apparent surface of the Sun appears to have reduced brightness at the edges relative to the center. The effect is more pronounced as the wavelength becomes smaller. At a wavelength of 534 nm, the solar intensity drops to 79 percent at 0.75 of the solar radius from the center and to 55 percent at 0.95 of the solar radius from the center [3]. The terrestrial disk reaches out to 0.92 of the solar radius from the center (when aligned with the Sun). If the integrated intensity reduction over the visible solar annulus is taken to be 50 percent, limb darkening provides an additional 3-dB reduction of

solar interference whenever the Earth disk is centered directly over the Sun.

The position of the communications terminal will be slightly perturbed by the orbiting of the Earth and the Moon about their barycenter. The amount of thrust and the quantity of energy required for station keeping have yet to be analyzed in detail, but they are expected to be small. The mass of the Moon is only 1.2 percent of the Earth's mass, and the radius of the Moon's orbit is only one-quarter of the corevolution distance. These circumstances reduce the Moon's influence considerably.

A deep-space probe will be able to locate the near-Earth optical communications terminal more easily if the terminal is at the corevolution point rather than in an Earth-centered orbit. Once the probe locates the Sun and the Earth, it has two points that establish a line. The corevolution point is always the same distance along the line from Earth away from the Sun. The probe must compute only the foreshortening of that distance according to the epoch and its own position relative to the ecliptic. In contrast, if the terminal is in an Earth-centered orbit, the probe still needs accurate information on six more orbital elements to locate the terminal after it has located the Earth and the Sun.

IV. Corevolution Theory

A small body placed behind a planet on the line from the Sun through the planet would experience centripetal acceleration from both the planet and the Sun. Close behind the planet, the centripetal acceleration is much greater than the acceleration of the planet toward the Sun. As the distance from the planet increases, centripetal acceleration decreases. At a certain point on the line, herein called the corevolution point, the centripetal acceleration is sufficient to keep the body moving in a circular path around the Sun at exactly the same angular rate as the planet. (Technically speaking, there are two other corevolution points. One lies between the planet and the Sun, while the other lies on the opposite side of the Sun from the planet. Neither point has any relevance to the present discussion.)

A. The Restricted Three-Body Problem

Consider three masses placed on a straight line and revolving steadily about their common center of mass. Let the bodies be arranged in order of decreasing mass, with the mass of the third body negligible compared with the masses of the other two. Let R be the distance between the first two, and let C be the distance between the last two. The center of mass will lie between the first two bodies at a distance $X = Rm_2/$ $(m_1 + m_2)$. The square of the angular revolution rate is $G(m_1 + m_2)/R^3$, where G is the universal gravitational constant.

The amount of centripetal acceleration required to keep the third mass moving in a circular path is the square of the angular revolution rate times the distance to the center of mass, $R + C - X$. This centripetal acceleration is supplied by gravitation from the first two masses and may be reduced by an outward thrust or tether tension T directed away from the two massive bodies. Dividing Newton's law of forces and accelerations for the third body by G and m_3 yields

$$\frac{m_1}{(R+C)^2} + \frac{m_2}{C^2} - \frac{T}{Gm_3} = \frac{m_1 + m_2}{R^3}(R + C - X)$$

Let $x = C/R$ and let $M = m_1/m_2$. If T is set to zero, one may solve the following quintic equation for x:

$$f(x) = 1 + 2x + x^2$$

$$- [(1 + 3M)x^3 + (2 + 3M)x^4 + (1 + M)x^5] = 0$$

B. Calculations

The quintic equation is easily solved using Newton's method for the approximation of roots. The value of M is large for any planet and $x \ll 1$, so an initial approximation for x is $(m_2/3m_1)^{1/3}$. Only a few refinements are required to find x when the planet has the combined masses of the Earth and Moon and the Earth's distance from the Sun. Sample iterations of the solution are shown in Table 1 using Sun, Earth, and Moon data found in Table 2. The distance C is 1.5073×10^6 km. This point is in the penumbra. Geometrically, 84.20 percent of the area of the Sun's disk is blocked by the Earth. These calculations may be performed with a SuperCalc program called COREV.CAL, written by the author.

C. Stability

Steady solutions have been sought for the motion of three bodies placed colinearly and revolving about the common center of mass. This configuration is stable in the angular directions but not in the radial direction. A corevolution point is therefore a saddle point, and we may expect to find no natural body at the corevolution point of any planet. However, an active system could stabilize itself at a corevolution point, expending energy and fuel only for station-keeping purposes to compensate for small perturbations from natural satellites or distant planets. The thrusts required are expected to be small.

D. Tabulation for All of the Planets

The corevolution distance C from planet center to satellite is given in Table 2 for each planet on the basis of planet mass and planet mean radius R from the Sun. The mass of a planet is usually much larger than the total mass of its natural satellites, and the natural satellite masses may be neglected. Earth is the only exception recognized here. The relative distance x is computed first from the quintic equation using Newton's method.

The percentage block is the area ratio of the planet to solar disk as seen from the corevolution point. This area ratio (if less than 100 percent) may be used to give a geometric reduction factor in the solar flux on the satellite. The flux will actually be reduced further by solar limb darkening. This further reduction is due to absorption in the solar atmosphere on slant paths near the limbs. It depends on the wavelength and is difficult to calculate exactly. Block percentages greater than 100 percent represent the amount of sky obscured by the planet relative to the solar obscuration at the satellite location.

E. Corevolution at a Distance Closer than the Corevolution Point

If one desires to place a satellite closer to the Earth in order to be completely within the umbra, one may choose the value of x and solve for the additional constant thrust or outward tension required to make the satellite corevolve. Suppose, for example, x is chosen to be 0.0112597 so that the apparent diameter of the Earth will be 75 percent greater than the apparent diameter of the solar disk. The distance xR now corresponds to 786,556 km. The thrust or tension required for a 10,000-kg satellite is 5.60 newtons (1.26 pounds). This amount seems small, but there is no practical way to provide it (see Appendix A).

V. Conclusion

Mars is the best sun shield for a space telescope. A telescope placed at the Mars corevolution point would experience no sunlight interference or thermal cycling and could view all but 0.00024 percent of the sky at any time.

The Earth corevolution point is a good location for the near-Earth end of an optical communications data link to deep-space probes. It is an easy point to locate. Thermal cycling of the telescope is reduced by 51 percent to 84 percent, and interference from scattered sunlight is reduced from 3 dB to 11 dB. All but from 0.00057 percent down to 0.00053 percent of the sky is visible there at any time.

References

[1] E. L. Kerr, "Fraunhofer Filters to Reduce Solar Background for Optical Communications," *TDA Progress Report 42-87*, vol. July–September 1986. Jet Propulsion Laboratory, Pasadena, California, pp. 48–55, November 15, 1986.

[2] K. R. Symon, *Mechanics* (3rd ed.), Reading, Massachusetts: Addison-Wesley, pp. 286–291 and 490–497, 1971.

[3] G. Abetti, *The Sun*, New York: Macmillan, pp. 276–283, 1957.

[4] W. A. Baracat and C. L. Butner, *Tethers in Space Handbook*, Washington, D. C.: National Aeronautics and Space Administration, August 1986.

Table 1. Sample iterations of the solution of the quintic equation when finding the Earth corevolution point

n	x_n	$f(x_n)$	$x_n - x_{n-1}$
0	0.0100442	0.0101104	
1	0.0100778	−0.000034	0.0000336
2	0.0100777	-3.9×10^{-10}	-1.14×10^{-7}
3	0.0100777	-2.2×10^{-16}	-1.3×10^{-12}
4	0.0100777	-2.2×10^{-16}	0

Table 2. Corevolution distance and solar blockage for all of the planets (solar and lunar data are also included)

| Planet	m, kg	r, km	R, km	x	C, km	Block, %
Mercury	3.181×10^{23}	2433	5.795×10^7	0.0037670	2.183×10^5	86.77
Venus	4.883×10^{24}	6053	1.081×10^8	0.0093795	1.014×10^6	87.61
Earth	6.053×10^{24}	6371	1.496×10^8	0.0100777	1.507×10^6	84.20
Mars	6.418×10^{23}	3380	2.278×10^8	0.0047616	1.085×10^6	105.02
Jupiter	1.901×10^{27}	69,758	7.781×10^8	0.0697847	5.430×10^7	236.10
Saturn	5.684×10^{26}	58,219	1.427×10^9	0.0463373	6.612×10^7	356.82
Uranus	8.682×10^{25}	23,470	2.870×10^9	0.0246016	7.061×10^7	197.27
Neptune	1.027×10^{26}	22,716	4.500×10^9	0.0260302	1.171×10^8	165.53
Pluto	1.256×10^{22}	1100	5.909×10^9	0.0012817	7.574×10^6	152.47
Sun	1.991×10^{30}	695,950				
Moon	7.354×10^{22}	1738	3.844×10^5	(from the Earth)		
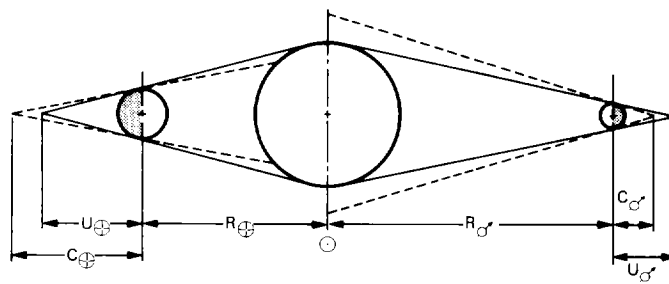
Fig. 1. Umbral and corevolution points of Earth (left) and Mars (right) in relation to the Sun (the relative diameters of Earth and Mars are drawn to scale, and their relative distances from the Sun are shown to a different scale; all other distances are exaggerated or reduced for clarity). Solid lines bound umbras; broken lines project planet limbs.

Appendix A

Corevolution Within the Earth Umbra

If it were desirable to locate an optical communications terminal entirely within the Earth umbra and still have it corevolve with the Earth, one might do so in two ways: by providing a constant thrust away from the Earth or by tethering the observatory to another body located beyond the corevolution point. The second body would be in the penumbra, where sufficient light would be available to generate power and send it to the optical communications terminal via a pair of wires that would serve as the tether.

The second option is not practical because of the length and mass of the tether. Suppose, for example, that the optical-communications-terminal mass is 10,000 kg and the solar-power generator is ten times more massive. (Additional mass at the generator would be required to balance the mass of the tether, as its center of mass does not coincide with the corevolution point.) The observatory may be located 790,000 km from the Earth's center, where the Earth subtense is 75 percent more than the solar subtense. The corevolution point is at 1.51 million kilometers, and the generator is located at 1.68 million kilometers. The tension in the pair of wires would be 5.60 newtons (1.26 pounds). A steady direct current in the wires would maintain their separation by the magnetic field, and the voltage would be limited only by the insulation and

vacuum gap at the connection points. Even so, the length of 898,000 km would make the mass of two aluminum wires 0.5 mm in diameter equal to 813,000 kg. The taper required for constant stress is very slight and therefore unnecessary. However, the resistance of the wires would be very large even in the cold of space unless a relatively high temperature extrudable superconducting material were found for them.

Alternatively, the power could be supplied by other means and the tether could be a plastic filament. If the filament diameter were 0.12 mm and the density were 1450 kg/m^2, the filament mass could be reduced to 14,000 kg, still very large.

A number of space applications for tethers are identified in [4]. The longest tether proposed there is 526,000 km long, so the present suggestion may be a record. The length of this tether by itself would require the listing of its "potential for technology demonstration" as "far-term."

The first option is also impractical. Supplying even a fraction of an ounce of thrust for one year would require a mass of fuel about equal to the mass of the optical communications terminal.